CONTRIBUTION TO THE THEORY OF MAGNETOACOUSTIC RESONANCE IN METALS

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A new effect of resonance absorption of ultrasonic waves in a magnetic field is studied theoretically.* In many respects the resonance mechanism is similar to cyclotron resonance, but is related to the spatial rather than temporal periodicity of the field in the metal. The calculations are performed for closed as well as for open electron trajectories. The positions, widths, and heights of the resonance peaks are determined as a function of the frequency, magnetic field, Fermi surface structure, magnetic field orientation and direction of propagation of the sound relative to the crystallographic axes. A sharp angular dependence of the absorption coefficient has been detected. An investigation of the effects under consideration may yield information on the topology and a number of other important characteristics of Fermi surfaces in metals. The results of the experiments are in good agreement with the predictions of the theory.

1. MECHANISM OF RESONANCE ABSORPTION

SOUND oscillations produce in a metal a spatially-periodic field. In the presence of a magnetic field $\mathbf{H}(0, 0, H)$, this periodicity leads to a non-monotonic dependence of the coefficient of ultrasound absorption α on H. This effect was first explained by Pippard,[1] who determined the period of the oscillations of α from qualitative considerations.

V. Gurevich[2] developed the theory of this phenomenon for closed Fermi surfaces and showed that the periodic variation of α as a function of H^{-1} can be of two kinds, namely harmonic oscillations (which we shall call nonresonant) and smooth increments that occur periodically. These periodic variations of α with H^{-1} have no resonant character in the sense that the relative widths and the forms of the absorption 'lines' remain practically unchanged as the mean free path l tends to infinity.

In the present paper we consider an essentially new effect of resonance absorption of ultrasound in metals. The mechanism of this magnetoacoustic resonance is analogous in many respects to the mechanism of cyclotron resonance in metals,[2] but is connected not with the temporal but with the spatial periodicity of the field in the metal. The magnetoacoustic resonance should take place when the average value (over the period T of the motion in the magnetic field) of the electron velocity along the wave vector $\mathbf{k}$ differs from zero. For closed trajectories this can be realized only when $\mathbf{k}$ is not perpendicular to $\mathbf{H}$. On the open periodic trajectories, resonance should be observed also when $\mathbf{k} \perp \mathbf{H}$, provided the vector $\mathbf{k}$ is not parallel to the direction of the open trajectory.

If the condition $\mathbf{k}\cdot\overline{\mathbf{v}}T \approx 2\pi n$ is satisfied [$n = 1, 2, 3 \ldots$ is an integer, and the bar denotes averaging over the period of the trajectory $p_z = \text{const}$ on the Fermi surface $\epsilon(\mathbf{p}) = \mu_0$], the electron is periodically accelerated by the field in the equal-phase planes, where it remains for a relatively longer time than in the remaining parts of the trajectory. Clearly, the longer the mean free path, the more effective will be the interaction between the electron and the field in these planes.

The dependence of $\mathbf{k}\cdot\overline{\mathbf{v}}T$ on p_z is, however, quite appreciable. If $\mathbf{k}\cdot\overline{\mathbf{v}}T$ is independent of p_z, then the condition $\mathbf{k}\cdot\overline{\mathbf{v}}T = 2\pi n$ will be satisfied simultaneously for all electrons. This is precisely the case realized on open periodic trajectories, when $\mathbf{k} \perp \mathbf{H}$ and the angle θ between $\mathbf{k}$ and the direction of the open trajectory is not small, so that $\mathbf{k}\cdot\overline{\mathbf{v}}T \sim kr \sin\theta \gg 1$. Here the resonance effect will be greater than in the case when $\mathbf{k}\cdot\overline{\mathbf{v}}T$ depends appreciably on p_z. The dependence on p_z leads to a smearing and reduction in the height of the resonance maximum. It is obvious that the electrons for which $\mathbf{k}\cdot\overline{\mathbf{v}}T$ has an extremum with

*A preliminary report on this phenomenon was published earlier.[11]

respect to p_z will be relatively more numerous than the other electrons, and therefore the amplitude of resonant absorption will be a maximum precisely for such "extremal" electrons, although its magnitude will be still less than when $\mathbf{k}\cdot\overline{\mathbf{v}}T = \text{const}$.

We emphasize that Gurevich[2] did in fact not consider this effect.

Along with magnetoacoustic resonance, the absorption coefficient should be highly anisotropic when the vectors **k** and **H** become slightly nonperpendicular, and also when the vector **k** deviates slightly from the direction of the open periodic trajectory. The reason for this anisotropy is that when $\mathbf{k}\cdot\overline{\mathbf{v}} \equiv 0$ all the electrons fall periodically into the equal-phase plane and participate effectively in the absorption, whereas the absorption when $\mathbf{k}\cdot\overline{\mathbf{v}} \not\equiv 0$ is essentially determined only by the electrons that drift most slowly along **k**. This circumstance brings about a fast reduction in α.

The strong angular dependence of the absorption when $\mathbf{k} \perp \mathbf{H}$, and the presence of magnetoacoustic resonance in this case, are closely linked with the presence of open trajectories, and do not take place in metals with closed Fermi surfaces.

2. GENERAL FORMULA FOR THE ABSORPTION COEFFICIENT

Let us turn to a quantitative examination of the problem. As was shown by Gurevich[2] and Akhiezer, Kaganov, and Lyubarskiĭ,[4] the coefficient of absorption α is connected with the rate of dissipation of energy Q by the relation

$$\alpha = Q/W, \qquad Q = h^{-3}\int d\tau_{\mathbf{p}}\nu\,|\chi^2|\,\delta(\varepsilon-\mu_0), \tag{2.1}$$

where Q is the dissipation function, $\chi\delta(\epsilon-\mu_0)$ is a small non-equilibrium addition to the Fermi distribution function f_0, $\nu(\mathbf{p})$ is the collision frequency [$\nu = t_0^{-1}$ where $t_0(\mathbf{p})$ is the time between collisions], $d\tau_{\mathbf{p}} = dp_x dp_y dp_z = eHc^{-1}d\epsilon\, dt\, dp_z$ is an element of momentum space, $\epsilon(\mathbf{p})$ is the energy, $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is the velocity, t the time of motion along the orbit, $-e$ is the charge, μ_0 is the electron end-point energy, h is Planck's constant, and c is the velocity of light.

