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ON THE NUCLEON-NUCLEON POTENTIAL

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A convenient method is proposed for setting up a potential in the form of the series $U(x) = \Sigma U^{(n)}(x)$, where $U^{(n)} \sim e^{-nx}$ for $x \to \infty$, starting from the relativistic meson-theoretical scattering amplitude expressed as an expansion in the number of exchange mesons (x is the distance in the units $1/\mu$, where μ is the meson mass). In the case of peripheral nucleon interaction this method yields a two-meson potential with a broad locality region $p^2/m^2 \ll 1$ (nonrelativistic region). The two-meson potential consists essentially of attractive tensor and central forces which depend weakly on the isotopic state.

In reference 1 a nucleon-nucleon potential was found for large distances $x > 1$ (in the units $1/\mu$, where μ is the mass of the π meson), which in first Born approximation is equivalent to the two-meson scattering amplitudes.[1-3] This potential (which we shall call the pseudopotential) is convenient for practical applications, since it permits us to make again use of the first Born approximation in various problems. A characteristic feature of the pseudopotential is its strong dependence on the energy in the region $E_{lab} \gtrsim 40$ Mev, $p^2/\mu^2 \gtrsim 1$ (p is the momentum in the system of the center of inertia). This "anomalous" energy dependence (nonlocality) is due to the infinite discontinuity in the absorptive part of the two-meson amplitude $\Delta M^{(2)}(q^2)$ at the point* $q^2 = -4\mu^2(1 + \mu^2/4p^2)$ (the so-called Karplus singularity) and also to the terms of the type $1/(p^2 + \mu^2)$. The anomalous nonlocality has a particularly marked effect in states with the isotopic spin $T = 0$; this must be taken into account in the analysis of the experimental data with the help of phenomenological potentials.†

The strong nonlocality of the pseudopotential, however, deprives it of its theoretical value, and it is still an open question whether it is possible to construct a "genuine" (local) potential with a broad region of locality $p^2 \ll m$ (nonrelativistic region), i.e., a potential which would, with the help of nonrelativistic techniques (Schrödinger equation), lead to the relativistic meson-theoretical scattering amplitude with an accuracy up to the "normal" corrections $\sim p^2/m^2$ (m is the nucleon mass).

*We use the notation of reference 1; in particular $q^2 = (p' - p)^2$ is the square of the momentum transfer.

†In the analysis of the experimental data with the help of phenomenological models one uses practically only the first Born approximation. Therefore, the majority of authors are dealing with pseudopotentials.

Charap and Fubini[4] showed on the example of scalar particles that an equivalent local potential for a given scattering amplitude can be constructed in the energy region $p^2 \ll m\mu + \mu^2/4$. In the construction of one- and two-meson potentials (i.e., potentials with the asymptotic behavior e^{-x} and e^{-2x}) this method is equivalent to the use of the Born approximation as in reference 1, if in addition one includes the second iteration of the one-meson potential.

In the present paper we show that a local potential in the form of the expansion

$$U(x) = \sum_{n=1}^{\infty} U^{(n)}(x), \qquad U^{(n)}(x) \sim e^{-nx} \quad \text{as } x \to \infty, \quad (1)$$

corresponding to an expansion of the amplitude in the number of exchange mesons,[5,2] can be constructed correctly from a given scattering amplitude, using the Born expansion. In this method we retain the possibility of making concrete estimates of the accuracy with which the potential reproduces the relativistic scattering matrix. Application of the method in the case of the two-meson interaction of the nucleons leads to a local potential with a wide locality region $p^2 \ll m^2$. This indicates that the anomalous nonlocality of the pseudopotential is equivalent to the description of the higher Born approximations by a "genuine" potential and has, therefore, no physical meaning.

