STRUCTURE OF LOW INTENSITY SHOCK WAVES IN MAGNETOHYDRODYNAMICS

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A general expression has been deduced for the width of low intensity shock waves in magnetohydrodynamics. The damping coefficient for small amplitude waves is determined and its relation to the discontinuity width is established.

THE problem of the determination of the structure of shock waves in magnetohydrodynamics is mathematically very cumbersome and can be solved in the general case only by numerical methods. At the present time the structure of perpendicular shock waves (i.e., traveling strictly perpendicular to the magnetic field) has been studied in sufficient detail.[1-4] However, such a wave is only one of the simplest types of shock waves in magnetohydrodynamics. So far as shock waves of the general types are concerned, the so-called oblique shocks, their structure has been considered only under certain simplifying assumptions, namely, under consideration of Joule dissipation only.[5] Even in this case, one has to resort to numerical integration, which complicates the investigation of the dependence of the solution on its parameters.

In this connection, it is interesting to investigate the structure of low intensity shock waves by a method developed by Landau and Lifshitz[6] for a shock wave in ordinary hydrodynamics, and by one of the present authors[4] for a perpendicular shock wave in magnetohydrodynamics. Although this method does not make it possible to include in the discussion such peculiarities characteristic of strong shocks as isothermal and isomagnetic discontinuities, it nevertheless does make it possible to solve the problem in the general case for waves of an arbitrary type with consideration of all dissipative processes. This is especially important in the study of the dependence of the solution on its parameters and on the special features arising therein.

1. THE EQUATION OF A LOW INTENSITY SHOCK WAVE

In what follows, it is convenient to select a set of coordinates[7] in which the lines of the magnetic field and the streamlines of the liquid are parallel at great distances from the discontinuity. This set of coordinates can be introduced for all shock waves with the exception of the strictly perpendicular, in which $\mathbf{v} \perp \mathbf{H}$. However, as we shall see below, the result does not depend on the choice of coordinates and therefore it will be useful for the perpendicular shock wave also.

Let us consider a plane shock in which all the quantities depend only on x. The general equation for stationary one-dimensional flow can be written in the following form:[4]

$$j = v_n / V = j_1, \tag{1}$$

$$H_n = H_{n1}, \tag{2}$$

$$jV\mathbf{H}_\tau - H_n \mathbf{v}_\tau - \beta\, d\mathbf{H}_\tau / dx = 0, \tag{3}$$

$$j\mathbf{v}_\tau - H_n \mathbf{H}_\tau - \eta\, d\mathbf{v}_\tau / dx = j\mathbf{v}_{\tau 1} - H_n \mathbf{H}_{\tau 1}, \tag{4}$$

$$p + j^2 V + \tfrac{1}{2} H_\tau^2 - \left(\tfrac{4}{3}\eta + \zeta\right) j\, dV/dx = p_1 + j^2 V_1 + \tfrac{1}{2} H_{\tau 1}^2, \tag{5}$$

$$\frac{1}{2} j^2 V^2 + \frac{1}{2} v_\tau^2 + w - \left(\frac{4}{3}\eta + \zeta\right) jV \frac{dV}{dx} - \frac{\eta}{2j}\frac{dv_\tau^2}{dx} - \frac{\varkappa}{j}\frac{dT}{dx} = \frac{1}{2} j^2 V_1^2 + \frac{1}{2} v_{\tau 1}^2 + w_1. \tag{6}$$

Here the following notation is introduced: w, p, T, and V are, respectively, the heat function of a unit mass of the substance, the pressure, the temperature and the specific volume of the medium; η and ζ are the first and second viscosity coefficients, κ is the coefficient of thermal conductivity of the medium, and $\beta = c_0^2/4\pi\sigma$ is the magnetic viscosity (σ is the electrical conductivity of the medium and c_0 is the speed of light). The rationalized system of units has been introduced for the intensity of the magnetic field. The index 1 denotes the value of the corresponding quantities at a large distance in front of the discontinuity. The indices n and τ respectively denote the components of the vector velocity **v** and the intensity of the magnetic field **H** normal and tangential to the surface of discontinuity.