The time t is determined by the equation of motion of the electron in a magnetic field[5]

$$d\mathbf{p}/dt = -(e/c)\,[\mathbf{vH}]. \tag{2.2}*$$

The quantity W is the energy density in the sound wave

$$W = \tfrac{1}{2}\rho\omega^2\,|\mathbf{u}^2|, \tag{2.3}$$

where ρ is the density of the metal, $\omega = ks$ is the frequency, s is the speed, $\lambda = 2\pi/k$ is the wavelength of sound, and $\mathbf{u}(\mathbf{r}, t) = \mathbf{u}_0 \exp(i\omega t - i\mathbf{k}\cdot\mathbf{r})$ is the displacement vector. The definition (2.1) of the absorption coefficient is certainly valid for $\omega \ll \nu$, which is assumed to be satisfied.

The solution of the linearized kinetic equation for the function χ has the form[2,6]

$$\chi = \int_{-\infty}^{t} dt_1 g(t_1)\exp\left[\int_{t}^{t_1}(\nu - i\mathbf{k}\mathbf{v})\,dt'\right]; \tag{2.4}$$

$$g = \Lambda_{ik}\dot{u}_{ik} - e\mathbf{E}'\mathbf{v}, \qquad \Lambda_{ik} = \lambda_{ik}(\mathbf{p}) - \langle\lambda_{ik}\rangle,$$

$$\langle\lambda\rangle = \int d\tau_{\mathbf{p}}\lambda\delta(\varepsilon-\mu_0)\Big/\int d\tau_{\mathbf{p}}\delta(\varepsilon-\mu_0),$$

$$u_{ik} = 2^{-1}(\partial u_i/\partial x_k + \partial u_k/\partial x_i),$$

$$e\mathbf{E}' = e\mathbf{E} - (e/c)\,[\dot{\mathbf{u}}\mathbf{H}] - \nabla\langle\lambda_{ik}\rangle u_{ik}. \tag{2.5}$$

Here $\lambda_{ik}(\mathbf{p})$ is the deformation potential, which determines the variation of the dispersion law in the field of the acoustic wave: $\delta\epsilon = \lambda_{ik}u_{ik}$, where u_{ik} is the deformation tensor. In order of magnitude we have $|\lambda_{ik}| \sim |\Lambda_{ik}| \sim \mu_0$. The quantity $\langle\delta\epsilon\rangle = \langle\lambda_{ik}\rangle u_{ik}$ represents the change in the chemical potential of the electrons in the field of the sound wave. The small terms connected with the Stewart-Tolman effect and the change in the temperature can be neglected.[4]

The electric fields induced by the ultrasound should be determined from Maxwell's equations (the field components transverse to the wave vector) and from the conditions for the electroneutrality of the metal, $\mathbf{j}\cdot\mathbf{k} = 0$. For this purpose it is necessary to calculate the current density

$$\mathbf{j} = -2eh^{-3}\int d\tau_{\mathbf{p}}\mathbf{v}\chi\delta(\varepsilon-\mu_0).$$

We shall not deal, however, with the determination of the electric fields, since it can be shown (see reference 2) that in the range of the values of H concerned here the field make the same (or smaller) contribution to the absorption as does the deformation potential, which in turn is known only in order of magnitude. These fields are contained in α only through the slowly varying function g, the form of which does not influence essentially the features of the investigated effects.

Substituting the expression for χ in the dissipative function and integrating by parts, we readily obtain

$$\alpha = h^{-3}W^{-1}\,\mathrm{Re}\int d\tau_{\mathbf{p}}g^*\chi\delta(\varepsilon-\mu_0), \tag{2.6}$$

where g^* is the complex conjugate of g.

*$[\mathbf{vH}] = \mathbf{v}\times\mathbf{H}$.

Magnetoacoustic resonance takes place when the trajectories of the electrons in momentum space are periodic in t. The motion on the closed trajectories is always periodic. Periodic open trajectories occur on open Fermi surfaces when the vector **H** lies in the crstallographic plane.[5-7]

Let us calculate the coefficient of absorption for the case when $T\overline{\nu} \ll 1$, and $\max\{kvT\} \gg 1$. This system of inequalities is equivalent to

$$\lambda \ll 2\pi r \ll l. \qquad (2.7)$$

We can therefore calculate α by the method of stationary phase. It is readily seen that the stationary phase points in the integrals with respect to t form a line of points $\mathbf{k}\cdot\mathbf{v}(t, p_z) = 0$ on the Fermi surface. Simple calculation leads to

$$\alpha \sim eHh^{-3}W^{-1}c^{-1}\operatorname{Re}\int dp_z \sum_{0<t_\alpha<T} \sum_{-\infty<t_\beta\leq t_\alpha}' g_\alpha^* g_\beta J_\alpha^* J_\beta \times \exp\Big[\int_{t_\alpha}^{t_\beta}(\nu - iq)\,dt\Big]; \qquad (2.8)$$

$$J_\alpha = \int_{-\infty}^{\infty} d\tau \exp\left[-\tfrac{1}{2}iq_\alpha'\tau^2 - \tfrac{1}{6}iq_\alpha''\tau^3\right]. \qquad (2.9)$$

Here $q = \mathbf{k}\cdot\mathbf{v}$, the points $t_\alpha(p_z)$ are all the solutions of the equations $q(t_\alpha, p_z) = 0$, $q_\alpha' = dq(t_\alpha)/dt$, $q_\alpha'' = d^2q(t_\alpha)/dt^2$, and $g_\alpha = g(t_\alpha)$. The summation with respect to t_α is over all the t_α in the interval $(0, T)$, while the sum with respect to t_β denotes summation over all the roots $t_\beta \leq t_\alpha$, while the first term of the sum, $t_\beta = t_\alpha$, should be multiplied by ½, a fact designated by the prime. Integration with respect to p_z is over those values of p_z for which solutions of the equations $q(t, p_z) = 0$ exist.