1. GENERAL METHOD

In setting up the potential we start from the sequence of relativistic amplitudes* $M^{(n)}(q^2)$

*If spin and isospin variables are included, the amplitudes and the potential are to be replaced by the corresponding operators in spin and isospin space. We then investigate the analytic properties of the scalar functions which multiply certain spinor invariants (which, themselves, still depend on the direction of the vector **q**).

which are obtained from meson theory by an expansion in the number of exchange mesons, or, more precisely, by successive separation of the contributions with the nearest singularity in q^2 (with fixed energy) in the points $q^2 = -\mu^2, -4\mu^2, \ldots, -(n\mu)^2, \ldots$ We restrict the discussion to the interaction of identical particles (of mass m) due to the exchange of mesons with mass μ only. In this case only the exchange graphs appearing as a consequence of the symmetrization of the amplitude have singularities on the right semi-axis. Therefore, these can in general be left out of the discussion. Moreover, we assume that we do not encounter any anomalous graphs with nearest singularities $q^2 \neq (n\mu)^2$, as, for example, the "square" in Fig. 1 with the condition $m^2 > \mu_1^2 + \mu_2^2$ at the vertex.

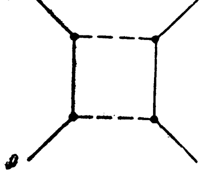

FIG. 1

To find the equations connecting the potential (1) with the sequence $M^{(n)}$ we consider the iteration solution of the Schrödinger equation with the potential (1). Denoting the nonrelativistic scattering amplitude in the j-th Born approximation by T_j, we find

$$T_1 = -\frac{m}{4\pi} U(\mathbf{q}), \qquad U(\mathbf{q}) = \int e^{-i\mathbf{qr}} U(\mathbf{r})\, d\mathbf{r},$$

$$T_j = -\frac{m}{(2\pi)^3} \int \frac{U(\mathbf{p}' - \mathbf{k})\, T_{j-1}(\mathbf{k} - \mathbf{p}, p^2)}{p^2 - k^2 + iO} d\mathbf{k} \text{ for } j \geqslant 2. \quad (2)$$

The Born expansion (2) can be used for separating from the nonrelativistic amplitude the contributions $T^{(n)}$ with the nearest singularity at $q^2 = -\mu^2$, $-4\mu^2$, etc. Considering that the nearest singularity of the Fourier component of the n-meson potential lies at $q^2 = -(n\mu)^2$, we obtain for the first iteration

$$T_1^{(n)} = -(m/4\pi)\, U^{(n)}(\mathbf{q}). \quad (3)$$

For the classification of the contributions from the following iterations we use the "displacement theorem" for the singularities: the nearest singularity of a function of the form

$$f(q^2, p^2) = \int \frac{f^{(n_1)}[(\mathbf{p}' - \mathbf{k})^2]\, f^{(n_2)}[(\mathbf{k} - \mathbf{p})^2]}{p^2 - k^2 + iO} d\mathbf{k}$$

is located at $q^2 = -(n_1 + n_2)^2\mu^2$, where $-(n_1\mu)^2$ and $-(n_2\mu)^2$ are the locations of the nearest singularities for $f^{(n_1)}(q^2)$ and $f^{(n_2)}(q^2)$. For the proof of this theorem it suffices to write the functions in the form of spectral integrals

$$f^{(n)}(q^2) = \int_{(n\mu)^2}^{\infty} \frac{\rho(\varkappa^2)\, d\varkappa^2}{q^2 + \varkappa^2}$$

and, after changing the order of integration, to find the nearest singularity of the function

$$\int [(\mathbf{p}' - \mathbf{k})^2 + \varkappa_1^2]^{-1} [(\mathbf{p} - \mathbf{k})^2 + \varkappa_2^2]^{-1} [p^2 - k^2 + iO]^{-1} d\mathbf{k}.$$

With the help of the methods proposed by Landau for relativistically invariant integrals[6] we easily find the value

$$q^2 = -(\varkappa_1 + \varkappa_2)^2 \mu^2 \leqslant -(n_1 + n_2)^2 \mu^2.$$

In particular, it follows from this theorem that the amplitude T_j has the nearest singularity* at $q^2 = -(j\mu)^2$, i.e., the contributions $T_j^{(n)}$ are different from zero only for $j \leq n$.

