CONSERVATION LAWS IN THE GENERAL THEORY OF RELATIVITY

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To determine the integral of a vector function in Riemannian geometry, and a vector field corresponding to a displacement of the origin of coordinates, a geometric operation of "harmonic" translation of the vector is introduced, which is defined in a unique manner by means of the first-order generally-covariant linear differential equations (13). The covariant vector components do not change during harmonic translation in a harmonic coordinate system, and this enables one to integrate the vectors by components. Therefore the energy-momentum vector, energy-momentum pseudotensor, energy density, and Hamiltonian of the system should be computed in a harmonic system. For the canonical energy-momentum tensor a unique expression is obtained which goes over to the Landau-Lifshitz pseudotensor after symmetrization.

1. STATEMENT OF THE PROBLEM

Two points are unclear from a mathematical point of view about conservation laws in the general theory of relativity:

1. The definition of an integral over a vector function. In the energy-momentum vector of a system[1]

$$P_i = \int t_i^k dS_k \tag{1}$$

the expression under the integral is a vector. However, the operation of addition of two vectors at different points in space is not defined.[2] It is true that the generalized Stokes theorem[3] is treated in differential geometry; however, the integrand in the generalized Stokes theorem is a scalar (skewsymmetric differential form).

2. The conservation laws are a consequence of invariance under a continuous group of coordinate transformations — displacements or rotations.[4] If $\delta\omega^j$ are the parameters of an element of this group in the neighborhood of unity, then the corresponding coordinate transformation has the form[4,1]

$$\delta x^i = \xi^i(x) = x_j^i(x)\,\delta\omega^j, \tag{2}$$

however, it is not defined which functions $\xi^i(x)$ correspond to displacements.

Integrals of type (1) arise in the general theory of relativity in investigations of the conservation laws. If t_i^k is the energy-momentum pseudotensor, and the summation (integration) is carried out component-wise, while the coordinates are Euclidean at infinity, then the integral does not depend on the coordinate system. Similarly, if the quantities ξ^i tend to become constant as $x^\alpha \to \infty$, the integrals of the different energy-momentum tensors generated by (2) agree with each other. However, it is an unsatisfactory situation when mathematical operations are used which are not defined, and which acquire meaning only through the physical nature of the integrand. In our opinion the definitions of the integral and of the displacement are purely geometrical ones, and should be given independently of the physical content of the problem.

There are two points of view about this possibility: a) a covariant definition of the integral and the displacement is possible in the general theory of relativity, b) a generally covariant definition of the integral does not exist in a Riemannian space. In the present work we develop the first point of view for an isolated mass distribution, when the metric becomes Euclidean at infinity.

2. FREE-VECTOR FIELDS

In arbitrary curvilinear coordinates only vectors defined at the same point can be added. Therefore before performing the integration, all vectors must be transferred to one point, which we shall call the observation point x_0. It is then natural to define the integral by

$$P_i(x_0) = \int dP_i(x, x_0) = \int t_i^k(x, x_0)\,dS_k. \tag{3}$$

The integration in (3) is to be carried out over the points with coordinates x.

Consequently we must define an operation of transfer of a vector, which we shall call "harmonic" translation,* and which has the following properties: 1) The "harmonic" translation is unique and consequently must not depend on the path; 2) it is defined by a generally covariant linear differential equation of first order; 3) In Euclidean space the "harmonic" translation is identical with the usual parallel transport.

An arbitrary vector $P_i(x_0)$ which is given at some point can be transferred to any other point with the help of the harmonic translation. It thus defines a vector field $P_i(x)$:

$$P_i(x) = \hat{C}P_i(x_0), \tag{4}$$

where $\hat{C}$ denotes the operator of harmonic translation. Thus condition 1) implies the existence of the vector field (4), which we may call a field of free-vectors.