We have retained two terms in the exponent of formula (2.9) for J_α, for $q_\alpha'(p_z)$ can vanish in certain important cases (see Sec. 3, item B). The term with q_α'' can be neglected if $|q_\alpha'|^3 \gg |q_\alpha''|^2$. In this case

$$J_\alpha = (2\pi/|q_\alpha'|)^{1/2}\exp\left(-\tfrac{1}{4}\pi i s_\alpha\right), \qquad s_\alpha = q_\alpha'/|q_\alpha'|. \qquad (2.10)$$

In the opposite limiting case

$$J_\alpha = (6/|q_\alpha''|)^{1/3}\,\Gamma(1/3)\,3^{-1/2}. \qquad (2.11)$$

Equation (2.8) can be transformed to

$$\alpha = \alpha_0 + \alpha_1, \qquad \alpha_0 = (eH/2h^3Wc)\int dp_z \sum_{0<t_\alpha<T} |g_\alpha J_\alpha|^2, \qquad (2.12)$$

$$\alpha_1 = (eH/h^3Wc)\operatorname{Re}\int dp_z\,[1 - \exp(2\pi i\beta - 2\pi\gamma)]^{-1} \times \sum_{0<t_\alpha<T} g_\alpha^* J_\alpha \sum_{0<t_\alpha - t_\beta\leq T} g_\beta J_\beta \exp\Big[\int_{t_\alpha}^{t_\beta}(\nu - iq)\,dt\Big]. \qquad (2.13)$$

Here $2\pi(\gamma - i\beta) = T(\overline{\nu} - i\overline{q})$. The bar denotes averaging over the period T:

$$\overline{\psi} = \frac{1}{T}\int_0^T \psi\,dt.$$

Contributions to α_0 are obviously made by all values of p_z. It is therefore easy to show, by using the expression (2.10) for J_α, that α_0 coincides with the absorption coefficient for $H = 0$, obtained by Akhiezer et al.[4]:

$$\alpha_0 = \pi h^{-3}W^{-1}\int d\tau_p\,|g|^2\,\delta(\varepsilon - \mu_0)\,\delta(\mathbf{kv}) \sim N\mu_0\omega/\rho s v, \qquad (2.14)$$

where N is the density of the electrons in the metal.

The resonance effects, like the sharp anisotropy of the absorption, are due to the presence of the resonance factor $B(p_z) = [1 - \exp(2\pi i\beta - 2\pi\gamma)]^{-1}$.

3. RESONANT ABSORPTION OF ULTRASOUND ON CLOSED TRAJECTORIES

Let us consider magnetoacoustic resonance on closed trajectories, for which $\overline{v}_x = \overline{v}_y \equiv 0$ and $\overline{v}_z \neq 0$. When $k_z \neq 0$ the function $B(p_z)$ has a series of peaked maxima. For fixed values of the magnetic field and of the frequency, these maxima correspond to values $p_z = p_{zn}$ for which $\beta(p_{zn}) = k_z\overline{v}_zT/2\pi = n$ (n is an arbitrary integer or zero).

The values of p_{zn} do not occur, generally speaking, when $\beta(p_z)$ has extrema. However, if H and ω are such that at some value of n the condition $\beta(p_z) = n$ is satisfied for a value of $\beta(p_z)$ which is extremal with respect to p_z, then the maximum of $B(p_z)$ will be considerably broader than the others, and its contribution to the integral will be greater. Let us show that this maximum leads to resonance oscillations, and not merely to increments in the absorption coefficient, as stated by Gurevich.[2] These increments, as will be made clear in what follows, pertain to the edges of the resonance line and describe its asymmetry.

A. The absorption near the extremum points p_s of the function $\beta(p_z)$ has the form

$$\alpha \sim \frac{eH}{h^3Wc}\left|\sum_{t_\alpha} g_\alpha J_\alpha e^{-iA_\alpha}\right|^2_{p_s} \operatorname{Re}\int_{-\infty}^{\infty} dp_z\{1 - \exp \times (2\pi i\Delta + \pi i\beta''(p_z - p_s)^2 - 2\pi\gamma)\}, \qquad (3.1)$$

where

$$\Delta = \beta_{ext} - n, \qquad 2\pi|\Delta| \ll 1,$$

$$2\pi|\gamma| \ll 1, \qquad A_\alpha = \int_0^{t_\alpha} q\,dt$$

and the small quantities $\nu(t_\beta - t_\alpha)$ have been neglected.

Expanding the exponential in a series and integrating, we obtain

$$\alpha = (eH/2h^3Wc)\,|\beta''|^{-1/2}\Big|\sum_{t_\alpha} g_\alpha J_\alpha e^{-iA_\alpha}\Big|^2 M,$$

$$M = \gamma(\gamma^2+\Delta^2)^{-1/2}\,[(\gamma^2+\Delta^2)^{1/2}+\sigma\Delta]^{-1/2}. \qquad (3.2)$$

The values of all the functions are taken at $p_z = p_s$, and $\sigma = |\beta''|/\beta''$.

Using (2.10), we obtain the following estimate for the height of the n-th maximum:

$$\alpha_n \sim \alpha_0 (kr\gamma)^{-1/2} \sim \omega^{1/2} H_n \bar{v}^{-1}. \qquad (3.3)$$

The relative width of the n-th maximum is $\Delta_n \sim \gamma$ or, expressed in terms of the magnetic field, $\Delta H/H_n \sim (kl)^{-1}$, is independent of n and is determined by the range. The greater l, the narrower the maximum and the higher the peak. Figure 1 shows schematically the dependence of the resonant absorption on H. With increasing n, the amplitude at the maximum decreases as n^{-1}, i.e., varies linearly with the field ($H_n \approx H_1 n^{-1}$).

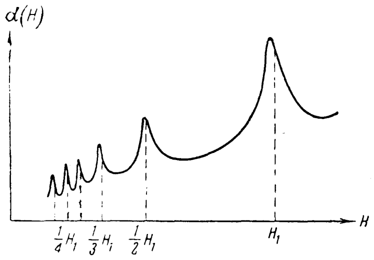


FIG. 1. Approximate course of resonant absorption vs. magnetic field for $\beta'' < 0$.