We now require that the non-relativistic techniques reproduce the sequence of meson-theoretical amplitudes $M^{(n)}$ with an accuracy up to relativistic corrections. Comparing consecutively the contributions $T^{(n)}$ and $M^{(n)}$ for $n = 1, 2, 3, \ldots$, we obtain a system of recurrence relations for the stepwise construction of the sequence $U^{(n)}$:

$$U^{(1)}(\mathbf{q}) = -4\pi m^{-1} M^{(1)}(\mathbf{q}, p^2),$$
$$U^{(2)}(\mathbf{q}) = -4\pi m^{-1} [M^{(2)}(\mathbf{q}, p^2) - T_2^{(2)}(\mathbf{q}, p^2)],$$
$$U^{(3)}(\mathbf{q}) = -4\pi m^{-1} [M^{(3)}(\mathbf{q}, p^2) - T_2^{(3)}(\mathbf{q}, p^2) - T_3^{3}(\mathbf{q}, p^2)], \quad (4)$$

The left-hand sides of Eqs. (4) are independent of p^2 by definition, so that one can set $p^2 = 0$ in the right-hand sides, i.e., retain only the basic terms of the expansions in p^2. If, on the other hand, we retain the corrections $\sim p^2$ on the right-hand sides, we also obtain nonlocal corrections to the potential and can in this way estimate the accuracy of the local potential.

Going over to the coordinate representation, we can write $U^{(n)}$ as the residue of the pole $q^2 = -\mu^2$ (for $n = 1$) or as an integral of the discontinuity $\Delta M^{(n)}(q, 0) - \Delta T^{(n)}(q, 0)$ across the cut $q^2 \leq -(n\mu)^2$ for $n \geq 2$ (reference 1); for the construction of the potential it therefore suffices practically to know only the absorptive parts of the amplitudes in q^2. This leads us to certain conditions which have to be fulfilled in order that the potential can be expanded into the series (1). Indeed, the two-meson potential is expressed in the form of an integral over $q^2 \leq -4\mu^2$, but the region of integration $q^2 \leq -9\mu^2$ gives a contribution of the same

*This result was obtained earlier by Bowcock and Martin[7] using a Yukawa potential.

type as the three-meson potential $U^{(3)}$, etc. On the other hand, it is obvious that this method enables us to set up a unique potential without expanding it into a series at all. For this purpose we must consider successively the regions of integration

$$-4\mu^2 \geqslant q^2 > -9\mu^2, \qquad -9\mu^2 \geqslant q^2 > -16\mu^2, \ldots,$$

collecting all non-vanishing contributions of the absorptive part of $\Delta M - \Delta T$. This shows that the system of equations (4) is equivalent to the Charap-Fubini method, slightly modified and generalized to the case of particles with spin.

Our method is formally analogous to that used earlier (see, for example, the paper of Klein and McCormick[8]) in the construction of an adiabatic potential in the form of an expansion in the number of exchange mesons, but differs by the parallel use of the nonrelativistic formalism and the relativistic scattering amplitude (the importance of this difference was noted in reference 4). We emphasize that in our method the Born approximation is only used as a means of classification of the nonrelativistic amplitudes according to the extent of the region of analyticity, and corresponds to the expansion in the number of exchange mesons in meson theory. In other words, the series (1) represents an expansion in the "degree of peripheralness" and not in the coupling constant, which can have an arbitrary value.

2. PERIPHERAL NUCLEON-NUCLEON POTENTIAL

Substituting the one-meson nucleon-nucleon amplitude (see, for example, reference 9) in (4) and going to the x representation, we obtain the well-known static one-meson potential

$$U^{(1)}(x) = \tfrac{1}{4}\mu g^2 \varepsilon^2 (\boldsymbol{\tau}^{(1)}\boldsymbol{\tau}^{(2)}) (\boldsymbol{\sigma}^{(1)}\nabla)(\boldsymbol{\sigma}^{(2)}\nabla) e^{-x}/x,$$

$$\varepsilon^2 = \mu^2/m^2. \qquad (5)$$

The inclusion of the nonlocal corrections to the one-meson potential introduces the factor $(1 + p^2/m^2)^{-1/2}$; the static potential (5) can therefore be used in the energy region $p^2/m^2 \ll 1$.

