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ELECTRIC MONOPOLE TRANSITIONS IN THE THEORY OF NONAXIAL NUCLEI

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It is demonstrated that if the coupling between rotation and β vibrations is taken into account, electric monopole transitions become possible between nuclear rotational states possessing the same momenta and parities. The transition matrix elements between such states are calculated. The results are compared with the experiments.

THE theory of nonaxial nuclei, proposed by Davydov and Filippov[1] and developed in subsequent works, explains adequately the positions and many properties of the low-lying levels of even-even nuclei. According to this theory, for each non-zero J there exist several rotational states of given momentum and parity. It is interesting to consider the probability of electric monopole transitions between such states with emission of internal-conversion electrons. Church and Weneser[2] have shown that the E0-transition operator can be expanded in powers of the parameters $\alpha_{2\mu}$ of quadrupole deformation. Since the principal term in the expansion is a constant and does not produce any transitions (in view of the orthogonality of the wave functions of the initial and final states of the nucleus) and there is no linear term, it is necessary to take account of terms of higher order in $\alpha_{2\mu}$. The E0 transitions between the lower vibrational states with $J \neq 0$ are due here to terms of third and higher order in $\alpha_{2\mu}$. As noted by Grechukhin,[3] the E0-transition operator is a scalar, i.e., it is independent of the Euler angles that characterize the orientation of the nucleus in space, and consequently E0 transitions between rotational states are strictly forbidden in the adiabatic theory.

We investigate in this work the probability of E0 transitions between rotational states of nonaxial nuclei, with allowance for the coupling between the rotation and the β oscillations. It is assumed that the rotation and the β oscillations are adiabatically slow compared with the γ oscillations, but account is taken of the dependence of the equilibrium value of the nonaxiality parameter γ on β, indicated in the papers of Davydov and Filippov[4] and Wang Ling.[5]

The wave functions of the initial and final states of the nucleus are obtained from the equation

$$(\hat T_\beta + V_\beta + \hat H^r_{\beta\gamma} - E)\Psi = 0, \tag{1}$$

where

$$\hat T_\beta = \hbar^2 (2B\beta^2)^{-1} \frac{\partial}{\partial\beta}\left(\beta^2 \frac{\partial}{\partial\beta}\right), \qquad V_\beta = \frac{1}{2} C (\beta - \beta_0)^2,$$

$$\hat H^r_{\beta\gamma} = \hbar^2 (8B\beta^2)^{-1} \sum_{\varkappa=1}^{3} \hat J_\varkappa^2 \left[\sin\left(\gamma - \frac{2\pi\varkappa}{3}\right)\right]^{-2}. \tag{2}$$

If we replace in the operator $\hat H^r_{\beta\gamma}$ in Eq. (1) the values of β and γ in terms of the equilibrium values β_0 and $\gamma_0 \equiv (\beta_0)$, we can separate the variables in this equation, viz.,

$$(\hat T_\beta + V_\beta + \hat H^r_{\beta_0\gamma_0} - E^v_\nu - E^r_{J\tau})\, u_\nu(\beta)\, \varphi_{J\tau}(\theta_i) = 0, \tag{3}$$

where the vibrational wave function has the form

$$u_\nu(\beta) = \beta^{-1} H_\nu\left(\delta \frac{\beta - \beta_0}{\beta_0}\right) \exp\left[-\delta^2 (\beta - \beta_0)^2 / 2\beta_0^2\right], \tag{4}$$

and the vibration energy is $E^V_\nu = \hbar\omega_V(\nu + 1/2)$ (see, for example, reference 6). Here $\delta = (\hbar^{-2}BC)^{1/4} = (B\beta^2\hbar^{-1}\omega_V)^{1/2}$ is a dimensionless parameter, which can be determined if the energy of the first excited state with spin 0^+ is known; H_ν is the Hermite function of the first kind. The parameter ν is obtained from the condition that the wave function is bounded when $\beta = 0$, i.e., from the condition $H_\nu(-\delta) = 0$. The value of the rotation energy $E^r_{J\tau}$ and the rotational wave functions $\varphi_{J\tau}$ are given in the papers by Davydov and Filippov[1] and by Davydov and the author.[7]

Since the total momentum J is conserved, we seek the solution of (1) in the form of a superposition of rotation-vibration functions with given J:

$$\Psi_{\nu J\tau} = \sum_{\nu',\tau'} A^{\nu J\tau}_{\nu' J\tau'} u_\nu(\beta)\, \varphi_{J\tau}(\theta_i). \tag{5}$$

Then in the first approximation of perturbation theory

$$A^{\nu J\tau}_{\nu' J\tau'} = (E^v_\nu - E^v_{\nu'} + E^r_{J\tau} - E^r_{J\tau'})^{-1} \langle u_\nu \varphi_{J\tau} | \hat{H}^r_{\beta\gamma} - \hat{H}^r_{\beta_0\gamma_0} | u_{\nu'} \varphi_{J\tau'} \rangle, \ \nu'\tau' \neq \nu\tau; \quad A^{\nu J\tau}_{\nu J\tau} = 1. \tag{6}$$