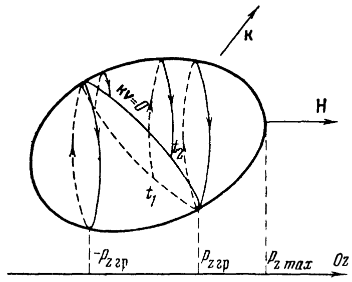
The form of any individual resonance line is determined by the factor M. Its maximum, of value $3^{3/4}2^{-1}\gamma^{-1/2}$, is reached when $\Delta = -3^{-1/2}\gamma^{-1/2}$, i.e., is shifted somewhat with respect to $\Delta = 0$.

The sharp jumps in the absorption coefficient, referred to by Gurevich,[2] pertain in this case actually to the edges of the resonance lines, and correspond to the limiting case $|\Delta| \gg \gamma$ of the general formula (3.2). These jumps describe the asymmetry of the resonance lines away from resonances when $\sigma\Delta > 0$ we have $M \approx \gamma|\Delta|^{-3/2}$ and when $\sigma\Delta < 0$ the value of M is on the order of $|\Delta|^{-1/2}$ and is $|\Delta|/\gamma \gg 1$ times greater than when $\sigma\Delta > 0$.

In evaluating the integral (3.1) near resonance we have assumed that the numerator, which contains the rapidly oscillating function $\exp\{iA_\alpha(p_z)\}$, varies slowly over the interval $\delta p_z \sim p_s(kl)^{-1/2}$. This assumption is justified if $\alpha_n/\alpha_0 \sim (kr\gamma)^{-1/2} \gg 1$, i.e., in the case of sharply pronounced resonance. In other words, sufficient sharpness of the resonance ensures the correctness of the limiting formula (3.3) If the inequality $\alpha_n \gg \alpha_0$ is not satisfied sufficiently well, then the resonance maxima will be shifted somewhat and "diffuse." In principle it is possible to take this effect into account by determining more accurately the positions of the stationary points in the integral with respect to p_z.

B. We proceed to estimate the contribution due to the limiting value $p_z = p_{z\,\lim}$. The latter is that maximum (minimum) value of p_z on the closed Fermi surface (Fig. 2) for which a unique point $\mathbf{k}\cdot\mathbf{v} = 0$ exists on the curve $p_z = p_{z\,\lim}$, $\epsilon(\mathbf{p}) = \mu_0$. For the sake of simplicity we consider a convex surface $\epsilon(\mathbf{p}) = \mu_0$ with a symmetry center. When $|p_z| < |p_{z\,\lim}|$ the electron trajectory has only two points t_α ($t_1 < t_2$) at which $\mathbf{k}\cdot\mathbf{v}(t) = 0$, and there are no such points at all when $|p_z| > |p_{z\,\lim}|$. The contribution from the limiting value must be estimated with caution, for when $p_z = p_{z\,\lim}$ we have $q'_\alpha = 0$ and expression (2.10) cannot be used for J_α. For small $\Delta p_z = p_{z\,\lim} - p_z$ we have $q'_\alpha \sim |\Delta p_z|^{1/2}$.

FIG. 2. Schematic form of electron trajectories and line $\mathbf{k}\cdot\mathbf{v}$ = const on closed convex surface $\varepsilon(\mathbf{p}) = \mu_0$.



The resonant part of the absorption coefficient can be written as

$$\alpha = \frac{eH}{h^3Wc}\,\mathrm{Re}\int dp_z\,\frac{|g_1J_1 + g_2J_2\exp(-iA_{21})|^2}{1-\exp(2\pi i\beta - 2\pi\gamma)}. \qquad (3.4)$$

We have neglected in the numerator the small quantities $\nu(t_\beta - t_\alpha)$, and replaced $\exp(2\pi i\beta)$ by unity.

Let us consider a case when $\beta'_{\lim}\,(d\beta/dp_z)_{\lim} \neq 0$ and $|\Delta| = |\beta_{\lim} - n| \ll (2\pi)^{-1}$. The relative width of the interval $\delta p_z/p_{z\,\lim}$ which makes an appreciable contribution to the integral is on the order of $(|\Delta| + \gamma)(kr)^{-1}$. Here

$$|q'^3_\alpha/q''^2_\alpha| \sim kr\,(\delta p_z/p_{z\,\lim})^{3/2} \ll 1,$$

and expression (2.11) must be used for J_α.

In this case the function $B(p_z)$ is "sharper" than $\exp(-iA_{21})$, since $A_{21}(p_z) = A_2 - A_1$ changes over the interval $\delta p_z \sim (kr)^{-1}p_{z\,\lim}$. Recognizing that $g_1J_1 = g_2J_2$ when $p_z = p_{z\,\lim}$, we obtain for small $|\Delta|$ and γ

$$\alpha_{\lim} = \frac{eH}{3h^3Wc}\,\frac{|g^2|}{|\beta'|}\left(\frac{6}{|q''|}\right)^{2/3}\Gamma^2(1/3)\left\{1+\frac{2\sigma}{\pi}\,\mathrm{arctg}\,\frac{\Delta}{\gamma}\right\}, \qquad (3.5)^*$$

*arctg = $\tan^{-1}$.

where $\sigma = (\beta'|/\beta')_{\lim}$. The factor preceding the curly bracket is of the order of $\alpha_0(kr)^{-2/3}$. Formula (3.5) describe periodic increments in the absorption coefficient, viz. the value of the expression in the curly brackets is 2 when $\sigma\Delta > 0$ and $|\Delta| \gg \gamma$, and is small ($\sim \gamma/|\Delta|$) when $\sigma\Delta < 0$.

The plot of α vs. H, given by Eq. (3.5), has a sawtooth shape with the "tooth" height decreasing with decreasing field. The magnitude of each jump is

$$\Delta\alpha = \frac{2eH\Gamma^2(1/3)\,|g|^2 6^{2/3}}{3h^3Wc\,|\beta'|\,|q''|^{2/3}} \sim \alpha_0(kr)^{-2/3} \sim \omega^{1/3}H_n^{2/3}. \tag{3.6}$$

The absorption increases with increasing H if $\beta' < 0$ and with decreasing H if $\beta' > 0$. On a closed convex Fermi surface, in particular, $\beta' > 0$ and α acquires increments when the magnetic field is decreased.