We now turn to Eq. (2). By the $\xi^i(x)$ in (2) we now mean a vector field that satisfies the following conditions: (a) $\xi^i(x, x_0)$ is a unique function of two points, the field point x, and the point x_0 from which the translation originated; (b) in Euclidean space the vectors $\xi^i(x)$ are parallel to each other. Clearly these conditions are satisfied by the field

$$\xi^i(x) = \hat{C}\xi^i(x_0). \tag{5}$$

Thus both problems posed in Sec. 1 have been reduced to a single geometric problem, that of finding the harmonic translation.

The harmonic translation must be defined uniquely, at least in topologically Euclidean spaces. It should be stressed that the harmonic translation differs from the parallel transport (Π transport), which is defined by somewhat different conditions: a) Π transport conserves the scalar product

$$\Pi(P_sQ^s) = (\Pi P_s)(\Pi Q^s); \tag{6}$$

b) In Euclidean space the Π transport is the usual parallel transport.

3. DEFINITION OF HARMONIC TRANSLATION

Consider first Euclidean space and introduce a Cartesian coordinate system. In this case the free vector clearly has constant components:

$$\partial P^s(x)/\partial x^k = 0; \quad s, k = 0, 1, 2, 3. \tag{7}$$

Equation (7) is of first order; therefore it is natural to demand that in the general case the harmonic translation also be defined by differential equations of first order. These equations must be linear: if ξ_1^i and ξ_2^i are two vector fields corresponding to a displacement of the origin, then the sum of these fields must correspond to a displacement of the origin by the vector $\xi_1^i(x_0) + \xi_2^i(x_0)$. The requirement of linearity also follows from the linearity of the operation of integration.

*This nomenclature was suggested by V. A. Fock.

In curvilinear coordinates in pseudo-Euclidean space, Eq. (7) takes the form

$$\nabla_k P^s(x) = 0, \tag{8}$$

where ∇_k denotes the covariant derivative. $P^s(x)$ is obtained by integrating Eq. (8); to do this, boundary conditions must be imposed, namely boundedness at infinity.

Let us study Eq. (8) in Riemannian geometry. Since the indices k and s are independent, Eq. (8) constitutes 16 conditions. If (8) is fulfilled, the curvature tensor vanishes because of the noncommutation of the covariant derivatives,

$$(\nabla_k\nabla_s - \nabla_s\nabla_k)P_i(x) = R^m_{iks}P_m = 0. \tag{9}$$

In the general theory of relativity condition (8) must therefore be replaced by weaker ones, which can be fulfilled even when $R^m_{iks} \neq 0$.

From the second-rank tensor $\nabla_k P_i$ one can construct the invariant

$$\zeta = \nabla_k P^k \equiv \operatorname{div} P \tag{10}$$

and introduce the symmetric and skewsymmetric parts

$$\xi_{ik} = \nabla_k P_i + \nabla_i P_k, \quad \eta_{ik} = \nabla_k P_i - \nabla_i P_k. \tag{11}$$

Thus the following generally covariant conditions can be imposed on the vector field $P_i(x)$:

$$\xi_{ik} = 0 \text{ (10 conditions)}. \tag{12}$$

These conditions have been investigated by Fock.[5] They can be fulfilled only in spaces of constant curvature, $\nabla_s R = 0$, and therefore must be rejected.

$$\zeta = 0, \ \eta_{ik} = 0 \text{ (7 conditions)}. \tag{13}$$

These conditions can be fulfilled for arbitrary R^m_{iks}. The solution of Eq. (13) is

$$P_k = \nabla_k\varphi = \partial\varphi/\partial x^k; \ \Box\varphi = 0. \tag{14}$$

At the point x_0 we must have

$$\left.\frac{\partial\varphi}{\partial x^k}\right|_{x=x_0} = P_k(x_0), \tag{15}$$

in order that

$$P_k(x) = \hat{C}P_k(x_0). \tag{16}$$

To calculate the integral (1) one must translate the integrand harmonically from the field point to the point x_0. Hence Eq. (14) must be integrated for every field point x. This is a rather cumbersome operation.

In Euclidean space a vector with vanishing curl and divergence, and bounded at infinity, is constant. Therefore the requirement (2) for "harmonic" translation is also fulfilled. In Euclidean space the harmonic translation is identical with parallel transport. The definition of harmonic translation introduced above is feasible only if one confines attention to first-order linear differential conditions.

