ISOTOPIC INVARIANCE IN PROCESSES INVOLVING ANTIHYPERONS

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A number of relations between cross section for reactions involving antihyperons is derived on basis of the isotopic invariance hypothesis. Some additional relations between the cross sections arise if interaction of π and K mesons with baryons predominates in states with definite isotopic spin values.

THE experimental investigation of processes in which antihyperons participate is of great interest, since it can be useful in the gathering of various information concerning interactions between elementary particles. As indicated by Amati and Vitale,[1] to obtain antihyperons it is advantageous to use the reaction

$$K + N \rightarrow N + \overline{Y} + N, \qquad (1)$$

which can occur in a K-meson beam extracted from an accelerator. Its threshold energy is on the order of 4 Bev, and K mesons of such energy can be obtained in nucleon-nucleon collisions. It can be assumed that various processes in an antiproton beam, for example of the type

$$\overline{p} + N \rightarrow Y_1 + \overline{Y}_2. \qquad (2)$$

can be used to obtain antihyperons (particularly low-energy ones).

Assuming that experiments with antihyperons will become feasible in the future, we obtain here, on the basis of the charge-invariance hypothesis, several relations between the cross sections of various processes in which antihyperons participate. An experimental verification of such relations would enable us not only to judge whether the charge-invariants hypothesis can be extended to include phenomena involving interactions of strange particles (in particular, antihyperons) but also, should this hypothesis prove correct, to obtain some information on the interaction between elementary particles in states with definite isotopic spins. We consider here several reactions that are expected in the collision between antihyperons and nuclei having isotopic spins 0 and $1/2$.

1. Let us consider the reactions

$$\overline{\Sigma}^+ + d \rightarrow n + \pi^- + K^+, \quad \overline{\Sigma}^0 + d \rightarrow p + \pi^- + K^+,$$
$$\rightarrow n + \pi^0 + K^0, \qquad \rightarrow p + \pi^0 + K^0.$$
$$\rightarrow p + \pi^- + R^0,$$

The corresponding charge-symmetrical reactions have equal cross sections. Reactions with $\overline{\Sigma}^+$ antihyperons were considered in detail by Amati and Vitale[1] and by Matinyan and Khutsishvili.[2] We derive here a few additional relations, including the cross sections of the reactions induced by neutral $\overline{\Sigma}^0$ antihyperons.

Assuming charge invariance, we can show that the following equations hold true:

$$d\sigma(\overline{\Sigma}^+ \rightarrow n\pi^0 K^0) = d\sigma(\overline{\Sigma}^0 \rightarrow p\pi^- K^+),$$
$$d\sigma(\overline{\Sigma}^+ \rightarrow n\pi^- K^+) + d\sigma(\overline{\Sigma}^+ \rightarrow p\pi^- K^0)$$
$$= d\sigma(\overline{\Sigma}^+ \rightarrow n\pi^0 K^0) + 2\, d\sigma(\overline{\Sigma}^0 \rightarrow p\pi^0 K^0). \qquad (3)$$

When experimental data on the cross sections of the reactions induced by negatively charged $\overline{\Sigma}^+$ antihyperons become available, these relations will yield information on reactions induced by neutral $\overline{\Sigma}^0$ antihyperons, which are very difficult to investigate experimentally, because of the short lifetime of the $\overline{\Sigma}^0$.

In addition to the inequalities established in reference 2, we can derive also the following:

$$d\sigma(\overline{\Sigma}^+ \rightarrow n\pi^0 K^0) + d\sigma(\overline{\Sigma}^0 \rightarrow p\pi^0 K^0) \geqslant \tfrac{2}{3} d\sigma(\overline{\Sigma}^+ \rightarrow n\pi^- K^+),$$
$$d\sigma(\overline{\Sigma}^+ \rightarrow p\pi^- K^0) + \tfrac{1}{3} d\sigma(\overline{\Sigma}^+ \rightarrow n\pi^0 K^0) \geqslant d\sigma(\overline{\Sigma}^0 \rightarrow p\pi^0 K^0),$$
$$d\sigma(\overline{\Sigma}^0 \rightarrow p\pi^- K^+) + d\sigma(\overline{\Sigma}^+ \rightarrow p\pi^- K^0) \geqslant \tfrac{1}{3} d\sigma(\overline{\Sigma}^+ \rightarrow n\pi^- K^+). \qquad (4)$$