The dependence of the derivative $d\alpha/dH$ on H should display resonance peaks

$$H\,d\alpha/dH \sim \alpha_0(kr)^{1/3}\sigma\gamma/(\gamma^2 + \Delta^2), \tag{3.7}$$

with an amplitude at the maximum amounting to $(d\alpha/dH)_{\max} \sim \alpha_0(\gamma H)^{-1}(kr)^{1/3}$ and with a position determined by the condition $\beta_{\lim} = n$.

In the case of a quadratic dispersion law, as for any convex closed surface, β' does not vanish anywhere, and the resonance oscillations take place on the $d\alpha'/dH$ curve and are connected only with the limit point.

For a non-convex Fermi surface, the limit point may coincide with the 'extremal' point at some inclination of the vector **k** relative to **H**, i.e., $\beta'_{\lim}$ may vanish. In this case resonant oscillations can occur* when $\gamma(kl)^{1/4} \ll 1$:

$$\alpha_{\lim} = \frac{6^{2/3}\Gamma^2(1/3)\,eH\,|g^2|}{3h^3Wc\,|\beta''|^{1/2}\,|q''|^{2/3}}\,M. \tag{3.8}$$

Their amplitude is treater than in the case (3.3):

$$\alpha_n/\alpha_0 \sim (kr)^{-1/6}\gamma^{-1/2} \sim \omega^{5/6}H_n^{2/3}\nu^{-1/2}.$$

The relative shape and width of the resonance curve is the same as in case A.

C. Let us estimate the contribution to the absorption from the points p_{zn} at which $\beta(p_{zn}) = n$, and $\beta'(p_{zn}) \neq 0$. Near these points, when inequalities (2.7) are satisfied, we have

$$\operatorname{Re} B(p_z) \approx \pi\delta(\beta - n). \tag{3.9}$$

Consequently, the contribution of the points p_{zn} is given by the equation

$$\delta\alpha = \frac{\pi eH}{h^3Wc}\sum_n \int_{\Delta p_{zn}} dp_z\,\delta(\beta - n)\left|\sum_{t_\alpha} g_\alpha |q'_\alpha|^{-1/2} \times \exp(-\pi i s_\alpha/4 - iA_\alpha)\right|^2 \tag{3.10}$$

Integration with respect to p_z is carried out in the vicinity of those points ($|p_{zn}| < |p_{z\,\lim}|$) at which $\beta(p_z) = n$, $\beta' \neq 0$, and $q'_\alpha \neq 0$.

It is seen from (3.10) that $\delta\alpha$ has no resonant character. The value of the non-oscillating part of $\delta\alpha$ is on the order of α_0, and the amplitudes of the individual oscillations are of the order of $\alpha_0(kr)^{-1}$ and less than the amplitude of the resonance or of the jumps in the absorption. When the number of terms in (3.10) is large, $\delta\alpha$ is an irregular function of H, owing to the large number of oscillating components with periods and amplitudes of the same order of magnitude.

Along with these singularities, the coefficient of absorption should also acquire smooth increments, connected with the extremal values of p_z (with $p_{z\,\max}$ on a closed convex surface), which were investigated by Gurevich.[2]

D. We investigate finally the sharp anisotropy of absorption when the vectors **k** and **H** deviate little from perpendicularity. We consider for simplicity trajectories on which $\mathbf{k}\cdot\mathbf{v} = 0$ at two points. The absorption can be represented as a sum of a monotonic and an oscillating part:

$$\alpha_{\text{mon}} = h^{-3}W^{-1}\int d\tau_p\,\delta(\varepsilon - \mu_0)\,\delta(\mathbf{kv})\,|g^2|\,\gamma(\gamma^2 + \beta^2)^{-1}, \tag{3.11}$$

$$\alpha_{\text{osc}} = eH(\pi ch^3W)^{-1}\operatorname{Re}\int dp_z\,\gamma(\gamma^2 + \beta^2)^{-1}\,g_1g_2^*J_1J_2^*e^{iA_{21}}, \tag{3.12}$$

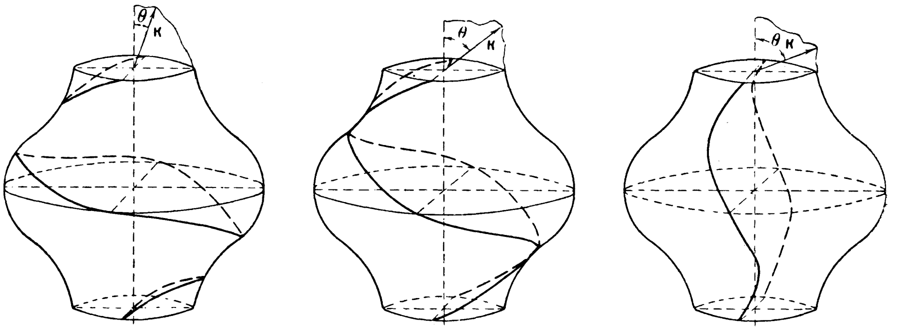
where $\beta = k_z\bar{v}_zT/2\pi$, and $k_z = k\sin\varphi$. When $\varphi = 0$, $\beta = 0$ and $\alpha_{\text{mon}} \sim \alpha_0\gamma^{-1}$, $\alpha_{\text{osc}} \sim \alpha_{\text{mon}}(kr)^{-1/2} \sim \alpha_0\gamma^{-1}(kr)^{-1/2}$. This case was investigated by Gurevich.[2] When $\varphi \ll (kl)^{-1}$ the average displacement of the electron in the direction of sound propagation is small ($|\beta| \ll \gamma$), and the absorption has the same form as when $\varphi = 0$.