Equations (5) and (6) contain only the squares of the tangential components $\mathbf{H}_\tau$ and $\mathbf{v}_\tau$. Since Eqs. (3) and (4) are linear in $\mathbf{v}_\tau$ and $\mathbf{H}_\tau$, then they can be transformed into equations containing only the squares H_τ^2 and v_τ^2. This transformation makes it possible to consider as low intensity discontinuities not only discontinuities in which all the physical quantities change but slightly, but also discontinuities in which the absolute values of all physical quantities undergo little change while the directions of the vectors v and H can change appreciably.

The latter can be the case if the change of the vectors v and H inside the discontinuity are determined by their rotation about the normal to the discontinuity. For rotational discontinuities, the angle of rotation can be arbitrary. However, for shock waves, the boundary equations require that the vectors v and H in front of the discontinuity and behind it lie in a single plane. Therefore, in the case of a shock wave, the rotation of the vectors v and H inside the discontinuity can exist only at an angle which is a multiple of π.

So far as we know, this interesting case of shock waves has not been discussed to date. The difficulty here lies in the fact that the problem of the structure of shock waves ought to be solved for nonplanar motion. However, in the approximation of weak shock waves considered below, this problem does not differ from the problem for plane motion.

For low-intensity shock waves, the differences of the physical quantities

$$\delta p = p - p_1, \quad \delta V = V - V_1, \quad \delta T = T - T_1,$$
$$\delta H_\tau^2 = H_\tau^2 - H_{\tau 1}^2, \quad \delta v_\tau^2 = v_\tau^2 - v_{\tau 1}^2$$

are small and we can limit ourselves in the equation to terms of no higher than second order of smallness in δp, δV and so forth. Moreover, we make use of the fact that the inverse of the discontinuity width $1/l$, as will be seen from the results, has the same order of smallness as the discontinuities in the quantities δp, δV and so forth at the discontinuity, and consequently differentiation with respect to x increases the order of smallness by unity.

Making use of Eqs. (3) and (4), we express the discontinuities δH^2 and δv^2 under these assumptions in terms of δV, with accuracy up to terms of second order:

$$\delta H_\tau^2 = a_1 \delta V + b_1 (\delta V)^2 + c_1 dV/dx, \qquad (7)$$

$$\delta v_\tau^2 = a \delta V + b (\delta V)^2 + c dV/dx; \qquad (8)$$

$$a = V_1 a_1, \quad b = V_1 b_1 + a_1/2, \quad c = V_1 c_1 + (\eta/j) V_1 a_1,$$
$$a_1 = 2j^2 H_{\tau 1}^2/\Delta, \quad b_1 = 3j^4 H_{\tau 1}^2/\Delta^2,$$
$$c_1 = -2jH_{\tau 1}^2 \Delta^{-2}(j^2\beta + H_n^2\eta), \quad \Delta = H_n^2 - j^2 V_1. \qquad (9)$$

We note here that in the solution of Eqs. (3), (4), (7), and (8) with respect to $a, a_1, \ldots$, division is carried out by the factor Δ which vanishes for rotational ($H_n^2 = \rho v_n^2$) discontinuities. The appearance of a singularity in Eqs. (7) — (9) for $\Delta = 0$ signifies the absence of a stationary structure in these discontinuities. In fact, for rotational discontinuities, the boundary equations require an equality of density, pressure and, consequently, entropy on both sides of the discontinuity. This requirement is in contradiction with the increase in entropy as the result of dissipation. Therefore, the rotational discontinuities cannot have a stationary character, and are smeared out with passage of time, as was shown by Landau and Lifshitz[8] for an incompressible fluid.

