LATERAL DISTRIBUTION OF HIGH ENERGY NUCLEAR-ACTIVE PARTICLES IN THE CORE OF EXTENSIVE ATMOSPHERIC SHOWERS

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The lateral distribution of high-energy nuclear-active particles in the core of extensive atmospheric showers is considered. The mean-square radius for nuclear-active particles with energies $\geq 5 \times 10^{11}$ ev is computed from the angular distribution of secondary particles emitted in multiple-production processes, as predicted by the Landau hydrodynamic theory. It is shown that the mean-square radius depends not only on the angles of emission of the secondary particles during multiple production but also on the diffraction scattering of the nuclear-active particles by nuclei of air atoms.

1. There are many known experimental papers devoted to the lateral characteristics of the nuclear-active component of extensive atmospheric showers (EAS) of cosmic rays.[1,2] One of the tasks of these investigations is to study the angular distribution of the secondary particles produced when high-energy nuclear-active particles collide with the nuclei of air atoms. For this purpose, the form of the lateral-distribution function of the flux density of the nuclear-active particles of energy higher than specified is determined in different experiments. Since the nuclear-active cascade is accompanied by a large number of electrons produced by the π^0 mesons which are created in the same interaction events as the $\pi^\pm$ mesons and the nucleons, one might think that an investigation of the lateral characteristics of the electron-photon component would explain several detailed features of the elementary act. However, it has been shown by many authors (see, for example, reference 3) that the lateral distribution of the electron-photon component is practically independent of the lateral and angular distributions of the π^0 mesons. It follows therefore that the lateral distribution of the π^0 mesons is very narrow, and consequently a study of the lateral characteristics of the electron-photon component (except perhaps at very short distances from the axis) does not make it possible to evaluate the angular distribution of the secondary nuclear-active particles in the elementary act.

The situation is different with $\pi^\pm$ and nucleons, the lateral distributions of which (in the case of high energies) are determined by the angular distribution of the particles during the acts of multiple generation and by elastic scattering on the nuclei of the air atoms (diffraction scattering).

2. Let us consider the passage of high-energy nuclear-active particles through the atmosphere. Let $P(E, t, \mathbf{r}, \theta)$ be the flux density of the nuclear-active particles of the EAS. Here E is the energy of the nuclear-active particles, t the height of observation in nuclear-interaction ranges, $\mathbf{r}$ the radius vector in a plane perpendicular to the shower axis, and θ the vector of direction of motion of the particle. The function P satisfies the following kinetic equation:[4]

$$\partial P(E, t, \mathbf{r}, \theta)/\partial t + \theta\,\partial P(E, t, \mathbf{r}, \theta)/\partial \mathbf{r} = -P(E, t, \mathbf{r}, \theta) + \int_E^\infty \int_\Omega P(E', t, \mathbf{r}, \theta+\chi)\,\varphi_L(E', E, \theta+\chi, \theta)\,dE'd\Omega + \int_\Omega [P(E, t, \mathbf{r}, \theta+\chi) - P(E, t, \mathbf{r}, \theta)]\,d\sigma(\chi), \qquad (1)$$

where $\varphi_L(E', E, \theta + \chi, \theta)$ is the probability that a particle of energy E', traveling at an angle $\theta + \chi$ to the shower axis, will have an energy E after colliding with the nucleus of the air atom, and will be deflected by an angle χ; $d\sigma(\chi)$ is the cross section for the deflection of a nuclear-active particle by an angle χ as a result of diffraction scattering.*

We shall consider nuclear-active particles of very high energies ($E \gg Mc^2$), and can therefore put $\theta \ll 1$ and $\chi \ll 1$.

For the function φ_L, which characterizes the multiple particle production processes, we use an

*We neglect the diffraction-generation effect.

expression that follows from the hydrodynamic theory of interaction of particles of very high energies,[5] with allowance for the fact that the distribution of the secondary particles along the direction $\theta + \chi$ has azimuthal symmetry:

$$\varphi_L(E', E, \theta+\chi, \theta)\,dE'\,d\Omega$$
$$\approx \frac{2}{3} N_{E'} \frac{\exp\{-(\eta-\eta_c')^2/2L\}\,dE'd\Omega}{\sqrt{2\pi L}\;2\pi p_\perp c}\;\delta\left(\chi - \frac{p_\perp c}{E}\right),$$
$$\eta \approx \ln\frac{E}{Mc^2}, \qquad L \approx 0.5\,\eta', \qquad N_{E'} \approx 2.3\,e^{\eta'/n},$$
$$\eta_c' \approx \frac{1}{2}\left[\eta' - \ln\frac{n_0+1}{2}\right],$$

n_0 is the average number of nucleons in a "tunnel" of the air-atom nucleus, and $p_\perp$ is the momentum acquired by a secondary particle in the direction perpendicular to the direction of motion of the primary particle. According to Milekhin,[5] $p_\perp \approx 3\mu c$,* where μ is the pion mass. The function φ_L is normalized to the total number of particles produced by a primary particle of energy E'.

