PAIRING FORCES AND PAIR CORRELATIONS IN THE Pb^{206} *NUCLEUS*

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The residual pairing forces acting between the nucleons in the nucleus are determined. The importance of taking these pairing forces and the pair correlations into account in determining the position and the properties of the levels is demonstrated. The energies, eigenfunctions, and transition probabilities of the levels of the Pb^{206} nucleus are derived and compared with experiment.

THE current models of the nucleus are based on the assumption, confirmed by experiment, that there exists a self-consistent field in the nucleus in which the nucleons move. However, the self-consistent field is not the only force acting on the individual nucleon; there also exist the so-called residual forces. The consideration of the residual forces is essential in the study of the behavior of the nucleons in the neighborhood of the Fermi energy surface. The importance of this circumstance follows from the application of the ideas of the theory of superconductivity to the study of the nucleus.

The present paper is devoted to the determination of the forces, including the residual ones, capable of explaining and describing all properties of the nuclear ground and excited states up to an excitation energy of 3 Mev.

CHOICE OF FORCES

Let us first consider a nucleus with one particle above the filled proton and neutron shells. The interaction of the extra particle with the particles in the core is described by the sum of a central potential $V_C(r, s\cdot l)$ and a quadrupole potential $V_S(r, \theta, \varphi)$. More precisely, V_C and V_S are the first two terms in the expansion of the self-consistent potential. This is shown most easily by the example of two-particle forces: $V(r_i) = \sum_k V(r_i - r_k)$. Expanding $V(r_i - r_k)$ in terms of the Legendre polynomials $P_l(\cos\omega_{ik})$ and averaging over the states of all particles except the i-th, we find that the term with $l = 0$ gives the central potential V_C and the term with $l = 2$ the quadrupole potential V_S. So far there has been no evidence that the higher multipole terms must be included in the potential for the outer nucleons; these terms will therefore be regarded as small. The potentials V_C and V_S represent the long range part of the total self-consistent potential, while the neglected terms give the short range part.

The central potential V_C is best determined from the calculations of the level spectra of nuclei with a single nucleon above closed shells or with a single hole. This was done by one of the authors and Volchok,[1] who found a potential of the form

$$V_C(r, ls) = V(r) - \lambda\left(\frac{\hbar}{2Mc}\right)^2 \frac{ls}{r}\frac{\partial V(r)}{\partial r}, \quad (1)$$

where

$$V(r) = -V_0/(1 + e^{\alpha(r-r_0)}). \quad (2)$$

The parameters V_0, r_0, α, and λ turned out to be nearly the same over the range of nuclei from O^{17} to Pb^{209}. The potential V_C can therefore be regarded as known.

The potential V_S is usually written in the form

$$V_S(r_k) = \pm \varkappa(r_k)\sum_\mu \alpha_{2\mu} Y_{2\mu}(\theta_k, \varphi_k); \quad (3)$$

The — and + signs correspond to particles and holes, respectively. The potential V_S can be regarded formally as an interaction of the outer particle with the remaining ones which leads to a deformation of the potential surface of the core. For small deformations of the potential surface,

$$\varkappa(r) = r\partial V(r)/\partial r, \quad (4)$$

where $V(r)$ is given by (2), we can compute the matrix elements

$$\langle nl\,|\varkappa(r)|\,n'l'\rangle = \int_0^\infty R_{nl}(r)\,R_{n'l'}(r)\,r^3\frac{\partial V}{\partial r}dr, \quad (5)$$

where $R_{nl}(r)$ are the radial functions of the particle in the potential (1), as derived in reference 1.

It turns out that, except for small deviations,

$$\langle nl|\varkappa(r)|n'l'\rangle \approx (-1)^{n+n'} \cdot 40 \text{ [Mev]}. \quad (5')$$

for all states of nuclei in the neighborhood of lead. This value will be used in the following.

The other parameter of the interaction with the surface, α_μ, can be represented in the form[2]

$$\alpha_\mu = \sqrt{\hbar\omega/2C}\,(b_\mu + (-1)^\mu b^*_{-\mu}), \quad (5'')$$

where C is the effective surface tension. Here we assume that the surface of the core undergoes ellipsoidal oscillations with the phonon energy $\hbar\omega$; b_μ and b^*_μ are annihilation and creation operators for a phonon with spin 2 and spin component μ along the z axis.

Let us now turn to a nucleus in which there are two extra particles or holes in addition to closed shells. The potential in which the extra nucleons move is

$$V = V_C + V_S + V_p(1,2). \quad (6)$$

The first two potentials, V_C and V_S, are single-particle potentials, whereas V_p is a two-nucleon potential. The potential V_p cannot be included in the first two terms, because its main short range part plays an important role. For in the case of two extra nucleons the potential V_p not only gives a contribution to the energy but also changes the character of the states. The pair interaction V_p is taken of the form

$$V_p = -[v_t\pi_t + v_s\pi_s]\exp\{-|\mathbf{r}_1 - \mathbf{r}_2|^2/\rho^2\}. \quad (7)$$

Here v_t and v_s are the parameters of the triplet and singlet interactions, and π_t and π_s are the corresponding operators:

$$\pi_t = \tfrac{1}{4}(3 + \sigma_1\sigma_2), \qquad \pi_s = \tfrac{1}{4}(1 - \sigma_1\sigma_2),$$

ρ is the effective range of the pair interaction.