Equations (5) and (6), with account of (7) and (8), reduce to the form

$$\delta p + \left(j^2 + \tfrac{1}{2} a_1\right) \delta V + \tfrac{1}{2} b_1 (\delta V)^2 + \left[\tfrac{1}{2} c_1 - j\left(\tfrac{4}{3}\eta + \zeta\right)\right] dV/dx = 0, \qquad (10)$$

$$\delta w + \left(j^2 V_1 + \tfrac{1}{2} a\right) \delta V + \tfrac{1}{2}(j^2 + b^2)(\delta V)^2 + \left[\tfrac{1}{2} c - jV\left(\tfrac{4}{3}\eta + \zeta\right)\right] dV/dx - (\eta a/2j) dV/dx = (\varkappa/j) dT/dx. \qquad (11)$$

It is convenient to solve the systems (10) and (11) for δp. For this purpose, multiplying (10) by V_1 and subtracting the result from (11) we get the following equation:

$$\delta w - V_1 \delta p + \tfrac{1}{2}\left(j^2 + \tfrac{1}{2} a_1\right)(\delta V)^2 = (\varkappa/j) dT/dx. \qquad (12)$$

Further, taking the pressure p and the entropy of a unit mass s as independent variables, and representing δw, δV and δT in Eqs. (12) in the form of series in δp and δs, we get for the first two terms (the contributions are similar to those obtained by Landau and Lifshitz[6]):

$$\delta s = \frac{\varkappa}{jT}\left(\frac{\partial T}{\partial p}\right)_s \frac{dp}{dx}. \qquad (13)$$

Equation (13) shows that for low intensity discontinuities in magnetohydrodynamics, with the exception of the vicinity of the singular point $\Delta = 0$, at which the expansions (7) and (8) are inappropriate, the change in entropy inside the discontinuity is small in comparison with the change in pressure. Therefore, it suffices in what follows to limit ourselves to the account of terms of first order in δs. In this approximation considering (13), we have

$$\delta V = \left(\frac{\partial V}{\partial p}\right)_s \delta p + \frac{1}{2}\left(\frac{\partial^2 V}{\partial p^2}\right)_s (\delta p)^2 + \frac{\varkappa}{jT}\left(\frac{\partial T}{\partial p}\right)_s \left(\frac{\partial V}{\partial s}\right)_p \frac{dp}{dx}. \qquad (14)$$

Substituting this expression in (10), we obtain a differential equation for the pressure p(x):

$$\left[1+\left(j^2+\frac{a_1}{2}\right)\left(\frac{\partial V}{\partial p}\right)_s\right]\delta p+\frac{1}{2}\left[\left(j^2+\frac{a_1}{2}\right)\left(\frac{\partial^2 V}{\partial p^2}\right)_s + b_1\left(\frac{\partial V}{\partial p}\right)_s^2\right](\delta p)^2=-\left\{\frac{j^2+a_1/2}{T}\frac{\varkappa}{j}\left(\frac{\partial T}{\partial p}\right)_s\left(\frac{\partial V}{\partial s}\right)_p + \left[\frac{c_1}{2}-j\left(\frac{4}{3}\eta+\zeta\right)\left(\frac{\partial V}{\partial p}\right)_s\right]\right\}\frac{dp}{dx}, \tag{15}$$

where $\delta p = p(x) - p_1$.

In the case of the absence of a magnetic field, Eq. (15) reduces to the equation for $p(x)$ in ordinary hydrodynamics, introduced by Landau and Lifshitz,[6] inasmuch as the coefficients a_1, b_1, c_1 tend to zero along with the intensity of the magnetic field.

2. DAMPING OF SMALL AMPLITUDE WAVES IN MAGNETOHYDRODYNAMICS

Equation (15) can be used directly for the determination of the damping coefficient of waves of small amplitude in magnetohydrodynamics. For this purpose, it will be sufficient to limit ourselves to the linear approximation, omitting from (15) the term with $(\delta p)^2$, and neglecting the difference between V and V_1 in the expression $j = v_n/V$. Moreover, taking into consideration the thermodynamic relations