We note that one of the relations in (4) is the consequence of the other two. In addition,

$$|\{2\, d\sigma(\overline{\Sigma}^+ \rightarrow p(n)\pi^\pm K^0)\}^{1/2} - \{2\, d\sigma(\overline{\Sigma}^0 \rightarrow p\pi^0 K^0)\}^{1/2}|$$
$$\leqslant \{d\sigma(\overline{\Sigma}^0 \rightarrow p\pi K^+)\}^{1/2} \leqslant \{2\, d\sigma(\Sigma^{\mp} \rightarrow p(n)\, \pi^\pm K^0)\}^{1/2}$$
$$+ \{2\, d\sigma(\overline{\Sigma}^0 \rightarrow p\pi^0 K^0)\}^{1/2},$$

$$|\{2\, d\sigma(\overline{\Sigma}^+ \rightarrow p(n)\pi^\pm K^0)\}^{1/2} - \{d\sigma(\overline{\Sigma}^0 \rightarrow p\pi^- K^+)\}^{1/2}|$$
$$\leqslant \{2 d\sigma(\overline{\Sigma}^0 \rightarrow p\pi^0 K^0)\}^{1/2} \leqslant \{2\, d\sigma(\overline{\Sigma}^+ \rightarrow p(n)\pi^\pm K^0)\}^{1/2}$$
$$+ \{d\sigma(\overline{\Sigma}^0 \rightarrow p\pi^- K^+)\}^{1/2},$$

$$|\{2\,d\sigma(\bar{\Sigma}^0 \to p\pi^0K^0)\}^{1/2} - \{d\sigma(\bar{\Sigma}^0 \to p\pi^-K^+)\}^{1/2}|$$
$$\leqslant \{2\,d\sigma(\bar{\Sigma}^+ \to p\,(n)\,\pi^\pm K^0)\}^{1/2} \leqslant \{2\,d\sigma(\bar{\Sigma}^0 \to p\pi^0K^0)\}^{1/2}$$
$$+ \{d\sigma(\bar{\Sigma}^0 \to p\pi^-K^+)\}^{1/2}. \qquad (5)$$

If the interaction of the π-N system in the state with T = $^3\!/_2$ predominates, a new relation arises

$$d\sigma(\bar{\Sigma}^+ \to n\pi^0K^0) = \tfrac{1}{2}\,d\sigma(\bar{\Sigma}^0 \to p\pi^0K^0). \qquad (6)$$

We can consider analogously reactions of the type

$$\bar{\Xi} + d \to N + K + K.$$

The charge invariance leads to the relation

$$d\sigma(\bar{\Xi}^- \to pK^+K^0) \geqslant \tfrac{1}{2}\,d\sigma(\bar{\Xi}^- \to nK^+K^+). \qquad (7)$$

At the present time the experimental data apparently point to a strong interaction of the K-N system in the state with T = 1. In this limiting case we obtain

$$d\sigma(\bar{\Xi}^- \to pK^+K^0) = 5\,d\sigma(\bar{\Xi}^- \to nK^+K^+). \qquad (8)$$

2. We now consider the reactions accompanied by the creation of two pions, for example

$$\bar{\Xi} + d \to N + \pi + \pi + K.$$

We construct the wave function of the final state with T = 1 from the wave functions of the system of two pions with $t_1 = 0$, 1, and 2, and the wave functions of the K-N system with $t_2 = 0$ and 1. If $A_{t_2}^{t_1}$ is the amplitude of the transition into states with isotopic spins t_1 and t_2, then the amplitudes of the reactions have the following form

$$M(\bar{\Sigma}^+ \to K^+p\pi^-\pi^-) = \left(\tfrac{3}{5}\right)^{1/2} A_1^2,$$
$$M(\bar{\Sigma}^+ \to K^0n\pi^0\pi^0) = -\left(\tfrac{1}{3}\right)^{1/2} A_1^0 + \left(\tfrac{1}{15}\right)^{1/2} A_1^2,$$
$$M(\bar{\Sigma}^+ \to K^0p\pi^-\pi^0) = -\tfrac{1}{2}A_0^1 + \left(\tfrac{1}{8}\right)^{1/2} A_1^1 - \left(\tfrac{3}{40}\right)^{1/2} A_1^2,$$
$$M(\bar{\Sigma}^+ \to K^0p\pi^0\pi^-) = \tfrac{1}{2}A_0^1 - \left(\tfrac{1}{8}\right)^{1/2} A_1^1 - \left(\tfrac{3}{40}\right)^{1/2} A_1^2,$$
$$M(\bar{\Sigma}^+ \to K^0n\pi^-\pi^+) = \left(\tfrac{1}{3}\right)^{1/2} A_1^0 - \tfrac{1}{2}A_1^1 + \left(\tfrac{1}{60}\right)^{1/2} A_1^2,$$
$$M(\bar{\Sigma}^+ \to K^0n\pi^+\pi^-) = \left(\tfrac{1}{3}\right)^{1/2} A_1^0 + \tfrac{1}{2}A_1^1 + \left(\tfrac{1}{60}\right)^{1/2} A_1^2,$$
$$M(\bar{\Sigma}^+ \to K^+n\pi^0\pi^-) = -\tfrac{1}{2}A_0^1 - \left(\tfrac{1}{8}\right)^{1/2} A_1^1 - \left(\tfrac{3}{40}\right)^{1/2} A_1^2,$$
$$M(\bar{\Sigma}^+ \to K^+n\pi^-\pi^0) = \tfrac{1}{2}A_0^1 + \left(\tfrac{1}{8}\right)^{1/2} A_1^1 - \left(\tfrac{3}{40}\right)^{1/2} A_1^2. \qquad (9)$$

