COVARIANT STATISTICAL THEORIES OF MULTIPLE PARTICLE PRODUCTION

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Possible covariant theories of multiple particle production are analyzed under the condition that the matrix element can be factorized. The multiplicity of the secondary particles is computed by assuming that the matrix element is a power function of the energy of the particles involved in the process.

1. The probability W_N that N particles with masses $m_1, m_2, \ldots m_N$ are produced in the collision of two particles is of the following form:

$$W_N = \int \cdots \int \Phi(K_0, k_1, \ldots, k_N)\,\delta^4 \times \Big(K_0 - \sum_{j=1}^{N} k_j\Big) \prod_{j=1}^{N} \delta(k_j^2 - m_j^2)\, d^4k_j, \qquad (1)$$

where $K_0(i\mathbf{P}_0, E_0)$ is the four-momentum of the initial state, $k_j(i\mathbf{p}_j, \epsilon_j)$ is the momentum of the j-th particle, and $\Phi(K_0, k_1, \ldots k_j)$ is an invariant function depending on the character of interaction of the particles.

We shall consider the case where the interaction between the particles is sufficiently small so that we can neglect the correlations. The function $\Phi(K_0, k_1, \ldots k_N)$ can then be represented in the form of the product

$$\Phi(K_0, k_1, \ldots, k_N) = \prod_{j=1}^{N} \Phi_j(K_0, k_j). \qquad (2)$$

In the following, we shall limit ourselves to invariant functions of $\Phi_j(K_0, k_j)$ of a special class, such that

$$\Phi_j(K_0, k_j) = C\,(K_{0\nu}k_{j\nu})^q / (\sqrt{K_{0\nu}K_{0\nu}})^s, \qquad (3)$$

where C is independent of K_0 and k_j, and is determined by the coupling constants, masses,* spins, and isotopic spins of the particles; q and s are integers.

Taking Eq. (2) into account, the relation (1) can be converted into the form

$$W_N = \frac{C^N}{2^N} \int \cdots \int \delta^4\Big(K_0 - \sum_{j=1}^{N} k_j\Big) \prod_{j=1}^{N} \Phi_j(K_0, k_j)\, \varepsilon_j^{-1} d^3p_j. \qquad (4)$$

We shall dwell in detail upon the important special cases:

$$\Phi_j(K_0, k_j) = C\,(K_{0\nu}k_{j\nu}) / (K_{0\nu}k_{0\nu}), \qquad (5)$$

$$\Phi_j(K_0, k_j) = C. \qquad (6)$$

The function (5) corresponds to the statistical Fermi theory,[1] and the function (6) to the theory of Shrivastava and Sudarshan[2] (SS theory).

The special cases based on the choice of Eq. (5) or Eq. (6) are of special interest because of the possibility of giving them a simple and clear interpretation. The physical interpretation of the Fermi theory is well known. The choice of $\Phi_j(K_0, k_j)$ in the form (6) can be treated in the following way: expanding the meson field of the nucleon (at sufficiently great distances from its center) in a Fourier integral, it is found[3,4] that the probability ω_j of a pseudoscalar meson having momentum in the interval $\mathbf{p}_j$, $\mathbf{p}_j + d\mathbf{p}_j$ is equal to

$$\omega_j = d^3p_j / \varepsilon_j. \qquad (7)$$

If ω_j is independent of the remaining ω_k ($k \neq j$), which is equivalent to the assumption of statistical independence of the particles, then the probability of N mesons being in the states with momenta $\mathbf{p}_j \ldots \mathbf{p}_N$ is equal to the product of corresponding probabilities for separate particles. Furthermore, in line with the Lewis, Oppenheimer, and Wouthuysen theory[4,5] (LOW theory), the process of multiple particle production can be interpreted as the breaking up of a meson cloud without any change in its internal state (i.e., conserving the distribution $\prod_{j=1}^{N} d^3p_j/\epsilon_j$).

Such an interpretation leads to Eq. (4), under condition (6). It is, however, necessary to mention that the SS theory is then not fully equivalent to the LOW theory, since the role of nucleons in the collision process is treated differently in the two theories. While, according to the LOW theory, it is necessary to assume that the nucleons lose a

*Moreover, the mass dependence should be such that W_N is a dimensionless quantity.

relatively small energy fraction, and consequently, have to be treated as separate particles from the energy point of view, the nucleons are treated equally with other particles in the relations (4) and (6). In essence, the mathematical formulation of the LOW theory is reduced to the following relation

$$W_N \sim \int \delta^4\Big(K - \sum_{j=3}^{N} k_j\Big) A(K_1, K_2) \prod_{j=3}^{N} d^3p_j / \varepsilon_j, \tag{8}$$

where $K = K_0 - K_1 - K_2$; K_1 and K_2 are the four-momenta of both nucleons determined by the mechanism of the process. The summation and multiplication is carried only over meson indices.