It must be especially emphasized that we assume that the π mesons and the nucleons interact in the same manner with the nuclei of the air atoms. It is possible that account should be taken of the difference between π-mesons interactions and nucleon interactions. This would lead to a system of kinetic equations which are genetically related, but such an analysis is not believed desirable in the present paper.

In addition to scattering that takes place during the nuclear-interaction events accompanied by multiple particle production, one must allow also for the so-called diffraction scattering, which occurs without loss of primary-particle energy, since it is found that the effective angles are of the same order of magnitude in diffraction scattering as in multiple-production events. The total cross section of diffraction scattering σ_{dif} is equal to the multiple-production cross section σ_{mp}:

$$\sigma_{dif} = \sigma_{mp} \approx \pi R^2 A^{2/3},$$

where A is the atomic weight of air and $R \approx 1.3 \times 10^{-13}$ cm.

The cross section σ_{dif} which we use for diffraction scattering contains one natural assumption, namely that the nucleus is a non-transparent "black ball" for fast nuclear-active particles.[6]

To simplify the kinetic equation (1), we use the condition $\chi \approx 1$ and expand the function $P(E, t, \mathbf{r}, \theta + \chi)$ in powers of χ:[7]

*By $p_\perp$ is meant the mean-squared value of the transverse momentum.

$$P(\theta+\chi) = P(\theta) + \frac{\partial P}{\partial\theta}\chi + \frac{1}{2}\left(\frac{\partial^2 P}{\partial\theta_x^2}\chi_x^2 + \frac{\partial^2 P}{\partial\theta_y^2}\chi_y^2 + \frac{\partial^2 P}{\partial\theta_x\partial\theta_y}\chi_x\chi_y\right) + \ldots \tag{2}$$

Here χ_x and χ_y are the components of the vector χ in a plane tangent to the unit sphere at the point θ:

$$\chi_x = \chi\cos\varphi, \quad \chi_y = \chi\sin\varphi\,.$$

Let us average Eq. (2) over φ with allowance for the azimuthal symmetry

$$\int_0^{2\pi} P(\theta+\chi)\,d\varphi \approx \int_0^{2\pi} P(\theta)\,d\varphi + \frac{\pi}{2}\chi^2\Delta_\theta P + \ldots, \tag{3}$$

where, in view of the smallness of χ, we discard terms with higher powers of χ.

We substitute (3) in the right half of (1). The integral that determines the multiple-production processes assumes the form

$$\int_E^\infty\int_\Omega P(E', t, \mathbf{r}, \theta+\chi)\,\varphi_L(E', E, \theta+\chi, \theta)\,dE'd\Omega$$
$$= L[P(E', t, \mathbf{r}, \theta)] + (p_\perp c/2E)^2\,\Delta_\theta L[P(E', t, \mathbf{r}, \theta)],$$
$$L[P(E', t, \mathbf{r}, \theta)] = \int_E^\infty P(E', t, \mathbf{r}, \theta)\,\varphi_L(E', E, \theta+\chi, \theta)\,dE'. \tag{4}$$

The integral that describes the diffraction scattering is transformed into

$$\int_0^\pi\int_0^{2\pi}[P(E, t, \mathbf{r}, \theta+\chi) - P(E, t, \mathbf{r}, \theta)]f(\chi)\sin\chi\,d\chi\,d\varphi$$
$$\approx \frac{\pi}{2}\;\Delta_\theta P(E, t, \mathbf{r}, \theta)\int_0^{\chi_m} f(\chi)\chi^3 d\chi. \tag{5}$$