$$\left(\frac{\partial V}{\partial p}\right)_s=-\frac{1}{\rho^2c^2},\qquad \left(\frac{\partial T}{\partial p}\right)_s\left(\frac{\partial V}{\partial s}\right)_p=\frac{T}{\rho^2c^2}\left(\frac{1}{C_v}-\frac{1}{C_p}\right),$$

where c is the speed of sound and $\rho = 1/V$ is the density of the medium, and also Eq. (9) for the coefficients a_1 and b_1, we get as a result

$$\frac{v_n}{\rho c^2}\left[\left(1+\frac{H_{\tau 1}^2}{\Delta}\right)\varkappa\left(\frac{1}{C_v}-\frac{1}{C_p}\right)+\left(\frac{4}{3}\eta+\zeta\right)+\frac{H_{\tau 1}^2}{\Delta^2}(\rho^2v_n^2\beta+H_n^2\eta)\right]\frac{dp}{dx} + \left[1-\frac{v_n^2}{c^2}\left(1+\frac{H_{\tau 1}^2}{\Delta}\right)\right]\delta p=0. \tag{16}$$

In the process of deriving this equation, we have omitted terms with higher derivatives in addition to linearizing in the amplitude. In the initial equations (3) — (6), terms with first derivatives in x contain as a factor one of the dissipative coefficients η, ζ, κ, or β. Therefore, in neglecting the products of these terms and terms with higher derivatives, we actually neglect products of the dissipation coefficients. This is equivalent to an assumption on the smallness of damping, which will be considered in the present work.

For perturbations whose time dependence has the form e^{ikx}, Eq. (16) reduces to the well-known relation between v_n and k:

$$v_n^4-v_n^2(c^2+u_n^2+u_\tau^2)+c^2u_n^2+ik\frac{v_n}{\rho}\left\{\left[\varkappa\left(\frac{1}{C_v}-\frac{1}{C_p}\right)+\left(\frac{4}{3}\eta+\zeta\right)\right](u_n^2-v_n^2)+u_\tau^2\varkappa\left(\frac{1}{C_v}-\frac{1}{C_p}\right) + \frac{u_\tau^2}{u_n^2-v_n^2}(\rho v_n^2\beta+u_n^2\eta)\right\}=0, \tag{17}$$

$u_n = H_n/\sqrt{\rho}$ is the Alfvén velocity, $u_\tau = H_\tau/\sqrt{\rho}$.

Equation (17) for $v_n = \omega/k$ is equivalent to the dispersion equation for small amplitude waves in magnetohydrodynamics with consideration of weak damping. In fact, the time-independent equation (16) can be obtained from the general linearized system of equations of magnetohydrodynamics by a formal transformation of coordinates: $x = x' + v_n t$, and by a corresponding transformation of the desired functions $e^{i(kx-\omega t)} \to e^{ikx'}$ (if $v_n = \omega/k$). Therefore, in the determination of the damping coefficient, one can start out immediately from Eq. (17), setting the phase velocity of the excitation $v = k^{-1}\mathrm{Re}\,\omega = \mathrm{Re}\,v_n$ and the damping coefficient $\gamma = \mathrm{Im}\,\omega = k\,\mathrm{Im}\,v_n$.

Without consideration of dissipative terms, Eq. (17) is reduced to the dispersion equation for small perturbations:[9]

$$v^4-v^2(c^2+u_n^2+u_\tau^2)+c^2u_n^2=0. \tag{18}$$

Solution of Eq. (17) in the case of weak damping can be represented in the form

$$v_n = v + iv_1 = v + i\gamma/k,$$

where $\gamma \ll kv$, and v satisfies Eq. (18). Substituting this solution in (17), we find the damping coefficient γ, which determines the decrease in the amplitude of the wave with time as $e^{-\gamma t}$,

$$\gamma = k^2 a, \tag{19}$$

$$a=\frac{1}{2(v^4-c^2u_n^2)}\left\{c^2(v^2-u_n^2)\left[\frac{\varkappa}{\rho}\left(\frac{1}{C_v}-\frac{1}{C_p}\right)+\frac{v^2}{\rho c^2}\left(\frac{4}{3}\eta+\zeta\right)\right]+(v^2-c^2)\left(v^2\beta+\frac{u_n^2}{\rho}\eta\right)\right\}. \tag{20}$$

