# ON MUTUAL FRICTION IN HELIUM II

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A possible method is indicated for deciding experimentally the question of the existence of a component parallel to the axis of rotation in the mutual friction force between the superfluid and normal components of rotating helium II.

THE mutual friction force acting on a unit mass of the superfluid component from the direction of the normal component in rotating helium II has been examined in a number of papers.[1-3] The force:

$$\mathbf{F}_{sn} = -\frac{\rho_n}{2\rho} B' \boldsymbol{\omega}\times[\mathbf{v}_n - \mathbf{v}_s] - \frac{\rho_n}{2\rho} B \left[\frac{\boldsymbol{\omega}}{\omega}[\boldsymbol{\omega}\times(\mathbf{v}_n - \mathbf{v}_s)]\right], \quad (1)$$

where $\boldsymbol{\omega} = \text{curl}\, \mathbf{v}_s$ and B and B′ are the coefficients of mutual friction of Hall and Vinen. Both terms in (1) are perpendicular to $\boldsymbol{\omega}$.

The purpose of this note is to indicate the possibility of an experimental resolution of the question of the existence of a third term in the mutual friction force, parallel to $\boldsymbol{\omega}$, i.e., of the existence of an additional term in $\mathbf{F}_{sn}$ of the form

$$\frac{\rho_n}{2\rho} B'' \frac{\boldsymbol{\omega}}{\omega} [\boldsymbol{\omega}\times(\mathbf{v}_n - \mathbf{v}_s)]. \quad (2)$$

The coefficient B′′ can be determined from the damping of the oscillations of a cylinder along its axis (coincident with the axis of rotation of the fluid).* The hydrodynamic equations for rotating helium II are solved[4] with the following boundary conditions, in order to derive the corresponding equations:

$$v_{nr}(R) = 0, \quad v_{n\varphi}(R) = \omega_0 R, \quad v_{nz}(R) = i\Omega z_0 \exp(i\Omega t), \quad (3)$$

where R is the radius of the cylinder, $\omega_0$ the angular velocity of rotation, $\Omega$ the frequency of the oscillations and $z_0$ their amplitude.

This leads to the following expression for the force acting on the surface of unit length of an infinite hollow thin-walled cylinder, oscillating in a boundless liquid:

$$F_z = i2\pi R\eta_n \Omega\varkappa \left[\frac{H_1^{(1)}(\varkappa R)}{H_0^{(1)}(\varkappa R)} - \frac{J_1(\varkappa R)}{J_0(\varkappa R)}\right] z_0 \exp(i\Omega t), \quad (4)$$

where

$$\varkappa^2 = -\frac{i\Omega}{\nu_n}\left[1 + \left(\frac{\omega_0}{\Omega}\right)^2 \frac{\rho_n\rho_s B''^2/\rho^2}{1 + (\omega_0/\Omega)^2(\rho_n B''/\rho)^2} - i\frac{\omega_0}{\Omega}\frac{\rho_s B''/\rho}{1 + (\omega_0/\Omega)^2(\rho_n B''/\rho)^2}\right]$$

$$(\text{Im}(\varkappa) > 0). \quad (5)$$

Here $\eta_n$ and $\nu_n$ are the dynamic and kinematic viscosities of the normal component, and H and J are the Hankel and Bessel functions.

Equation (5) leads to a penetration depth $1/\text{Im}(\kappa) \sim \sqrt{2\nu_n/\Omega}$. For R ~ 1 cm, the quantity $\kappa R$ can therefore be considered large, and taking the asymptotic expansions of the Bessel functions we easily obtain the following equation:

$$\frac{\gamma_2 - \gamma_1}{l_2 - l_1} = \frac{\pi R\sqrt{2\eta_n\rho\Omega}}{m}\left(1 + \frac{\omega_0}{2\Omega}\frac{\rho_s}{\rho}B''\right), \quad (6)$$

which is valid for

$$(\omega_0\rho_n B''/\Omega\rho)^2 \ll 1, \quad (\omega_0/\Omega)^2\rho_n\rho_s B''^2/\rho^2 \ll 1,$$

$$R/\text{Im}(\varkappa) \gg 1.$$

Here $\gamma_2$ and $\gamma_1$ are the damping coefficients for the cylinder immersed to the depths $l_2$ and $l_1$: m is the mass of the oscillating system. Edge effects are automatically removed by subtracting $\gamma_1$ from $\gamma_2$. It is assumed that the oscillating system is "heavy" ($\Omega_2 = \Omega_1 = \Omega$).

It is convenient to use the following equation for determining B′′:

$$(\gamma_2 - \gamma_1)/(\gamma_2 - \gamma_1)_{\omega_0=0} = 1 + (\omega_0\rho_s/2\Omega\rho)B''. \quad (7)$$

If B′′ = 0, the damping is independent of the speed of rotation. If this is not the case we shall find a linear increase in damping with increasing $\omega_0$.

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*According to I. L. Bekarevich and I. M. Khalatnikov (private communication) an additional term should also be introduced into the force $\mathbf{F}_{sn}$ containing the product of $\omega$ and curl$(\boldsymbol{\omega}/\omega)$. Actually, in the case considered of the oscillations of a cylinder along the vortex lines, curl$(\boldsymbol{\omega}/\omega) = 0$.

[1] H. E. Hall and W. F. Vinen, Proc. Roy. Soc. **A238**, 204, 215 (1956).

[2] E. M. Lifshitz and L. P. Pitaevskiĭ, JETP **33**, 535 (1957), Soviet Phys. JETP **6**, 418 (1958).

[3] L. P. Pitaevskiĭ, Thesis, Institute for Physics Problems, U.S.S.R. Academy of Sciences (1958).

[4] Yu. G. Mamaladze and S. G. Matinyan, JETP **38**, 184 (1960), Soviet Phys. JETP **11**, 134 (1960).

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154