Expanding the perturbation operator and retaining the first-order terms, we obtain

$$\hat{H}^r_{\beta\gamma} - H^r_{\beta_0\gamma_0} = (\beta - \beta_0)\beta_0^{-1}[-2\hat{H}^r_{\beta_0\gamma_0} + \varepsilon\hat{F}], \tag{7}$$

where

$$\varepsilon = \beta\partial\gamma / \partial\beta|_{\beta=\beta_0,\ \gamma=\gamma_0}, \qquad \hat{F} = \partial\hat{H}^r_{\beta\gamma} / \partial\gamma|_{\beta=\beta_0,\ \gamma=\gamma_0}.$$

As shown by Chaban,[8] the distinction between rotational and vibrational motion is meaningful only when $\delta > 2$. We confine ourselves to these values of δ. Then when $n < 3$ the Hermite function of the first kind, H_{ν_n}, differs little from the Hermite polynomials with corresponding integral indices, H_n, and the differences $\nu_{n+1} - \nu_n - 1$ are small and tend rapidly to zero with increasing δ. Therefore, in calculating the matrix elements we can assume

$$H_{\nu_{n+1}} = H_{\nu_n+1}. \tag{8}$$

The perturbation operator (7), which is proportional to $(\beta - \beta_0)/\beta_0$, has here non-vanishing matrix elements only when $\nu - \nu' = \pm 1$.

Let us proceed to calculate the matrix element of the E0 transition. For collective models of the nucleus, the operator of the E0 transition, accurate to second-order terms in $\alpha_{2\mu}$ (with allowance for the constant volume of the nucleus), has the form

$$\widehat{E0} = N\frac{3Z}{4\pi R^3}\int_V \left(\frac{r}{R}\right)^2 dV = N\frac{3Z}{4\pi}\left(\frac{4\pi}{5} + \beta^2\right),$$

$$N = \tfrac{1}{6}e^2\Phi_i(0)\Phi_f(0)R^2, \qquad \beta^2 = \sum_{\mu=-2}^{2}|\alpha_{2\mu}|^2. \tag{9}$$

Here N is a factor defined by the electron wave functions, Φ_i and Φ_f are the radial parts of the wave functions of the initial and final states of the electron, Z is the charge, and R is the radius of the nucleus.

The matrix elements of the transition between different rotational states, with allowance for the orthogonality of the rotational functions $\varphi_{J\tau}$, are written as

$$\langle \Psi_{pJ\tau_1} | \widehat{E0} | \Psi_{qJ\tau_2} \rangle = N\frac{3Z}{4\pi}\sum_{\nu,\nu',\tau} A^{pJ\tau_1}_{\nu J\tau} A^{qJ\tau_2}_{\nu' J\tau} \langle u_\nu | \beta^2 | u_{\nu'} \rangle. \tag{10}$$

Let us consider the transitions between two lower levels of the nucleus with a given non-zero spin J. According to the theory of nonaxial nuclei, these levels are referred to a single lower vibrational state ν_0. Taking condition (8) into account, we obtain from (10)

$$\langle \Psi_{\nu_0 J1} | \widehat{E0} | \Psi_{\nu_0 J2} \rangle = N\frac{3Z}{4\pi}\sqrt{2}\beta_0^2\delta^{-1}(A^{\nu_0 J1}_{\nu_1 J2} + A^{\nu_0 J2}_{\nu_1 J1}). \tag{11}$$

Using (6), (7) and (8) we find

$$A^{\nu_0 J1}_{\nu_1 J2} = -(\hbar\omega_v + E^r_{J2} - E^r_{J1})^{-1}(\delta\sqrt{2})^{-1}\varepsilon\langle\varphi_{J1}|\hat{F}|\varphi_{J2}\rangle. \tag{12}$$

The value of $A^{\nu_0 J_2}_{\nu_1 J_1}$ is obtained from (12) by interchanging the indices $\tau = 1$ and $\tau = 2$. Differentiating the equation $(\hat{H}^r_{\beta\gamma} - E^r_{J\tau})\varphi_{J\tau} = 0$ with respect to γ, we obtain

$$\langle\varphi_{J1}|\hat{F}|\varphi_{J2}\rangle = (E^r_{J2} - E^r_{J1})\langle\varphi_{J1}|\frac{\partial}{\partial\gamma_0}\varphi_{J2}\rangle. \tag{13}$$

Putting $x_J \equiv (\hbar\omega_v)^{-1}(E^r_{J_2}$ and $E^r_{J_1})$, we obtain finally

$$\langle \Psi_{\nu_0 21} | \widehat{E0} | \Psi_{\nu_0 22} \rangle = -N\frac{3Z}{4\pi}\beta_0^2\delta^{-2}\varepsilon\frac{2x_J}{1-(x_J)^2}\langle\varphi_{J1}|\frac{\partial}{\partial\gamma}\varphi_{J2}\rangle. \tag{14}$$

We note that (14) cannot be used for transitions between two lower levels with spin 0^+, since these levels pertain to different vibrational states. The matrix element of the transition between these levels, calculated in accordance with (10), is

$$\langle \Psi_{\nu_0 0} | \widehat{E0} | \Psi_{\nu_1 0} \rangle = 3ZN\sqrt{2}\beta_0^2\delta^{-1}/4\pi$$

and is independent of ϵ in first approximation.