4. A PREFERRED COORDINATE SYSTEM

Up to this point all equations were generally covariant. In Euclidean space a "preferred" coordinate system exists, namely, a Galilean system, in which the components of a vector remain unchanged during parallel transport. In this coordinate system addition and integration of vectors at different points can be performed component-wise. In the present section such a preferred system will be defined for spaces of arbitrary curvature.

Let us find the conditions that define the class of coordinate systems in which the components of a covariant field of "free" vectors [in the sense of Eq. (13)] are constant:

$$P_i(x) = \hat{C}P_i(x_0) = P_i(x_0). \quad (17)$$

Consider again Eq. (14). We have

$$\operatorname{div} P = \Box\varphi = \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^i}(\sqrt{-g}\,g^{ik}P_k) = 0, \quad \frac{\partial P_k}{\partial x^i} = 0, \quad (18)$$

hence

$$\partial(\sqrt{-g}\,g^{ik})/\partial x^i = 0, \qquad g^{im}\Gamma^k_{im} = 0. \quad (19)$$

Condition (19) defines the class of "harmonic" coordinate systems.[5] Subject to corresponding boundary conditions, these systems are defined uniquely up to Lorentz transformations.

We note that the components of a contravariant vector do not remain constant after the translation:

$$P^i(x) = \hat{C}P^i(x_0) = g^{is}(x)\,g_{si}(x_0)\,P^i(x_0). \quad (20)$$

One can show that in general no coordinate systems exist in which the contravariant components remain constant under harmonic translation.

In this section we have found the "preferred" coordinate system for a covariant vector. We write the word "preferred" in quotes since we mean by this only that addition of vectors at different points can be performed by components in this coordinate system. Thus in arbitrary coordinate systems the translation is generally covariantly defined by Eq. (13). In a harmonic coordinate system these differential conditions can be integrated and lead to formulas (17) and (20).

The situation with respect to "preferred" coordinate systems in the general theory of relativity is entirely analogous to the situation in electrodynamics. The equations of electrodynamics can be written down and solved in an arbitrary coordinate system. The calculation of integrals over vectors or tensors is carried out in the "preferred" Galilean coordinate systems, where vectors can be added by components.

The class of harmonic coordinate systems has been thoroughly investigated.[5] In particular, the uniqueness proof for harmonic translation reduces to the uniqueness proof for harmonic coordinates. This question was investigated in detail by Fock (see reference 5, Sec. 93), and we shall not treat it here. The question of the existence and uniqueness of the solution of Eqs. (14) and (15) and the precise mathematical formulation of boundary conditions in finite-sized regions or in a space of non-Euclidean topology will not be considered in the present paper.

5. CONSERVED QUANTITIES

So far our reasoning was purely geometric in character and the formulas we developed were considered independent of the physical nature of the vectors. Consider now an integral of type (1)

$$P_i(x_0) = \int_\sigma t_i^k dS_k = P_i(x_0, \sigma), \quad (21)$$

where the integration extends over a hypersurface σ. The integrand is a function of two points: the point x_0, where all vectors are collected and where their component-wise summation is carried out, and the variables of integration, x. As was pointed out in Sec. 4, to be able to integrate by components, we must use a harmonic coordinate system.

The vector $P_i(x_0, \sigma)$ is said to be conserved if, for fixed x_0, it does not depend on the hypersurface σ. It is usual to require that P_i be independent of the hypersurface of integration; the stipulation concerning a fixed x_0 is necessary in general relativity, where measuring rods and clocks may change from point to point, i.e., where the units of space and time measurement, and, consequently, any units in which P_i is measured, may change. We note that the point x_0 may also lie outside the hypersurface σ.

It follows immediately from the definition of conserved quantities that the tensor t_i^k satisfies the condition

$$\partial t_i^k(x, x_0)/\partial x^k = 0. \quad (22)$$

Formula (22) is a consequence of integration by components and therefore is correct for non-translated quantities only in harmonic coordinates.