Here $f(\chi)$ is determined from the expression for the diffraction-scattering cross section $d\sigma(\chi) = f(\chi)\,d\chi$, and χ_m is the maximum angle of deflection, taking account of the transparency of the edge of the nucleus to nuclear-active particles. We note that the expression for $f(\chi)$, which follows from the "black ball" model,[6]

$$f(\chi) = \frac{1}{\pi}\left|J_1\left(\frac{E}{\mu c^2}\sin\chi\right)/\sin\chi\right|,$$

leads to a divergent expression for the mean-squared deflection angle $\overline{\chi^2}$. Taking into account the fact that we know the structure of the edge of the nucleus, we introduce the mean-squared angle of diffraction scattering $\overline{\chi^2} = (b\mu c^2/E)^2$, and then (5) is transformed to

$$(b\mu c^2/2E)^2\,\Delta_\theta P(E, t, \mathbf{r}, \theta). \tag{6}$$

The parameter b should be determined experimentally, for example, from emulsion data. We thus write finally expression (4) in the form

$$\frac{\partial P(E,\ t,\ \mathbf{r},\ \theta)}{\partial t} + \theta \frac{\partial P(E,\ t,\ \mathbf{r},\ \theta)}{\partial \mathbf{r}} = -P(E,\ t,\ \mathbf{r},\ \theta)$$

$$+ \left(\frac{b\mu c^2}{2E}\right)^2 \Delta_\theta P(E,\ t,\ \mathbf{r},\ \theta)$$

$$+ \left[1 + \left(\frac{p_\perp c}{2E}\right)^2 \Delta_\theta\right] L[P(E',\ t,\ \mathbf{r},\ \theta)]. \qquad (7)$$

3. Let us now determine the mean-squared angle and the radius of deflection of the nuclear-active particles. Following the usual procedure of calculating the moments of the function $P(E, t, \mathbf{r}, \theta)$,[7] we integrate (7) with respect to t from 0 to ∞ and then multiply by θ^2 and integrate over all of space and all the solid angles. As a result we obtain

$$P_1(E) - L[P_1(E')] = \left(\frac{b\mu c^2}{E}\right)^2 P_0(E) + \left(\frac{p_\perp c}{E}\right)^2 L[P_0(E')], \qquad (8)$$

$$P_0(E) = \int_0^\infty \int_{\mathbf{r}} \int_\Omega P(E,\ t,\ \mathbf{r},\ \theta)\, dt\, d\mathbf{r}\, d\theta,$$

$$P_1(E) = \int_0^\infty \int_{\mathbf{r}} \int_\Omega P(E, t, \mathbf{r}, \theta)\, \theta^2 dt\, d\mathbf{r}\, d\theta. \qquad (8')$$

Hence

$$\overline{\theta^2} = P_1(E) / P_0(E). \qquad (8'')$$

Next, multiplying (7) by $\theta \cdot \mathbf{r}$ and carrying out the same integrations, we obtain

$$P_2(E) - L[P_2(E')] = P_1(E), \qquad (9)$$

$$P_2(E) = \int_0^\infty \int_{\mathbf{r}} \int_\Omega P(E,\ t,\ \mathbf{r},\ \theta)(\theta \mathbf{r})\, dt\, d\mathbf{r}\, d\theta; \qquad (9')$$

and finally, multiplying (7) by $\mathbf{r}^2$ and integrating over all space and all the solid angles, we get

$$P_3(E) - L[P_3(E')] = 2P_2(E), \qquad (10)$$

$$P_3(E) = \int_0^\infty \int_{\mathbf{r}} \int_\Omega P(E,\ t,\ \mathbf{r},\ \theta)\, r^2 dt\, d\mathbf{r}\, d\theta. \qquad (10')$$

The mean square of the deviations $\overline{\mathbf{r}^2}$ is given by

$$\overline{r^2} = P_3(E) / P_0(E). \qquad (10'')$$

Thus, the problem of determining $\overline{\theta^2}$ and $\overline{\mathbf{r}^2}$ reduces to a successive solution of very complicated integral equations (8), (9), and (10). These equations can be solved if we know the function $P_0(E)dE$, i.e., if we know the differential energy spectrum of the nuclear-active particles in the EAS. For this function we can use the expression

$$P_0(E)\, dE = AE^{-2} dE, \qquad A = \text{const}, \qquad (11)$$

which follows both from an examination of the altitude variation of the EAS under a variety of assumptions regarding the character of the elementary act,[8] and from experimental data.[2,10]

After substituting (11) in (8) we obtain

$$P_1(E) - L[P_1(E')] = \left(\frac{b\mu c^2}{E}\right)^2 \frac{A}{E^2}$$

$$+ \left(\frac{p_\perp c}{E}\right)^2 \int_E^\infty \frac{A}{E'^2} \varphi_L(E',\ E)\, dE'. \qquad (12)$$

The solution of Eqs. (12), (9) and (10), carried out by the method of successive approximations, has made it possible to obtain the following expressions* for $\overline{\theta^2}$ and $\overline{\mathbf{r}^2}$:

$$\overline{\theta^2} \approx 1.1\, (\mu c^2 / E)^2 [b^2 + 0.7\, (p_\perp / \mu c)^2], \qquad (13)$$

$$\overline{r^2} \approx 3.0\, (\mu c^2 / E)^2 [b^2 + 0.7\, (p_\perp / \mu c)^2]. \qquad (14)$$

For a comparison with the experimental data, it is necessary to obtain an expression for the mean-squared radius of particles of energy greater than specified, $\overline{\mathbf{r}^2}(\geq \mathrm{E})$, since this quantity can be estimated experimentally.