If, as is customarily done, the summation is carried out over the states of the π mesons, we obtain from (9) the inequalities

$$d\sigma(\bar{\Sigma}^+ \to K^0p\pi^-\pi^0) + d\sigma(\bar{\Sigma}^+ \to K^+n\pi^-\pi^0)$$
$$\geqslant \tfrac{1}{2}\,d\sigma(\bar{\Sigma}^+ \to K^+p\pi^-\pi^-),$$
$$d\sigma(\bar{\Sigma}^+ \to K^0n\pi^+\pi^-) + d\sigma(\bar{\Sigma}^+ \to K^0n\pi^0\pi^0)$$
$$\geqslant \tfrac{1}{6}\,d\sigma(\bar{\Sigma}^+ \to K^+p\pi^-\pi^-), \qquad (10)$$

which could be verified experimentally.

If the K meson and the nucleon interact only in the state with t = 1, an additional relation appears

$$d\sigma(\bar{\Sigma}^+ \to K^0p\pi^-\pi^0) = d\sigma(\bar{\Sigma}^+ \to K^+n\pi^-\pi^0). \qquad (11)$$

If the pions are produced in states with even orbital momentum (we refer to their relative motion), then the following equality holds

$$d\sigma(\bar{\Sigma}^+ \to K^+n\pi^-\pi^0) = \tfrac{1}{4}\,d\sigma(\bar{\Sigma}^+ \to K^+p\pi^-\pi^-). \qquad (12)$$

The wave function of the final state with T = 1 can also be constructed by considering the functions of the subsystems (πK) and (πM) with isotopic spins t = $^1\!/_2$ and $^3\!/_2$. In the limiting case when the interactionof the π-N system in the state with t = $^3\!/_2$ predominates, we obtain

$$\tfrac{2}{5}\,d\sigma(\bar{\Sigma}^+ \to K^+n\pi^-\pi^0) + \tfrac{29}{10}\,d\sigma(\bar{\Sigma}^+ \to K^+p\pi^-\pi^-)$$
$$+ d\sigma(\bar{\Sigma}^+ \to K^0n\pi^0\pi^0) = \tfrac{2}{3}\,d\sigma(\bar{\Sigma}^+ \to K^0n\pi^-\pi^+)$$
$$+ d\sigma(\bar{\Sigma}^+ \to K^0p\pi^0\pi^-),$$
$$\tfrac{63}{25}\,d\sigma(\bar{\Sigma}^+ \to K^+p\pi^-\pi^-) + \tfrac{3}{2}\,d\sigma(\bar{\Sigma}^+ \to K^0n\pi^0\pi^0)$$
$$= \tfrac{1}{3}\,d\sigma(\bar{\Sigma}^+ \to K^0n\pi^-\pi^+) + \tfrac{2}{25}\,d\sigma(\bar{\Sigma}^+ \to K^+n\pi^-\pi^0)$$
$$+ d\sigma(\bar{\Sigma}^+ \to K^0p\pi^-\pi^0). \qquad (13)$$

If suitable experimental data become available, relations (10) — (13) will enable us to obtain information on the cross sections of reactions with one and two π^0 mesons in the final state, which are very difficult to identify experimentally.

The foregoing analysis can be readily generalized to include heavier nuclei with zero isotopic spin (provided the isotopic spin is a good quantum number for such nuclei).

3. As in the case of nucleon-antinucleon collisions, we can expect in the interaction between antihyperons and nucleons the annihilation to be accompanied by the creation of at least two π^- and K mesons. Such processes will naturally take place also in interactions between antihyperons and nucleons bound in the nucleus.