2. We shall calculate the probability W_N for the class of theories indicated in Sec. 1. We have previously developed a method of calculating the quantity (4) for the Fermi theory [assumption (5)].[6] Here, we shall apply this method for calculating Eq. (4) with Φ_j in the form (3) with arbitrary q and s; in particular, we shall obtain formulas for the SS and LOW variants.

In order to simplify the calculations, we shall consider the problem in the c.m.s. ($P_0 = 0$). Since the probability W_N depends only on the invariants $E_0^2 - P_0^2$ and m_j, it is necessary to make the substitution $E_c^2 \to E_0^2 - P_0^2$ in the final expressions in order to go over to the general case $P_0 \neq 0$. Using the Fourier transform of the δ function, we transform (4) to the form

$$W_N = C^N [2^N E_c^{(s-q)N} (2\pi)^4]^{-1} \int_{-\infty-i\delta}^{+\infty-i\delta} \exp[-i\tau_1 E_c]\, d\tau_1 \int_{-\infty-i\delta}^{+\infty-i\delta} d\tau$$
$$\times \prod_{j=1}^{N} \iiint_{-\infty}^{+\infty} (p_j^2 + m_j^2)^{(q-1)/2} \exp[i(\tau_1 \sqrt{p_j^2 + m_j^2} + \tau p_j)]\, d^3p_j. \tag{9}$$

The integral over p_j can easily be transformed to the form

$$J_j = -\frac{2\pi}{\tau i^q} \frac{d^q}{d\tau_1^q} \frac{d}{d\tau} \int_{-\infty}^{\infty} \exp[i(\tau_1 \sqrt{p_j^2 + m_j^2} + \tau p_j)]$$
$$\times (p_j^2 + m_j^2)^{-1/2} dp_j = -\frac{2\pi}{\tau i^q} \frac{d^q}{d\tau_1^q} \frac{d}{d\tau}\, i\pi H_0^1 (m_j \sqrt{\tau_1^2 - \tau^2}) \tag{10}$$

[where $H_0^{(1)}(z)$ is the Hankel function]. The phase φ of argument $m_j\sqrt{\tau_1^2 - \tau^2}$ is chosen in the following way:

$$\varphi = 0 \text{ for } \tau_1 > \tau, \qquad \varphi = i\pi/2 \text{ for } -\tau < \tau_1 < \tau,$$
$$\varphi = i\pi \text{ for } \tau_1 < -\tau. \tag{11}$$

Carrying out the differentiation, and using the recurrent formula for the Hankel function, we obtain

$$J_j = \frac{2\pi^2}{i^{q+1}} \frac{m_j^{q+1} \tau_1^q}{(\tau_1^2 - \tau^2)^{(q+1)/2}} H_{q+1}^{(1)} (m_j \sqrt{\tau_1^2 - \tau^2}). \tag{12}$$

For future calculations, similarly to what was done by us earlier,[6] we shall expand the product of the Hankel functions in a series and integrate it term by term. As a result, we obtain the expression for (9) in the form of a non-power series of a small parameter ν_j

$$\nu_j = m_j / E_c. \tag{13}$$

In particular, the first term of this series, independent of ν_j, (which corresponds to ultra-relativistic particles), is of the form

$$W_N = \frac{C^N \pi^{N-1} (q!)^N}{2^{N(q+1)-1}}$$
$$\times \frac{(2N(q+1)-4)!\, E_c^{N(2+2q-s)-4}}{[N(q+2)-4]!\,[N(q+1)-1]!\,[N(q+1)-2]!}. \tag{14}$$

We shall estimate the variation of the most-probable value of $\bar{N}$ with energy. If N is sufficiently large so that we can use the Stirling formula, we obtain

$$\bar{N} \backsim E_c^{(2q-s+2)/(q+3)}. \tag{15}$$

In the case of the SS or LOW theory ($q = 0$, $s = 0$), we obtain the following expression for W_N

$$W_N = C^N \Big(\frac{\pi}{2}\Big)^{N-1} (E_c^2)^{N-2} \Big\{ \frac{1}{(N-1)!\,(N-2)!} - \frac{1}{(N-2)!\,(N-3)!}$$
$$\times \Big[\sum_{j=1}^{N} \nu_j^2 \ln \frac{1}{\nu_j^1} - \Big(\sum_{m=1}^{N-2} \frac{1}{m} + \sum_{m=1}^{N-3} \frac{1}{m} - 1 \Big) \sum_{j=1}^{N} \nu_j^2 \Big] + \ldots \Big\}. \tag{16}$$

The first term of this series has been obtained earlier.[4,7] In addition to the first term, Yakovlev[7] has, by different methods, obtained the expressions for W_N for N = 3, 4, 5. However, in our opinion, errors were committed in the derivation, and the expressions are not correct.

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