6. INFINITESIMAL COORDINATE TRANSFORMATIONS

Conservation laws are a consequence of the invariance of the action under infinitesimal coordinate transformations. Let the action be expressed by the integral

$$S = \int \mathscr{L}\, d^4x, \quad \mathscr{L} = \mathscr{L}_0 \sqrt{-g}, \tag{23}$$

where the Lagrangian density $\mathscr{L}_0$ is a function of the field variables and their first derivatives:

$$\mathscr{L}_0 = \mathscr{L}_0 (u^A_k;\, u^A_{,k}), \quad u^A_{,k} = \partial u^A / \partial x^k \tag{24}$$

and does not contain the coordinates x explicitly (the index A denotes the totality of the tensor indices of the field). Consider the infinitesimal coordinate transformation

$$\xi^i (x) = x^i_j (x, x_0)\, \xi^j (x_0), \tag{25}$$

corresponding to a displacement of the origin x_0 by the infinitesimal vector $\xi^j (x_0)$, which characterizes the direction of the displacement.

The change of the field functions at some point in space will be[1,4]

$$\delta^* u^A = (\psi^A_j - x^k_j\, \partial u^A / \partial x^k)\, \xi^j (x_0). \tag{26}$$

Here $\delta^* u^A$ is the variation of the field function at the original point in space, which has new coordinates after the transformation, ψ^A_j is the matrix that relates total variation of the field variables (due to polarization properties as well as due to the displacement to the new point) with the displacement of the origin.[1,2,4] The canonical energy-momentum tensor is then defined by the relation by the relation

$$\Theta^k_j (x, x_0)\, \xi^j (x_0) = -\delta^* u^A\, \partial \mathscr{L} / \partial u^A_{,k} - \mathscr{L} x^k_j \xi^j (x_0) \tag{27}$$

and satisfies Eq. (22):

$$\partial \Theta^k_j / \partial x^k = 0. \tag{28}$$

The integral over the hypersurface

$$P_j (x_0, \sigma) = \int \Theta^k_j\, dS_k \tag{29}$$

is conserved. The integration in (29) is carried out component-wise. The quantities Θ^k_j in Eqs. (28) and (29) are referred to the observation point x_0, where x_0 may be located anywhere, not only at infinity.

The physical significance of the relationships obtained in this way depends on the coordinate transformation being considered. If one adopts the point of view expressed above, that the coordinate transformation corresponding to displacement or rotation must be uniquely defined, then the tensor Θ^k_j will also be unique.

7. THE GRAVITATIONAL FIELD

The general formula (27) is correct for an arbitrary field. Let us apply it to the gravitational field. The simplified action for the gravitational field is

$$\mathscr{L}_0 = g^{ik} (\Gamma^s_{im} \Gamma^m_{ks} - \Gamma^s_{ik} \Gamma^m_{sm}). \tag{30}$$

According to Sec. 4, the operation of harmonic translation can be written in a finite, non-differential form only in a system of harmonic coordinates. Therefore $\mathscr{L}_0$ should also be calculated in harmonic coordinates. The second term in (30) the vanishes, and

$$\mathscr{L}_0 = g^{ik} \Gamma^s_{im} \Gamma^m_{ks}. \tag{31}$$

Since harmonic coordinates form an affine group, $\mathscr{L}_0$ is a true scalar in harmonic coordinates. The components of the metric tensor g^{ik} are the field variables. The quantity $\Theta^0_0 (x)$ is the energy density of the system, or the Hamiltonian density. We note that Dirac[6] has pointed out the necessity of fixing the coordinates for a Hamiltonian formulation.

For infinitesimal coordinate transformations[2] we have

$$\delta^* g^{ik} = -(\nabla^i \xi^k_{\cdot} + \nabla^k \xi^i) = -\xi^{ik}, \tag{32}$$

where ξ^i is defined by Eq. (2). In the literature the case $\xi^i = \text{const}$ is usually discussed, and the coordinate system is not restricted. This leads to Einstein's canonical pseudotensor

$$t^k_i = g^{ml}_{,i}\, \partial \mathscr{L} / \partial g^{ml}_{,k} - \mathscr{L} \delta^k_i. \tag{33}$$

However, the vector field $\xi^i = \text{const}$ cannot be defined generally covariantly by means of first-order differential conditions.