For a parallel shock wave, the phase velocity of the wave $v = c$ and the coefficient a is given by

$$a=a_\parallel=\frac{1}{2\rho}\left[\varkappa\left(\frac{1}{C_v}-\frac{1}{C_p}\right)+\left(\frac{4}{3}\eta+\zeta\right)\right], \tag{21}$$

which coincides with the expression obtained by Landau and Lifshitz.[6]

For a weak rotational (Alfvén) discontinuity ($v = v_n$) we obtain

$$a = a_A = \tfrac{1}{2}(\beta + \eta/\rho). \tag{22}$$

As is seen from (22), the dissipation in this case, as in the case of an incompressible fluid,[8] is due only to the viscosity and the conductivity of the medium.

For a weak oblique wave, considered as the limit for $u_n^2 = c^2$, $u_\tau^2 \to 0$, we find from (18) and (20) that $v^2 = u_n^2 = c^2$, and

$$a_0=\frac{1}{4}\left[\frac{\varkappa}{\rho}\left(\frac{1}{C_v}-\frac{1}{C_p}\right)+\frac{1}{\rho}\left(\frac{4}{3}\eta+\zeta\right)+\left(\beta+\frac{\eta}{\rho}\right)\right]. \tag{23}$$

We note that in this case the damping coefficient is equal to half the sum of the coefficients in the ordinary sound wave and in a weak rotational discontinuity.

For a perpendicular wave, in which $u_n^2 = 0$, but $u_T = H_T^2/\rho \neq 0$, we find $v^2 = u_T^2 + c^2$ and

$$a = a_\perp = \frac{1}{2(1 + u_\tau^2/c^2)}\left[\frac{\varkappa}{\rho}\left(\frac{1}{C_v} - \frac{1}{C_p}\right) + \left(1 + \frac{u_\tau^2}{c^2}\right)\frac{1}{\rho}\left(\tfrac{4}{3}\eta + \zeta\right) + \frac{u_\tau^2}{c^2}\beta\right]. \tag{24}$$

3. WIDTH OF THE DISCONTINUITY

The width of the discontinuity can be determined from Eq. (15). At large distances the pressure on the left and right of the discontinuity is equal to p_1 and p_2 respectively, while the right hand side of the equation vanishes along with dp/dx. Therefore, the roots of the quadratic three terms on the left hand side will be p_1 and p_2, and Eq. (15) is equivalent to the equation

$$\frac{dp}{dx} = -\frac{2}{A}(p - p_1)(p - p_2). \tag{25}$$

The coefficient A is equal to double the ratio of the coefficients for dp/dx and $(\delta p)^2$ in (15); with the aid of the relations used in the derivation of Eq. (16), this coefficient can be written in the form

$$A = 4\frac{c^2}{\rho v} \times \frac{(1 + H_\tau^2/\Delta)\varkappa(1/C_v - 1/C_p) + (4\eta/3 + \zeta) + (H_\tau/\Delta)^2(\rho^2 v^2\beta + H_n^2\eta)}{\rho^2 c^4(1 + H_\tau^2/\Delta)(\partial^2 V/\partial p^2)_s + 3v^2 H_\tau^2/\Delta^2}. \tag{26}$$

The quantity $\Delta = H_n^2 - j^2 V_1 = \rho(u_n^2 - v^2)$ appearing here depends on the velocity of the medium v relative to the surface of discontinuity. For this velocity we can substitute in Eq. (26) (in the approximation under discussion) the velocity of small excitations determined from the dispersion equation (18). Then A, with consideration of (20), takes the following form:

$$A = \frac{8c^2\rho}{v}\frac{(v^4 - c^2 u_n^2)\,a}{\rho^3 c^6(v^2 - u_n^2)(\partial^2 V/\partial p^2)_s + 3(v^2 - c^2)v^2}. \tag{27}$$

Integration of Eq. (25) shows (see, for example, reference 6) that the change in pressure takes place essentially in a layer of thickness

$$l \approx A/(p_2 - p_1), \tag{28}$$

that is, l is the effective thickness of the discontinuity.