To find the two-meson potential we must calculate the second iteration

$$T_2^{(2)}(\mathbf{q}, p^2) = \frac{m^2}{2(2\pi)^4} \int \frac{U^{(1)}(\mathbf{p}' - \mathbf{k})\, U^{(1)}(\mathbf{k} - \mathbf{p})}{p^2 - k^2 + i0}\, d\mathbf{k}, \qquad (6)$$

or more precisely, the discontinuity $\Delta T_2^{(2)}$ on the cut $q^2 \leq -4\mu^2$. Substituting (5) in (6) and integrating over dk, we obtain for $q^2 \leq -4\mu^2$

$$\Delta T_2^{(2)} = i\pi g^4 \frac{(3 - 2\lambda_\tau)\varepsilon}{32 m \sqrt{1 - s^2}} \Big\{ \frac{(1 - 2s^2)^2}{v}$$

$$+ i\mathbf{S}\mathbf{n} p^2 \sin\theta \frac{1 - 2s^2}{p^2 + \mu^2(1 - s^2)} \Big(\frac{1 - 2s^2}{v} - 1 \Big)$$

$$+ [(\boldsymbol{\sigma}^{(1)}\mathbf{q})(\boldsymbol{\sigma}^{(2)}\mathbf{q}) - q^2 \boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}] \frac{1 - 2s^2 - v}{4(p^2 + \mu^2(1 - s^2))}$$

$$+ (\boldsymbol{\sigma}^{(1)}\mathbf{n})(\boldsymbol{\sigma}^{(2)}\mathbf{n}) \frac{\varepsilon^2 (1 - s^2) m^2}{p^2 + \mu^2(1 - s^2)}$$

$$\times \Big[v + \frac{(1 - 2s^2)^2}{v} - 2(1 - 2s^2) \Big] \Big\};$$

$$s^2 = 1 + q^2/4\mu^2, \qquad v = \sqrt{4s^2p^2 + \mu^2}/\mu,$$

$$\mathbf{S} = {}^1/_2 (\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)}), \qquad \mathbf{n} = [\mathbf{p}' \times \mathbf{p}] / |[\mathbf{p}' \times \mathbf{p}]|.$$

$\lambda_T = 1$ or -3 for the isotopic spin $T = 1$ or 0, respectively. The further calculations consist in the integration of the absorptive part of the potential

$$\Delta M^{(2)}(\mathbf{q}, p^2) - \Delta T_2^{(2)}(\mathbf{q}, p^2) \qquad (7)$$

along the cut $q^2 \leq -4\mu^2$ for $p^2 = 0$, which presents no essential difficulties.

Let us now examine to which extent the adiabatic absorptive part ($p^2 = 0$) is equivalent to the exact meson-theoretical amplitudes in the non-relativistic approximation $p^2 \ll m^2$, i.e., which corrections to $\Delta M^{(2)}(\mathbf{q}, p^2)$ must be discarded in order that the absorptive part (7) cease to depend on the energy. Starting from the expansion of the meson-nucleon amplitude in powers of the invariant ν for a given q^2 (see reference 2), one can show that, at least for $-4\mu^2 \geq q^2 > -9\mu^2$, only the fourth-order contribution of perturbation theory, $\Delta M_4^{(2)}$, corresponding to Fig. 1, has a significant energy dependence.

Neglecting corrections $\sim p^2/m^2$ to $\Delta M_4^{(2)}$, we can convince ourselves that the subtraction of $\Delta T_2^{(2)}$ leads to the cancellation of all terms having an appreciable energy dependence in the region $\mu^2 \lesssim p^2 \ll m^2$, and, in particular, to the disappearance of the discontinuity in the Karplus point $q^2 = -4\mu^2 \times (1 + \mu^2/4p^2)$. If we assume that the two-meson potential is determined only by the region $-4\mu^2 \geq q^2 > -9\mu^2$, this means that the potential $U^{(1)}(x) + U^{(2)}(x)$ is equivalent to the one- and two-meson amplitudes with an accuracy up to terms of order $\sim p^2/m^2$.