By definition

$$\overline{r^2}(\geqslant E) = \int_E^\infty P_3(E')\, dE' \Big/ \int_E^\infty P_0(E')\, dE'. \qquad (15)$$

Substituting in (15) the values of $P_3(E)$ and $P_0(E)$, we obtain

$$\overline{r^2}(\geqslant E) \approx (\mu c^2 / E)^2 [b^2 + 0{,}7\, (p_\perp / \mu c)^2]. \qquad (16)$$

We must qualify that the expressions obtained for $\overline{\theta^2}$, $\overline{\mathbf{r}^2}$, and $\overline{\mathbf{r}^2}(\geq \mathrm{E})$ are valid at very high energies ($E \gtrsim 5 \times 10^{11}$ ev), since we did not take into account the spontaneous decay of the pions.

To compare the value obtained for $\overline{\mathbf{r}^2}(\geq \mathrm{E})$ with the experimental data, it is necessary to know the value of the diffraction parameter b. The value of b cannot be determined theoretically since the structure of the nucleon is unknown, and consequently we do not know the character of the "transparency" of the edge of the nucleus to fast pions. It is sensible to assume, however, that $b \approx 3$ (this corresponds to a smearing of the nuclear edge $\sim \hbar/Mc$, where M is the nucleon mass). In this case $[\overline{\mathbf{r}^2}(\geq 10^{12}\ \mathrm{ev})]^{1/2} \approx 0.6$ m for an altitude of 3,860 m above sea level (Pamir). We assume here that $p_\perp \approx 3\mu c$, if we are to follow the hydrodynamic theory of multiple production.[5] This corresponds to a hydrodynamic system decay temperature $T_K \approx \mu c^2$ (the transverse momentum acquired by the particles through expansion of the hydrodynamic system at primary-particle energies $\sim 10^{13}$ ev is much less than the transverse momentum obtained in thermal motion[9]).

From the results obtained in the investigation of the energy characteristics of the nuclear-active

*The computational accuracy of (13) and (14) is not lower than 10%.

component in the region of the core of the EAS,[10] it follows that $[\overline{r^2}(\gtrsim 10^{12}\text{ ev})]^{1/2} \gtrsim 1$ m. Thus, the experimental and theoretical values of the mean-squared radius are quite close to each other, although the experimental value is somewhat higher. There is little likelihood of attributing this difference to an underestimate of b (when $b \approx 6$ we have $\overline{r^2_{\exp}} \approx \overline{r^2_{\text{theor}}}$). It is natural to assume that the transverse momentum $p_\perp$, acquired by the secondary particles during multiple production, is in fact higher for the faster particles than follows from the hydrodynamic theory, although $p \approx 3\mu c$ for the overwhelming majority of the secondary particles (this is confirmed by emulsion data[11]). In addition, the fastest particle can be a nucleon[12] with a transverse momentum considerably greater than $3\mu c$.

4. We see from the formula for $\overline{r^2}$ that for particles of energy $\gtrsim 5 \times 10^{11}$ ev, r enters into the lateral distribution function of the nuclear-active particles only in the combination rE. Since particles of such energies are observed only near the axis of the shower, we can use the Pomeranchuk-Migdal method[13,14] to calculate the lateral distribution function of the flux density of the nuclear-active particles. We seek a distribution function in the form

$$P(E, r, t) = P(E, t)\, F(rE / kE_\alpha), \tag{17}$$

where $P(E, t)$ is the total number of nuclear-active particles with energies in the interval E, E + dE, at a depth t; E_α determines the value of $(\overline{r^2})^{1/2}$, and k = const. The normalizing factor A is determined from the condition

$$2\pi A \int_0^\infty F(rE / kE_\alpha)\, r\, dr = 1,$$

hence

$$A = E^2 / 2\pi (kE_\alpha)^2. \tag{18}$$

For the flux density of nuclear-active particles with energy $\geq E$ we have

$$\rho(E, r, t) = \frac{1}{2\pi (kE_\alpha)^2} \int_E^{E_0} P(E', r, t) E'^2 F\left(\frac{rE'}{kE_\alpha}\right) dE'. \tag{19}$$