In the angle interval $(kl)^{-1} \ll \varphi \ll (kr)^{-1}$ the number of electrons with $|\beta| \lesssim \gamma$, and hence the absorption, is $kl\varphi$ times smaller than when $\varphi = 0$. In this case the formula for α_{mon} is obtained from (3.11) by making the substitution $\gamma(\gamma^2 + \beta^2)^{-1} \to \pi\delta(\beta)$:

$$\alpha_{\text{mon}} = \pi h^{-3}W^{-1}\int d\tau_p\,|g^2|\,\delta(\mathbf{kv})\,\delta(\beta)\,\delta(\varepsilon - \mu_0) \sim \alpha_0(kr\varphi)^{-1}. \tag{3.13}$$

The ratio of the "speeds" of the variations of $(\gamma^2 + \beta^2)^{-1}$ and of $\exp(iA_{21})$ in (3.12) is important for the estimate of α_{osc}. The oscillating factor near the central section, where $A_{21}(p_z)$ has an extremum, changes appreciably in the interval δp_1

*When this inequality is satisfied, the "sharpest" is the function $B(p_z)$, and Eq. (2.11) must be used for J_α.



a b c

FIG. 3. Line $\mathbf{k \cdot v} = 0$ on a "corrugated cylinder" type of surface. a – $\theta < \theta_0$, b – $\theta = \theta_0$, c – $\theta > \theta_0$ (θ_0 is the maximum angle of inclination of the "corrugation" to the cylinder axis).

$\sim p_0 (kr)^{-1/2}$, and $(\gamma^2 + \beta^2)^{-1}$ changes over a distance $\delta p_2 \sim p_0 (kl\varphi)^{-1}$ (p_0 is the characteristic momentum of the electron). δp_1 is proportional to the number of electrons for which $|A_{21}(p_z) - A_{21}(0)| \lesssim 1$, while δp_2 is proportional to the number of electrons with $|\beta| \le \gamma$. When $\delta p_2 \ll \delta p_2$ [i.e., $\varphi \ll (\gamma/kl)^{1/2}$] application of the method of stationary phase yields $\alpha_{osc} \sim \alpha_0 \gamma^{-1} (kr)^{-1/2}$. The period of oscillation is the same when $\varphi \ll 1$ as when $\varphi = 0$, and coincides with the extremal diameter of the Fermi surface in the direction of $\mathbf{k \times H}$. In the opposite limiting case, $\delta p_2 \ll \delta p_1$, we can replace $\gamma(\gamma^2 + \beta^2)^{-1}$ by $\pi\delta(\beta)$ and $\alpha_{osc} \sim \alpha_{mon} \sim \alpha_0 (kr\varphi)^{-1}$.

Thus, the monotonic part of the absorption falls off at small φ much more rapidly than the oscillating part, and when $(\gamma/kl)^{1/2} \ll \varphi \ll (kr)^{-1}$ the amplitude of the oscillations is comparable with the value of α_{mon} (remaining, naturally, smaller than the latter). With further increase in φ, the nonresonant oscillations are replaced by the resonant oscillations investigated above. The latter take place, generally speaking, for all $\varphi \gtrsim (kr)^{-1}$, and particularly also when $\varphi = \pi/2$ ($\mathbf{k} \parallel \mathbf{H}$). However, resonance will occur for $\varphi = \pi/2$ only when the curve $v_z = 0$ is not plane and consequently does not coincide with the curve $\bar{v}_z = p_z = 0$. If the two curves coincide (for example, in the case of quadratic dispersion), then the oscillations will take place only when $|\pi/2 - \varphi| \gg (kr)^{-1}$. Near $\varphi = \pi/2$ ($|\pi/2 - \varphi| < (kr)^{-1}$) the oscillations vanish, and $\alpha = \alpha_0$ il the range of fields under consideration.

All the foregoing statements regarding closed surfaces holds true for closed trajectories on open Fermi surfaces.

4. SINGULARITIES IN THE ABSORPTION OF ULTRASOUND ON OPEN PERIODIC TRAJECTORIES

Most metals have open Fermi surfaces.[5,8,9] When the vector H lies in the crystallographic plane, the open trajectories are periodic. Since the electrons absorb sound most effectively when entering the equal-phase plane, an important role is played by the position of the lines $\mathbf{k \cdot v} = 0$ on the surface $\epsilon(\mathbf{p}) = \mu_0$. Figure 3 shows the lines $\mathbf{k \cdot v} = 0$ on a surface of the "corrugated" cylinder of revolution type for different angles of inclination θ of the vector k to the cylinder axis x. When $\theta < \theta_0$ (θ_0 is the angle of inclination of the "corrugation" to x at the parabolic point) two closed non-intersecting lines $\mathbf{k \cdot v} = 0$ (Fig. 3a) exist in each cell of the reciprocal lattice. When $\theta = \theta_0$ they are in contact (Fig. 3b), and when $\theta > \theta_0$ they go into two open periodic curves, which are symmetrical about the plane passing through k and x (Fig. 3c). In the general case these lines are not planar, with the exception of $\theta = 0$ and $\theta = \pi/2$.

For a planar network of "corrugated cylinders,"* when the vector $\mathbf{k}$ is parallel to the cylinder axis, the lines $\mathbf{k \cdot v} = 0$ represent an aggregate of closed and open periodic curves (Fig. 4a). As the vector $\mathbf{k}$ is tilted away from the cylinder axis, the closed and open lines come closer and become tangent at a certain orientation of the vector $\mathbf{k}$ (Fig. 4b). Further deviation of the vector k causes the lines $\mathbf{k \cdot v} = 0$ to be closed (Fig. 4a). On the open trajectories we have $\bar{v}_x \equiv 0$ and $|\bar{v}_y T| = cb_x/eH$, where b_x is the period of the open trajectory in momentum space in the direction of x.

On the closed trajectories we have $\bar{v}_x = \bar{v}_y = 0$. The limit between the open and closed trajectories is a self-intersecting curve passing through the saddle point, at which the effective mass and the period T become infinite logarithmically.[8]

If the vector $\mathbf{k}$ makes, with the direction x of the open periodic trajectory an angle much less than $(kl)^{-1}$, the expression for α has the same form as in the case of closed trajectories when $\mathbf{k} \perp \mathbf{H}$ (see reference 2). The period of the oscilla-

*Such a surface is similar in its topological properties to the Fermi surface in tin.[10]

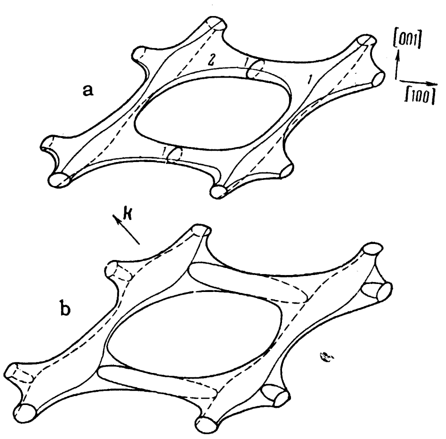


FIG. 4. Lines $\mathbf{k}\cdot\mathbf{v} = 0$ on a surface comprising a planar network of "corrugated cylinders," a) line 1 – k directed along the 100 axis; 2 – along the 001 axis; b) k in the (010) plane. The figure shows the instant of tangency between the open and closed lines $\mathbf{k}\cdot\mathbf{v} = 0$.

tions is determined by the extremal dimension of the open trajectory in the $\mathbf{k} \times \mathbf{H}$ direction (for a "corrugated cylinder" — by the height of the "corrugation" on the axial section).