We have already considered some of these reactions. However, in collision between antihyperons and nuclei, in addition to these usual annihilation processes, so-called "unusual" annihilation can take place, due to the absorption of part of the mesons produced by the remaining nucleons as a result of the elementary act, in analogy with the nucleon-antinucleon annihilation considered by Pontecorvo.[4] Thus, the following processes are possible

$$\bar{\Lambda} + d \to p + K^0, \quad \bar{\Sigma}^0 + d \to n + K^+, \quad \bar{\Sigma}^+ + d \to n + K^0.$$

along with the corresponding charge-symmetrical reactions with equal cross sections.

The isotopic invariance leads in this case to the equation $d\sigma(\bar{\Sigma}^0 \to nK^+) = \frac{1}{2} d\sigma(\bar{\Sigma}^+ \to nK^0)$. Interest attaches also to an experimental investigation of the inverse reactions, for this will afford a check on the spins of the $\bar{\Lambda}$ and $\bar{\Sigma}$ antihyperons, inasmuch as the ratio of the cross sections of the direct and inverse reactions contains the factor $(2S_N + 1)/(2S_{\bar{Y}} - 1)$, where S_N and $S_{\bar{Y}}$ are respectively the spins of the nucleon and the antihyperon.

Let us consider now several reactions of the "unusual" type, caused by the interaction between antihyperons and nuclei that are isotopic doublets (for example He^3 and H^3), i.e., reactions of the type $\bar{\Sigma} + X^3 \to N + N + K$. Denoting by A_{13} and A_{11} the amplitudes of the transitions in the states with T = 1 for the K-N system, and by A_{01} the amplitude with T = 0, we obtain

$$M(\bar{\Sigma}^- \to ppK^+) = A_{13},$$
$$M(\bar{\Sigma}^+ \to nnK^+) = \tfrac{1}{3} A_{13} - \tfrac{1}{3} A_{11} + \left(\tfrac{1}{3}\right)^{1/2} A_{01},$$
$$M(\bar{\Sigma}^+ \to npK^0) = \tfrac{1}{3} A_{13} + \tfrac{2}{3} A_{11},$$
$$M(\bar{\Sigma}^+ \to pnK^0) = \tfrac{1}{3} A_{13} - \tfrac{1}{3} A_{11} - \left(\tfrac{1}{3}\right)^{1/2} A_{01},$$
$$M(\bar{\Sigma}^0 \to ppK^0) = \left(\tfrac{2}{9}\right)^{1/2} A_{13} + \left(\tfrac{1}{18}\right)^{1/2} A_{11} - \left(\tfrac{1}{6}\right)^{1/2} A_{01},$$
$$M(\bar{\Sigma}^0 \to pnK^0) = \left(\tfrac{2}{9}\right)^{1/2} A_{13} - \left(\tfrac{2}{9}\right)^{1/2} A_{11},$$
$$M(\bar{\Sigma} \to npK^+) = \left(\tfrac{2}{9}\right)^{1/2} A_{13} + \left(\tfrac{1}{18}\right)^{1/2} A_{11} + \left(\tfrac{1}{6}\right)^{1/2} A_{01}. \quad (14)$$

Assuming the cross sections over the states of the nucleons, we obtain on the basis of (14) the well known equation[5]

$$d\sigma(\bar{\Sigma}^- \to ppK^+) + d\sigma(\bar{\Sigma}^+ \to pnK^0) + d\sigma(\bar{\Sigma}^+ \to nnK^+) = 2\,[d\sigma(\bar{\Sigma}^0 \to ppK^0) + d\sigma(\bar{\Sigma}^- \to pnK^+)]. \quad (15)$$

Using the expressions for the amplitudes $M(\bar{\Sigma}^- \to ppK^+)$, $M(\bar{\Sigma}^+ \to nnK^+)$, $M(\bar{\Sigma}^0 \to ppK^0)$, we obtain

$$|\{d\sigma(\bar{\Sigma}^+ \to nnK^+)\}^{1/2} - \{d\sigma(\bar{\Sigma}^- \to ppK^+)\}^{1/2}| \leqslant \{2d\sigma(\bar{\Sigma}^0 \to ppK^0)\}^{1/2} \leqslant \{d\sigma(\bar{\Sigma}^+ \to nnK^+)\}^{1/2} + \{d\sigma(\bar{\Sigma}^- \to ppK^+)\}^{1/2},$$
$$|\{2d\sigma(\bar{\Sigma}^0 \to ppK^0)\}^{1/2} - \{d\sigma(\bar{\Sigma}^+ \to nnK^+)\}^{1/2}| \leqslant \{d\sigma(\bar{\Sigma}^- \to ppK^+)\}^{1/2} \leqslant \{2d\sigma(\bar{\Sigma}^0 \to ppK^0)\}^{1/2} + \{d\sigma(\bar{\Sigma}^+ \to nnK^+)\}^{1/2},$$
$$|\{2d\sigma(\bar{\Sigma}^0 \to ppK^0)\}^{1/2} - \{d\sigma(\bar{\Sigma}^- \to ppK^+)\}^{1/2}| \leqslant \{d\sigma(\bar{\Sigma}^+ \to nnK^+)\}^{1/2} \leqslant \{2d\sigma(\bar{\Sigma}^0 \to ppK^0)\}^{1/2} + \{d\sigma(\bar{\Sigma}^- \to ppK^+)\}^{1/2}.$$