Let us consider in more detail the expressions (27) and (28) for the width of the discontinuity. For a parallel shock wave ($H_T^2 = 0$), Eq. (28) coincides with the width of the ordinary shock wave,[6] as it should, inasmuch as

$$A_\parallel = \frac{8V^2 a_\parallel}{c^3}\left(\frac{\partial^2 V}{\partial p^2}\right)_s^{-1} = \frac{4V^3}{c^3}\left(\frac{\partial^2 V}{\partial p^2}\right)_s^{-1}\left[\varkappa\left(\frac{1}{C_v} - \frac{1}{C_p}\right) + \left(\frac{4}{3}\eta + \zeta\right)\right]. \tag{29}$$

The set of coordinates we have chosen is generally not suitable for consideration of a perpendicular wave. However, Eq. (28) also contains the perpendicular wave as a limiting case $H_n \to 0$ and $H_T \neq 0$. In this case,

$$A = \frac{4c\left[\varkappa(C_v^{-1} - C_p^{-1}) + (4\eta/3 + \zeta)(1 + u_\tau^2/c^2) + \rho\beta u_\tau^2/c^2\right]}{\sqrt{1 + u_\tau^2/c^2}\left[\rho^3 c^4(\partial^2 V/\partial p^2)_s + 3u_\tau^2/c^2\right]} \tag{30}$$

and the width of the discontinuity coincides with that obtained earlier.[4]

As is seen from the derivation of Eqs. (10) and (11), the singular case $\Delta = 0$ is, strictly speaking, excluded from our consideration. In addition to the rotational discontinuity and discontinuities close to it, a singular oblique wave,[9] in which $H_T = 0$ on one side of the discontinuity and $\Delta = 0$ on the other, also corresponds to this case.

So far as the rotational discontinuity is concerned, such a discontinuity (as has been noted above) cannot have a stationary width in the presence of a dissipation. Formally, this case corresponds to an infinite width because of the vanishing of the denominator of Eq. (28). For discontinuities close to rotational, we get from (26), for $\Delta = 0$,

$$A = \frac{4}{3}\frac{c^2}{v^3}\left(\rho v^2\beta + \frac{H_n^2}{\rho}\eta\right). \tag{31}$$

However, as can be seen from Eqs. (7) — (9), in such discontinuities (i.e., for $H_T \neq 0$) δV, and therefore δp also, must approach zero along with Δ, and therefore Eq. (28) does not give a finite value for the width of the discontinuity. Thus one can conclude that both the rotational discontinuity and discontinuities close to it cannot have a stationary width.*

For the singular oblique wave, considered as the limit of a shock wave for $u_n^2 = c^2$ and $u_T^2 \to 0$, the expression for A reduces to the following:

$$A = \frac{4c\left[\varkappa(C_v^{-1} - C_p^{-1}) + (4\eta/3 + \zeta) + (\beta\rho + \eta)\right]}{\rho^3 c^4(\partial^2 V/\partial p^2)_s + 3}. \tag{32}$$

For such a limiting transition, the coefficients of the expansions (7) and (8) remain finite and,

*Furthermore, we note that, in discontinuities close to rotational, the tangential component of the magnetic field changes sign. Such shock waves, as has been pointed out by Polovin and Lyubarskiĭ,[10] are unstable relative to splitting (non-evolutionary).

consequently, the peculiarities which are characteristic of rotational discontinuities do not appear here.