When the vector **k** deviates from the xz plane ($\mathbf{H} \parallel z$), the open surface is equivalent to a closed non-convex surface, and the analysis given in Sec. 3 is fully applicable.

When $\mathbf{k} \perp \mathbf{H}$, the contribution to absorption from the open trajectories diminishes rapidly as the vector **k** tilts away from the x direction, since the quantity $|\beta| = (kcb_x/2\pi eH)\sin\theta$ (θ is the tilt angle) never vanishes for all the electrons on the unclosed trajectories. Consequently when $(kl)^{-1} \ll \theta \ll (kr)^{-1}$, both the monotonic and the oscillating parts of the absorption from the open trajectories are $(kl\theta)^2$ times smaller than when $\theta = 0$.*

When $\theta \gg (kr)^{-1}$, resonant oscillations are produced. Unlike the closed trajectories, resonant oscillations take place also when $\mathbf{k} \perp \mathbf{H}$. Since β is independent of p_z, it is possible near resonance to take $\mathrm{Re}\, B(p_z) \approx \gamma_0[\pi(\gamma_0^2 + \Delta^2)]^{-1}$ outside the integral sign in (2.13), and the absorption will have the form

$$\alpha = \frac{eH\gamma_0}{\pi h^3 W c\,(\gamma_0^2 + \Delta^2)} \int dp_z \left| \sum_{t_\alpha} g_\alpha J_\alpha e^{-iA_\alpha} \right|^2 \sim \frac{\alpha_0\gamma_0}{\gamma_0^2 + \Delta^2}, \qquad (4.1)$$

where γ_0 is the characteristic value of $\gamma(p_z)$. The integration in (4.1) is over the layer of open trajectories, on which there are points t_α. It is seen from (4.1) that at resonance $\alpha_n \sim \alpha_0\gamma_0^{-1}$ and is comparable with the non-oscillating part of the absorption on the closed trajectories. In this case the resonance is "sharper" than on the closed trajectories with $k_z \neq 0$, since the average displacement $|\bar{v}_y T|$ is the same for all the electrons and there is no additional averaging over p_z.

On these resonance oscillations are superimposed oscillations of relatively small amplitude $[\sim \alpha_0\gamma_0^{-1}(kr)^{-1/2}]$ from the central section (for $\theta < \theta_0$ in the case of a cylinder) with a period $\Delta H^{-1} = |2\pi ec^{-1}(k_x\Delta p_y - k_y\Delta p_x)_0|$, and oscillations from the self-intersecting trajectory (saddle point) with two periods (the periods are different on the sides of the open and closed trajectories), and with an amplitude which is roughly $(kr)^2$ times as small as the amplitude of the resonance oscillations. The phase of these last oscillations has a logarithmically slow dependence on the field.*

The resonant oscillations vanish when $\mathbf{k} \perp \mathbf{H}$ at an angle θ_k of between the vector k and x such that the angle $\mathbf{k}\cdot\mathbf{v} = 0$ shifts on the closed trajectories. In this case the analysis carried out in the preceding section is fully applicable.

The amplitude of the resonance oscillations depends quite sharply on the angle φ between the vector k and the plane $\mathbf{k} \perp \mathbf{H}$. The results given above are valid when $\varphi \ll (kl)^{-1}$. In the region $(kl)^{-1} \ll \varphi \ll (kr)^{-1}$ the amplitude of the resonance has at the maximum an order of $\alpha_0(kr\varphi)^{-1}$ and is kl times as small as when $\varphi = 0$. In the angle interval $(\gamma_0/kl)^{1/2} \ll \varphi \ll (kr)^{-1}$, when $\theta < \theta$, the amplitude of the resonant oscillations from the central section is comparable with the amplitude of the resonance, similar to what takes place on the closed trajectories (see Sec. 3, item D). In the region of angles $\varphi \gg (kr)^{-1}$, resonant oscillations take place, along with periodic increments in the coefficient of absorption, similar to those that exist on closed trajectories when $k \neq 0$.

5. ANISOTROPY OF THE ABSORPTION ON OPEN PERIODIC OR STRONGLY ELONGATED CLOSED TRAJECTORIES

Trajectories of this type are as a rule insignificant in the absorption of ultrasound. Exceptions are the almost-periodic trajectories with a period on the order of the reciprocal-lattice period. In

*When the vector **k** makes with x an arbitrary small angle $\kappa[\kappa \gg (kl)^{-1}]$, then the period of the oscillations is determined by the extremal dimensions of the trajectories in the y direction when $\kappa \ll (\gamma/kl)^{1/2}$, and by the dimensions of the trajectories with $\beta(p_z) \equiv (T/2\pi)(k_y\bar{v}_y + k_z\bar{v}_z) = 0$ when $\kappa \gg (\gamma/kl)^{1/2}$. We note that even at small κ these periods can differ greatly from each other.

*In view of the smallness of the amplitude of the oscillations connected with the saddle point, we do not give the exact formulas and limit ourselves to the remark that we must remember in the calculations that $T \to \infty$ logarithmically near the saddle point. The estimate in the text is valid if $\gamma_0 \ln kr \ll 1$.

order to make the following arguments clear, let us consider a specific example of a "corrugated cylinder" surface, when the vector **H** is almost perpendicular to the axis x of the cylinder.