In the foregoing discussion we have also assumed that besides the Karplus singularity from the graph of Fig. 1, there are no other singularities in the region $-4\mu^2 \geq q^2 > -9\mu^2$. The nearest omitted singularity is given by the graph shown in Fig. 2, which has a singularity at infinity for p^2

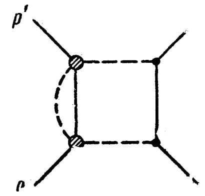

FIG. 2

$= m\mu + \mu^2/4$ (threshold for π meson creation) which quickly approaches $-4\mu^2$ as the energy increases. As long as this singularity lies beyond $-9\mu^2$ ($p^2/m^2 < 0.5$), it gives rise to the same type of effects as the three-meson amplitude. For higher energies the effects from this singularity must be included in the two-meson approximation; they can only be described with the help of a non-local two-meson potential. The condition $p^2/m^2 < 0.5$ ($E_{lab} < 900$ Mev) is, however, comprised in the non-relativistic approximation and does not lead to additional restrictions.

For the explicit calculation of the two-meson potential we use the approximation used in the determination of the pseudopotential[1] (expansion in terms of $1/x$, ϵ^2, and $\epsilon\sqrt{x}/2$). We find

$$U^{(2)}(x) = U_S^{(2)}(x) + \mathbf{LS}U_{LS}^{(2)}(x) + [(\boldsymbol{\sigma}^{(1)}\mathbf{n}_x)(\boldsymbol{\sigma}^{(2)}\mathbf{n}_x) - \boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}]\,U_T^{(2)}(x), \tag{8}$$

where

$$U_S^{(2)}(x) = -\frac{3g^4\varepsilon^2\mu}{4\sqrt{\pi}}\frac{e^{-2x}}{x^{5/2}}\left\{(\alpha-1)^2 + \varepsilon\sqrt{\pi x}\,(\alpha-1) + \varepsilon^2 x\left(1-\alpha+\frac{\lambda_\tau}{3}\right)\right\};$$

$$U_{LS}^{(2)}(x) = -\frac{3g^4\varepsilon^4\mu}{4\sqrt{\pi}}\frac{e^{-2x}}{x^{7/2}}\left\{(\alpha-1)^2 - \frac{4\lambda_\tau}{3}\left(\beta_2-\frac{\beta}{2}\right) + \varepsilon\sqrt{\pi x}\,\alpha - \frac{4\lambda_\tau}{3}\frac{\beta\sqrt{\pi}}{\varepsilon\sqrt{x}}\right\};$$

$$U_T^{(2)}(x) = -\frac{3g^4\varepsilon^4\mu}{4\sqrt{\pi}}\frac{e^{-2x}}{x^{5/2}}\left\{1 + \frac{2\lambda_\tau}{3}\beta_1\left(\frac{\sqrt{\pi}}{\varepsilon\sqrt{x}}-1\right) - \varepsilon\sqrt{\pi x}\,\frac{9+2\lambda_\tau}{24}\right\},$$

$$\alpha = 1.2\,(\Delta|\alpha| \leqslant 10\%), \qquad \beta_1 = 0.025, \qquad \beta_2 = -0.029,$$
$$(\Delta|\beta_1|, \Delta|\beta_2| \leqslant 20\%), \qquad \beta = -\beta_1 - \beta_2.$$

Expression (8) is the principal term of the asymtotic expansion in $1/x$ and gives the two-meson potential in the peripheral region $x > 1$ with an accuracy $\sim 1/x$. For relatively low energies, $E_{lab} \lesssim 100$ Mev, the region $x \gtrsim 1$ gives the main contribution for all orbital angular momenta $l \geq 1$. The potential (8) enables us therefore, at least qualitatively, to study the most important properties of the two-meson interaction. The expansion in $\epsilon^2 x/4$ practically leads to no further restrictions, since the region $\epsilon^2 x/4 \gtrsim 1$ ($x \gtrsim 180$) is of no practical interest. However, the presence of the parameter $\epsilon^2 x/4$ has theoretical significance, since it seems to indicate that the static approximation in meson theory is not correct (see also reference 4).