For the case $J = 2$ we use the specific form of $\varphi_{2\tau}$ and $E^r_{2\tau}$, obtained by Davydov and Filippov.[1] We find

$$\langle\varphi_{21}\ \frac{\partial}{\partial\gamma}\varphi_{22}\rangle = \frac{4\sin^2 3\gamma}{9-8\sin^2 3\gamma}, \qquad \delta^{-2} = x_2\ \frac{2\sin^2 3\gamma}{3[9-8\sin^2 3\gamma]^{1/2}},$$

$$\langle \Psi_{\nu_0 21} | \widehat{E0} | \Psi_{\nu_0 22} \rangle = -N\frac{3Z}{4\pi}\beta_0^2\varepsilon\frac{16\sin^4 3\gamma_0}{3[9-8\sin^2 3\gamma_0]^{1/2}}\frac{x_2^2}{1-x_2^2}. \tag{15}$$

The final result contains the parameter $\epsilon = \beta\partial\gamma/\partial\beta|_{\beta=\beta_0,\gamma=\gamma_0}$. Theoretical estimates of this parameter can be obtained from the papers of Davydov and Filippov[4] and of Wang Ling.[5] The plot given by Wang Ling of the dependence of the equilibrium value of the nonaxiality parameter γ on $s = \log(4TB\beta^3\hbar^{-2})$ can be extrapolated, with good accuracy, by means of the parabola $\gamma = (s^2 - 14s + 41)\pi/180$. This yields $\epsilon(\gamma_0) = -0.045 \times \sqrt{8 + \gamma_0}$ (γ_0 in degrees). When γ_0 varies from 0 to 30°, ϵ changes from 0.13 to 0.28, i.e., merely by a factor of two.

The table lists the theoretical values of the matrix element ρ of the E0 transition between two lower levels with spin 2^+, calculated from (15):

$$\rho \equiv N^{-1}\langle\Psi_{\nu_0 21}\ \widehat{E0}\ \Psi_{\nu_0 22}\rangle, \qquad N = \tfrac{1}{6}e^2\Phi_i(0)\Phi_f^*(0)R^2.$$

The literature data on the energy levels are indicated in the last column of the table and the values of β_0 are taken from the survey article by Davydov,[9] while γ_0 and δ are calculated on the

Nucleus	β_0	γ_0	δ	$-\varepsilon$	ρ_{theor}	Reference
Cd^{114}	0.20	24	1.85	0.26	0.095	[12]
Gd^{154}	0.30	13	2.6	0.21	0.046	[13]
Er^{166}	0.33	13	4.5	0.21	0.006	[12-14]
Os^{188}	0.18	19	3.0	0.23	0.014	[13]
Pt^{194}	0.15	30	2.4	0.28	0.040	[16]
Pt^{196}	0.13	30	2.4	0.28	0.030	[12]
Hg^{198}	0.11	22	1.9	0.25	0.045	[12,15]

basis of the paper by Chaban,[8] with the energy corrected for non-adiabatic rotation. For Pt^{196}, the 0^+ level is unknown. It follows from $E_{22}/E_{21} = 1.93 < 2$, that the non-adiabatic corrections are strong here, $\delta \sim 2$, and $\gamma_0 = 30°$. In the calculation of ρ_{theor} we used $\delta = 2.4$, obtained for the neighboring even-even nucleus Pt^{194}, which has a similar arrangement of the lower 2^+ levels.

The results obtained indicate that electric monopole transitions between rotational states of nonaxial nuclei are possible and can serve as a criterion for the applicability of the adiabatic approximation. It is seen from (14) and (15) that the E0 transition must be sought primarily among the strongly-deformed nuclei with low-lying 0^+ level and small δ (we note that the non-adiabatic nature of the rotation influences these nuclei most strongly), and with large non-axiality (i.e., with $E_{22}/E_{21} \sim 2$).

Let us compare our results with the experimental data for Pt^{196} obtained by Gerholm and Peterson.[10] To determine ρ_{exp} they used the ratio of the reduced probabilities of the electric quadrupole transitions

$$B(E2;\ 22 \rightarrow 21)/B(E2;\ 21 \rightarrow 0) = 2,$$

given by the "free oscillation" theory of Scharf-Goldhaber and Weneser.[11] If we use the value 10/7 given by the theory of nonaxial nuclei for $\gamma = 30°$, the experimental value of ρ is found to be in the limits $0.013 \leq \rho_{exp} \leq 0.04$. The theoretical value $\rho_{theor} = 0.030$ is in good agreement with experiment.

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