In addition to the closed trajectories contained within each reciprocal-lattice cell, there exist strongly elongated closed trajectories,[7] which include $\sim \psi^{-1}$ cells of the reciprocal lattice (ψ is the angle between **H** and the plane perpendicular to x). When **k** is perpendicular to the x axis, then the strongly-elongated trajectories have only two points with $\mathbf{k}\cdot\mathbf{v} = 0$. When $\psi \ll r/l$, the time of motion of the electron between the neighboring points where $\mathbf{k}\cdot\mathbf{v} = 0$ is considerably greater than the time of the free path, t_0. In this case the H-dependent contribution to the absorption from the considered trajectories is exponentially small, and the coefficient of absorption from the strongly elongated trajectories is the same as when H = 0.

When $r/l \ll \psi \ll 1$ and $\mathbf{k} \perp \mathbf{H}$, α_{osc} is on the order of $\alpha_0 (lr)(kr)^{-1/2}\psi^2$, and the order of magnitude of the period of oscillations is ψ^{-1} times as small as in the case of closed Fermi surfaces.

Let **k** be directed parallel to the x axis, and let $q'_\alpha > 0$ when $0 < t_\alpha < T/2$ and $q'_\alpha < 0$ when $T/2 < t_\alpha < T$. The number of points with $\mathbf{k}\cdot\mathbf{v} = 0$ on a strongly elongated closed trajectory is of the order of ψ^{-1}. To estimate the sum under the integral sign in (2.13) let us calculate

$$S = \sum_{0<t_j<t_l<T/2} g_j^* g_l J_j^* J_l \exp\Big[\int_{t_l}^{t_j} (\nu - iq)\,dt\Big]. \qquad (5.1)$$

The remainder of the sum is of the same order as S. Confining ourselves to linear terms in $\psi \ll (kr)^{-1}$, we obtain

$$A_{lj} = \int_{t_j}^{t_l} q\,dt = \begin{cases} A + A\psi(l-j) & (l-j = 2n+1) \\ A\psi(l-j) & (l-j = 2n), \end{cases} \qquad (5.2)$$

where $A(p_z) \sim kr$; n is an integer.

Using (5.2) and replacing summation over t_j and t_l in (5.1) by integration with respect to j and l, we obtain

$$S \sim \psi^{-2} |gJ|^2 (1 + e^{iA}) \{(2iA - \pi\gamma)^{-1} + (2iA - \pi\gamma)^{-2}(1 - e^{2iA - \pi\gamma})\}. \qquad (5.3)$$

With the aid of (5.3) we can readily estimate the contribution to from strongly elongated trajectories. The results of calculations are listed in the following table:

	$\varphi \ll (kl)^{-1}$	$(kl)^{-1} \ll \psi \ll (kr)^{-1}$; $\psi \ll r/l$	$r/l \ll \psi \ll (kr)^{-1}$
α_{mon}/α_0	l/r	$(kr\psi)^{-2}$	$(kr\psi)^{-2}$
α_{osc}/α_0	$(l/r)(kr)^{-1/2}$	$(kr)^{-3/2}\psi^{-1}$	$(kr\psi)^{-1}$

If the vector **H** is tilted away from a crystallographic plane with small rational indices (nm0) ($\sqrt{n^2 + m^2} \ll l/r$) by an angle less than $(kl)^{-1}$, then the open trajectories make the same contribution to the absorption of the ultrasound as the strictly periodic trajectories.

6. CONCLUSION

Magnetoacoustic resonance can be used to reconstruct the topology and the form of the Fermi surfaces in metals.

a) The presence of resonant oscillations of the non-harmonic type when $\mathbf{k} \perp \mathbf{H}$ is connected with the existence of open periodic trajectories for a given direction of **H**. The observed resonances at $\mathbf{k}\cdot\mathbf{H} \neq 0$ are evidence of the non-convexity of the Fermi surface, i.e., of a sharp deviation of the dispersion from a quadratic law.

b) If open periodic trajectories exist for a given direction of **H**, then the diagram of the rotation of the vector **k** in the plane $\mathbf{k} \perp \mathbf{H}$ should disclose a sharp maximum when the vector **k** is parallel to the direction of the open periodic trajectory. This maximum is the principal (greatest) one and its position is independent of the magnetic field, whereas the position of the lateral maxima shifts with changing H. For closed trajectories, the absorption is almost isotropic when $\mathbf{k} \perp \mathbf{H}$, and there are no resonant oscillations.

c) To determine the form of the Fermi surface we can use non-resonant oscillations of the harmonic type, the period of which is determined by the extremal dimensions of the Fermi surface in the $\mathbf{k} \times \mathbf{H}$ direction. Owing to the sharp anisotropy in the angular dependence of the amplitude of the nonresonant oscillations due to the open periodic trajectories, it is possible to determine the extremal dimensions of the open and closed trajectories separately.

The experimental researches of Galkin and Korolyuk[11] on single crystals of high-purity tin at $\omega/2\pi = 220$ Mc/sec have shown that resonant nonsinusoidal oscillations exist when $\mathbf{k} \perp \mathbf{H}$. All the main characteristics of the experimental curve (periodicity in the reciprocal of the field, vanishing of the oscillation phase, dependence of the width and height of the maximum on the field) confirm fully the theoretical deductions.[11] Galvanomagnetic measurements[9,10] show that the Fermi surface of tin is open in the (001) plane. The period of the reciprocal lattice as calculated from the period of these oscillations is in good agreement with the known crystallographic data (see reference 11).

Recently Morse and co-workers determined the extremal dimensions of the open Fermi surface of gold and silver in the (111) plane from the periods of the nonresonant oscillations. Their data are in good agreement with the results of galvanomagnetic measurements, the de Haas — van Alphen effect, and others.

Thus, an investigation of the absorption of ultrasound in metals in a magnetic field, along with galvanomagnetic and other effects, is a good method of reconstructing the topology of Fermi surfaces in metals. It yields not only the directions of the open trajectories, but also the anisotropy of the diameters and hence the form of the extremal sections on the Fermi surface. The absorption of ultrasound is more convenient than the other high-frequency properties of metals in that it is a volume rather than surface phenomenon, and consequently the need for a perfect surface finish is eliminated.

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