We demand that the vector field $\xi^i (x)$ be derived from the vector of the displacement of the origin $\xi^i (x_0)$ by harmonic translation. In harmonic coordinates

$$x^i_j (x, x_0) = g^{is} (x)\, g_{sj} (x_0). \tag{34}$$

The change in field variables must be calculated from Eq. (32):

$$\delta^* g^{ik} = -2\, \Gamma^{ik,\, l} (x)\, g_{lj} (x_0)\, \xi^j (x_0), \tag{35}$$

so that the energy-momentum tensor equals

$$t_j^k = 2\frac{\partial \mathcal{L}}{\partial g^{lm}_{,k}}\Gamma^{ml,s}(x)\,g_{sj}(x_0) - \mathcal{L}g^{ks}(x)\,g_{sj}(x_0)$$

$$= t^{ks}(x)\,g_{sj}(x_0). \qquad (36)$$

The infinitesimal coordinate transformation (34) can be written in the form

$$\xi^i(x) = g^{is}(x)\,\xi_s, \quad \xi_s = \text{const.} \qquad (37)$$

As Bergmann[7] showed, the transformation (37) leads to a tensor t^{ks} which, after addition of a strictly conserved quantity, becomes the tensor of Landau and Lifshitz. Thus the geometrical definition of the displacement leads to the tensor of Landau and Lifshitz, and not to that of Einstein.

As follows from the derivation, the quantities t_i^k and, consequently, the Hamiltonian[6] must be calculated in harmonic coordinates. Therefore the question raised again recently by Møller about the localization of the energy-momentum of the gravitational field makes sense, although, in our opinion, Møller's solution is wrong. Consider two mechanical oscillators much less than a wavelength apart, so that oscillation of one excites the other via gravitational interaction. The energy transfer calculated in the nonrelativistic approximation, using the canonical symmetric tensor, agrees with the result obtained from the equations of motion; the same calculation using Møller's tensor gives a zero result. For the Schwarzschild solution, Møller's energy density vanishes in any coordinate system, whereas the canonical energy density in harmonic coordinates is positive definite.

The principal objection against the Landau-Lifshitz energy-momentum tensor was, of course, the incorrect weight.[8,9] However, it is clear from Eqs. (33) and (36) that the canonical tensors have the same weights. Goldberg's conserved expressions of arbitrary weight are not connected with infinitesimal coordinate transformations and must be rejected.

8. THE GRAVITATIONAL FIELD AS A PERTURBATION

Let us analyze the case when the gravitational field can be considered a small perturbation, and compare the results of calculations in different coordinate systems. It is well known that the scattering angle of light in a centrally-symmetric field is given by

$$\theta = 4\varkappa M/c^2R, \qquad \theta \ll 1, \qquad (38)$$

where R is the distance of closest approach, which in general relativity depends sensitively on the coordinate system employed.[10] From (38) we obtain for the effective scattering cross section, for $\theta \ll 1$:

$$d\sigma = \frac{R(\theta)}{\sin\theta}\left|\frac{dR}{d\theta}\right| do = 16\left(\frac{\varkappa M}{c^2}\right)^2 \frac{do}{\theta^4}. \qquad (39)$$

There is a difference in principle between Eqs. (38) and (39). The latter depends only on the scattering angle θ, which is measured away from the gravitating masses, at infinity, where the metric is Galilean, whereas (38) contains the distance of closest approach, a quantity measured near the mass.