Analogously we can establish the following inequalities

$$d\sigma(\bar{\Sigma}^0 \to ppK^0) + d\sigma(\bar{\Sigma}^+ \to nnK^+) \geqslant \tfrac{1}{3} d\sigma(\bar{\Sigma}^- \to ppK^+),$$
$$d\sigma(\bar{\Sigma}^0 \to ppK^0) + d\sigma(\bar{\Sigma}^0 \to pnK^+) \geqslant \tfrac{2}{3} d\sigma(\bar{\Sigma}^- \to ppK^+),$$
$$d\sigma(\bar{\Sigma}^+ \to nnK^+) + d\sigma(\bar{\Sigma}^+ \to pnK^0) \geqslant \tfrac{1}{3} d\sigma(\bar{\Sigma}^- \to ppK^+). \quad (16)$$

With the aid of experimental data on the interaction between charged antihyperons, we could obtain from relations (16) and (17) information on the cross section of the reactions induced by neutral $\bar{\Sigma}^0$ antihyperons.

A similar analysis can be readily extended to the case of heavier nuclei (Li^7, Be^7), for which the isotopic spin is a good quantum number.

An interesting example of "unusual" reactions of processes of the type

$$\bar{\Sigma} + d \to \Sigma(\Lambda) + K + K.$$

The isotopic invariance gives rise to the relation

$$d\sigma(\bar{\Sigma}^+ \to \Sigma^- K^+ K^0) = d\sigma(\bar{\Sigma}^+ \to \Sigma^0 K^0 K^0) + d\sigma(\bar{\Sigma}^0 \to \Sigma^0 K^+ K^0). \quad (17)$$

If the K meson and Σ hyperon interact only in the state with T = ½, then

$$d\sigma(\bar{\Sigma}^+ \to \Sigma^0 K^0 K^0) : d\sigma(\bar{\Sigma}^+ \to \Sigma^- K^+ K^0) : d\sigma(\bar{\Sigma}^0 \to \Sigma^0 K^+ K^0) = 1:2:1. \quad (18)$$

In the opposite case of interaction in the state with T = 3/2,

$$d\sigma(\bar{\Sigma}^+ \to \Sigma^0 K^0 K^0) : d\sigma(\bar{\Sigma}^+ \to \Sigma^- K^+ K^0) : d\sigma(\bar{\Sigma}^0 \to \Sigma^0 K^+ K^0) = 1:5:4. \quad (19)$$

If, according to Gell-Mann and Pais, all the baryons are isotopic doublets

$$N_1 = \begin{vmatrix} \Sigma^+ \\ Y^0 \end{vmatrix}, \quad Y^0 = \frac{1}{\sqrt{2}}(\Lambda^0 - \Sigma^0),$$
$$N_2 = \begin{vmatrix} Z^0 \\ \Sigma^- \end{vmatrix}, \quad Z^0 = \frac{1}{\sqrt{2}}(\Lambda^0 + \Sigma^0),$$

we have the approximate relation

$$d\sigma(\bar{\Sigma}^+ \to \Sigma^0 K^0 K^0) \approx d\sigma(\bar{\Sigma}^+ \to \Lambda^0 K^0 K^0),$$

the accuracy of which is

$$\delta = (M_\Sigma - M_\Lambda)/M_\Sigma \approx 7\%.$$

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[1] D. Amati and B. Vitale, JETP **33**, 556 (1957), Soviet Phys. JETP **6**, 435 (1958).

[2] S. G. Matinyan and G. R. Khutsishvili, JETP **35**, 546 (1958), Soviet Phys. JETP **8**, 377 (1959).

[3] E. Fermi, Lectures on Pions and Nucleons (Transl.) IIL, 1956.

[4] B. Pontecorvo, JETP **30**, 947 (1956), Soviet Phys. JETP **3**, 966 (1956).

[5] L. I. Lapidus, JETP **31**, 865 (1956), Soviet Phys. JETP **4**, 740 (1957).

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