4. THE CONNECTION BETWEEN THE DAMPING COEFFICIENT AND THE WIDTH OF A LOW INTENSITY DISCONTINUITY

One can also arrive at Eqs. (27) and (28) for the width of the discontinuity by starting from the qualitative picture of shock wave formation. In fact, the stationary structure of the discontinuity is established as a result of the equilibrium of two opposing processes. The first of these consists in the smearing out of the jump under the action of viscosity, finite conductivity and thermal conductivity. The action of these dissipative processes can be described by a certain effective viscosity which for a wave of small amplitude is connected to the damping coefficient by the well-known relation $\gamma = ak^2$, and therefore is determined by Eq. (20) in magnetohydrodynamics. The smearing out of the discontinuity as the result of dissipation has a diffusion character and the velocity of such smearing out V_- can be estimated from the relation

$$l^2 \sim 4at, \qquad V_- \sim l/t \sim 4a/t, \tag{33}$$

where l is the width of the discontinuity and t is the time, measured from the moment of formation of the discontinuity.

As the opposing process, we have the "entanglement" of the discontinuity brought about by the different velocities of the excitation in front of the discontinuity and behind it. The "entanglement" tends to reduce the width of the shock which takes on a certain stationary value when both these processes are equal to one another, i.e., the rate of entanglement V_+ becomes equal to the rate of smearing out V_-. In this case the width of the discontinuity, in accord with (33), becomes equal in order of magnitude to

$$l \approx 4a/V_+. \tag{34}$$

The rate of "entanglement" V_+ is equal to the difference of the velocities of small disturbances in front of the discontinuity and behind it in a fixed system of coordinates:

$$V_+ = v' + \delta v_{\mathrm{av}} - v = \delta v + \delta v_{\mathrm{av}}, \tag{35}$$

where v' and v are the velocities of propagation of the waves of small amplitude under consideration relative to the medium on the two sides of the discontinuity, and δv_{av} is the jump in the normal component of the velocity of the medium in the shock wave.

In zero approximation (for magnetohydrodynamic "sound"), the velocities v' and v coincide and are equal to the velocity of the discontinuity. In the following approximation, one can determine $\delta v = v' - v$ from the dispersion equation (18), in which it is convenient to select as independent variables the density ρ and the tangential component H_τ:

$$v^4 - v^2(c^2 + H_n^2/\rho + H_\tau^2/\rho) + c^2H_n^2/\rho = 0. \tag{36}$$

Then

$$\delta v = \frac{\partial v}{\partial \rho}\delta\rho + \frac{\partial v}{\partial \mathbf{H}_\tau}\delta\mathbf{H}_\tau, \tag{37}$$

where we have taken it into consideration that H_n is continuous and the change in entropy is small in comparison with the change in the density. Therefore,

$$\delta c \approx \left(\frac{\partial c}{\partial \rho}\right)_s \delta\rho = \frac{c}{\rho}\left[\frac{\rho^3c^4}{2}\left(\frac{\partial^2 V}{\partial p^2}\right)_s - 1\right]\delta\rho. \tag{38}$$

Computing the derivatives $\partial v/\partial\rho$ and $\partial v/\partial \mathbf{H}_\tau$ by means of (18), and taking it into account that in a wave of small amplitude [see reference 9, Eqs. (2.21) and (2.22)]

$$\delta \boldsymbol{H}_\tau = [(v^2 - c^2)/H_\tau^2]\,\mathbf{H}_\tau\delta\rho, \tag{39}$$

$$\delta v_{\mathrm{av}} = v\delta\rho/\rho, \tag{40}$$

we finally obtain:

$$V_+ = \delta v + \delta v_{\mathrm{cp}} = \frac{v^2\delta\rho}{2\rho v\,[v^4 - c^2u_n^2]}\left[(v^2 - u_n^2)\,\rho^3c^6\left(\frac{\partial^2 V}{\partial p^2}\right)_s + 3v^2(v^2 - c^3)\right]. \tag{41}$$

This expression for the rate of "entanglement," together with (34) again leads to Eqs. (27) and (28) for the width of the discontinuity.

From the qualitative considerations that have been given, it is clear that the rotational discontinuity for which "entanglement" is absent ($\delta\rho = 0$) cannot have a stationary width.

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139