Let us find the effective scattering cross section using the methods of quantum-mechanical perturbation theory. The Lagrangian density for the electromagnetic field equals

$$\mathcal{L} = \frac{1}{8\pi}F_{ik}^2 = \frac{1}{8\pi}g^{is}g^{km}F_{ik}F_{sm}. \qquad (40)$$

For a weak field we have, in the notation of reference 2,

$$\mathcal{L} = \mathcal{L}_0 + \Delta\mathcal{L} = \frac{1}{8\pi}F_{ik}F^{ik} + \frac{1}{4\pi}h_m^kF^{im}F_{ik}. \qquad (41)$$

The second summand in (41) can be considered as the interaction Lagrangian of the electromagnetic and gravitational fields. In (41) the indices are raised and lowered with the zero-order metric tensor $\overset{(0)}{g}_{ik}$.

The small additions to the Lagrangian and Hamiltonian have opposite signs:[2]

$$\Delta\mathcal{H} = -\Delta\mathcal{L} = -\frac{1}{4\pi}h_m^kF^{im}F_{ik}. \qquad (42)$$

For a weak gravitational field in harmonic coordinates we have

$$h_\alpha^\beta = -2c^{-2}\delta_\alpha^\beta\varphi, \qquad h_0^0 = 2c^{-2}\varphi, \qquad h_\alpha^0 = 0, \qquad (43)$$

where φ is the gravitational potential. Thus we find, for a light wave

$$\Delta\mathcal{H} = \frac{1}{4\pi c^2}\varphi(\mathbf{E}^2 + \mathbf{H}^2) = \frac{2}{c^2}\frac{\varphi}{8\pi}(\mathbf{E}^2 + \mathbf{H}^2). \qquad (44)$$

The quantity $(\mathbf{E}^2 + \mathbf{H}^2)/8\pi c^2$ can be treated like an inertial and gravitational mass density of the photon. The factor 2 corresponds to the well-known fact that the angle of deflection of light in the general theory of relativity is twice as large as in the special theory.

We shall consider the interaction Hamiltonian (44) as a perturbation, where for F_{ik} we must substitute the wave functions in zeroth order, namely plane waves. If we let the normalization volume be Ω, we find the normalized wave functions:

$$\mathbf{E} = \sqrt{4\pi\hbar\omega/\Omega}\,\mathbf{e}_{\mathbf{k}} e^{i\mathbf{kr}},$$

$$\langle \Delta \mathscr{H}_{12} \rangle = \frac{2}{c^2} \frac{\hbar\omega}{\Omega} \int \exp\{i(\mathbf{k}_1 - \mathbf{k}_2)\,\mathbf{r}\}\, dv = \frac{8\pi}{\Omega} \frac{\varkappa M}{c^2} \frac{\hbar\omega}{k^2\theta^2}, \quad (45)$$

and we can easily see that the scattering cross section agrees with the classical result:

$$d\sigma = \frac{2\pi}{\hbar c} \langle \Delta \mathscr{H}_{12} \rangle^2 \frac{\Omega^2}{(2\pi)^3} k^2 \frac{dk}{\hbar\, d\omega}\, do = 16 \left(\frac{\varkappa M}{c^2}\right)^2 \frac{do}{\theta^4}. \quad (46)$$

The whole calculation could have been performed by perturbing the wave equation. The quantum notation was used only for brevity and because it is more familiar.

We show that $\langle \Delta \mathscr{H}_{12} \rangle$ depends on the coordinate system chosen. When applying perturbation theory one must use plane waves as wave functions in the zero-order approximation. In flat space and Galilean coordinates $h_{ik} \equiv 0$ and $\langle \mathscr{H}_{12} \rangle = 0$. After an infinitesimal coordinate transformation we have

$$\delta \langle \Delta \mathscr{H}_{12} \rangle = -\frac{1}{4\pi} \overset{(0)}{F}{}^{mi} \overset{(0)}{F}{}^{k}_{m\ldots} \int e^{i(\mathbf{k}_1 - \mathbf{k}_2)\mathbf{r}} (\nabla_i \xi_k + \nabla_k \xi_i)\, dV \neq 0, \quad (47)$$

and, therefore, an arbitrary coordinate system does not satisfy the correspondence principle, and we must use the harmonic system for the calculations as we have concluded above.

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