To determine the functions $P(E, t)dE$ we make use of the analytic expression derived by Fukuda, Ogita, and Ueda,[3]

$$P(E, t')\, dE = \frac{(1-\delta)\nu}{[4\pi\sqrt{(1-\delta)\nu t' y}]^{1/2}} \times \exp\{-t' + \delta y + 2[(1-\delta)\nu t' y]^{1/2}\}\, dy, \tag{20}$$

where $y = \ln(E_0/E)$, t' is the depth in units of range of nuclear interaction from the point of shower production to the observation level, ν is the ratio of charged nuclear-active particles to the total number of particles produced during the multiple-generation act, and δ is the fraction of the energy retained by the nucleus after each interaction; the values of ν and δ were chosen by comparing the altitude variation with experiment: $\nu \approx 2/3$, $\delta \approx 1/2$. The function $F(rE/kE_\alpha)$ was chosen to be

$$F(rE / kE_\alpha) = e^{-rE/kE_\alpha}.$$

Then $k = 1/\sqrt{6}$ [this follows from the condition $\overline{r^2} = (E_\alpha/E)^2$, and for $r \lesssim \sqrt{\overline{r^2}}$ the choice of the specific form of the function F is immaterial].

To compare the experimental data with the calculated lateral distribution, account must be taken of the fluctuations in the depth of the onset of the shower, namely, that showers with a total of N particles at the observation level can be produced by primary nuclear-active particles of different energies, interacting at different altitudes from the observation level. In calculating $\rho(E, r, t)$ we used the dependence of the total number of particles $N = \xi(E_0, t')$, which follows from calculations of the altitude variation of the EAS under the assumption that the hydrodynamic theory of multiple production is valid.[9] In this case the range for the interaction of nuclear-active particles with nuclei of air atoms was assumed to be 75 g/cm^2.

Figure 1 shows a comparison of the experimentally-obtained and theoretically-calculated lateral distributions for particles with energy $\gtrsim 5 \times 10^{11}$ ev, for the case $E_\alpha \approx 1.5 \times 10^9$ ev (corresponding to $b \approx 6$ and $p_\perp \approx 3\mu c$).

It must be noted that the total number of nuclear-active particles with energy $\geq 5 \times 10^{11}$ ev in a shower with a large number of particles, $N = 10^5$, obtained by integrating expression (20) with respect to the energy for the Pamir altitude, is about one-fourth the experimentally obtained value. The theoretically-calculated lateral distribution was therefore normalized to the experimental value relative to the total number of nuclear-active particles of energy $\geq 5 \times 10^{11}$ ev.

Let us consider now the dependence $\rho(E, r, t)$ for fixed values of r and t, i.e., let us determine the energy spectra of the nuclear-active particles at different distances from the shower axis. The results of the calculation are shown by the solid lines of Fig. 2, which shows also the experimental values of $\rho(\geq E)$ taken from the paper of Dovzhenko, Zatsepin et al.[10] The calculation was carried out for $E \approx 1.5 \times 10^9$ ev.

In conclusion, the authors consider it their

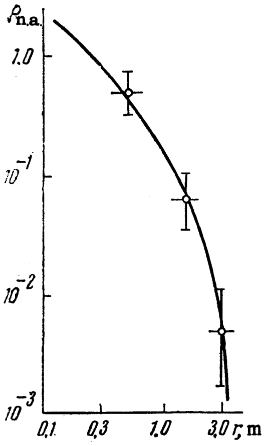

FIG. 1

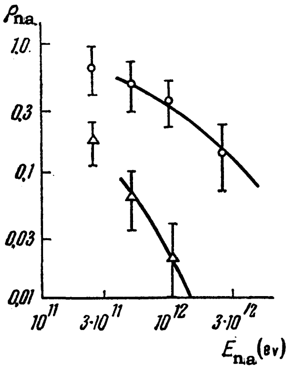

FIG. 2

FIG. 1. Lateral distribution of nuclear-active particles with energy $\geqslant 5 \times 10^{11}$ ev. The experimental points were borrowed from reference 10, and the solid curve $\rho_{n.a.}(r)$ is calculated by formulas (19) – (21).

FIG. 2. Energy spectra of nuclear-active particles for distances r from the shower axis ranging from zero to one meter (o) and from one to two meters (Δ) for a shower with a total number $N = 10^5$ particles.[10]

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