The interaction obtained above takes account of virtual scattering processes (rescattering corrections) with the help of the parameters α, β_1, and β_2, which have a very definite theoretical meaning,[2] and is thus a rigorous consequence of the pseudoscalar, symmetric meson theory. However, in obtaining the numerical values of these parameters we have used the experimental data on meson-nucleon scattering, so that the potential (8), with the indicated values of the parameters, is, in a certain sense, semi-phenomenological.

Let us consider briefly the properties of the two-meson interaction. The characteristic feature of the potential (8) which distinguishes it from the pseudopotential[1] is its weak dependence on the isotopic state. The scalar and spin-orbit forces are very different from those calculated by perturbation theory on account of the combination $\alpha - 1 \ll \alpha, 1$ which enters in the basic terms. The tensor forces and the forces of the type $\boldsymbol{\sigma}^{(1)}\boldsymbol{\sigma}^{(2)}$ have the additional smallness ϵ^2 in comparison with $U_S^{(2)}$, but their relative contribution is strongly increased on account of the additional compensation in $U_S^{(2)}$. In the complete expression for the potential these forces, however, play a minor role owing to the presence of large contributions of the same type from the one-meson potential. The forces of the type $(\boldsymbol{\sigma}^{(1)}\mathbf{L})(\boldsymbol{\sigma}^{(2)}\mathbf{L})$ have the additional smallness $\epsilon^4 x/4$ and have been omitted completely in the computations.

Of particular interest are the spin-orbit forces, for which several different expressions have been found,[10-13] and whose role in the interpretation of the experimental data is also not sufficiently clear (see references 14 to 16). The two-meson interaction $U_{LS}^{(2)}$ is considerably weaker than the strong phenomenological spin-orbit forces of Gammel and Thaler[14] and Signell and Marshak,[15] and can be neglected in the region $x \gtrsim 1$.* Calculations on the basis of the non-relativistic meson theory[10,11] lead to the same result, although the two-meson potential (8) differs quantitatively from the potentials obtained earlier (more precisely, their peripheral parts).

*In the paper of Grashin and Kobzarev presented at the Ninth Conference on High Energy Physics at Kiev, 1959, a spin-orbit interaction was given which was too strong by an order of magnitude, due to an error in the calculations. This error has been corrected in the published article.[1]

We note that for comparatively low energies the semi-phenomenological potentials without spin-orbit forces have been successfully employed by a number of authors for the interpretation of the experimental scattering data[16] and of certain properties of the nuclear interactions.[17] We believe, however, that it is difficult to explain the experimental data for energies of several hundred Mev without introducing strong spin-orbit forces; apparently, these must therefore be connected with potentials of shorter range corresponding to effective masses $m_{eff} = |q|_{eff} \geq 3\mu$.

Our calculations show that the only correction to the one-meson potential in the region $x \gtrsim 1$ is given by attractive central forces which are independent of the spin and isotopic spin states. The other types of forces give only small contributions and can be neglected. The interaction obtained in this way is in qualitative agreement with the phenomenological potentials used earlier,[16,17] even extrapolating into the region $x < 1$. One may assume that the addition of some phenomenological core at distances $x \lesssim \mu/m$ to the potential $U^{(1)} + U^{(2)}$ [formulas (5) and (8)] enables us to use it for the description of the interaction of a pair of nucleons at comparatively low energies.

It appears that the best method of introducing a phenomenological core is that of imposing a boundary condition on the logarithmic derivative, as proposed by Moszkowski and Scott.[18] The boundary condition must be chosen in such a way that the potential gives the correct S phases. To obtain better agreement with experiment one can also try to vary the parameters α, β_1, and β_2 within the limits of error $\Delta|\alpha|$, $\Delta|\beta_1|$, and $\Delta|\beta_2|$ with which they were determined from the experimental data on meson-nucleon scattering.

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134