Thus the spin dependent part of the interaction consists of a mixture of triplet and singlet forces. We leave out tensor forces, because there are no clear effects at low energies which would permit us to separate the tensor forces from the ordinary spin forces. The choice of a Gaussian shape for the radial dependence of the forces is somewhat arbitrary; any other strongly decreasing function would be just as good. Since the results depend on the integral effect, details cannot play any role, only the depth and the range of the potential well being of importance. We neglected the repulsive forces between the particles at distances $r_{12} \leq 0.5$ f, i.e., the so-called "hard core." The introduction of a hard core and the corresponding correction of the functions with the help of a cut-off factor of the type $1 - \exp(-r_{12}^2/r_c^2)$ give a negligible contribution to the energy. Hence the nature of the forces at small distances does not come into play in our problem.

METHOD OF CALCULATION

As the first nucleus to be studied we chose Pb^{206}, a nucleus with two neutrons missing from the filled shell. The choice of Pb^{206} was dictated by the fact that this nucleus has been investigated extensively and is therefore convenient for analysis. About 30 levels of the Pb^{206} nucleus are known, the spins are identified, and the transition probabilities or relative intensities have been determined. Several theoretical papers have been devoted to the study of the Pb^{206} nucleus with an analysis of the levels (see reference 3, and the literature quoted there); these can be used for an orientation in our calculations. We regard the Pb^{206} nucleus as a system consisting of two nucleons (holes) and the surface of the core. The Hamiltonian of this system is written as

$$\hat{H} = \hat{H}_S + \hat{H}_p(1) + \hat{H}_p(2) + \hat{V}_S(1) + \hat{V}_S(2) + \hat{V}_p\ (12), \quad (8)$$

where

$$\hat{H}_S = \hbar\omega\left(\tfrac{5}{2} + \sum_\mu b_\mu^* b_\mu\right), \qquad \hat{H}_p = -(\hbar^2/2M)\Delta + V_C;$$

V_C, V_S, and V_p are given by formulas (1), (3), and (7).

The eigenfunctions of the Hamiltonian $\hat{H}_S$ are functions of the quadrupole oscillation of the surface with the energy $\hbar\omega$ and the number of quanta N; the eigenfunctions of the Hamiltonian $\hat{H}_p$ are functions of the shell model potential (1). We therefore choose basis functions of the type

$$|j_1 j_2 J;\ NR;\ IM\rangle,$$

corresponding to a state where two nucleons with spins j_1 and j_2 are coupled with total angular momentum J; N phonons contribute the angular momentum R and together with J give the total angular momentum of the nucleus I with projection M.

We shall seek the eigenfunctions of the Hamiltonian $\hat{H}$ given by (8) in the form of an expansion in terms of the basis functions

$$|IM\rangle = \sum_{j_1 j_2 J;\ NR} c(j_1 j_2 J; NR)\,|j_1 j_2 J; NR; IM\rangle. \quad (9)$$

The energies and eigenfunctions of the single-particle states of Pb^{206} were determined in reference 1; the results needed here are shown in Fig. 1.

We must now calculate the various matrix elements. The matrix elements of the interaction with the surface oscillations were computed according to the scheme[3]

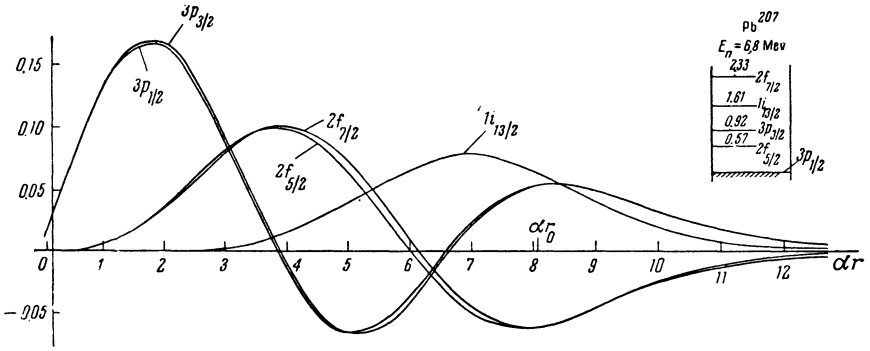

FIG. 1. Single-particle wave functions and energy levels of the Pb^{207} nucleus.

$$\langle j_1 j_2 J;\ NR; I \mid \sum_{1,2} \varkappa(r) \sum_{\mu} \alpha_{2\mu} Y_{2\mu}(\theta,\varphi) \mid j_1' j_2' J';\ N'R'; I\rangle$$
$$= \varkappa \sqrt{\hbar\omega/2C}\,(NR \| b^2 \| N'R')\ W(JRJ'R';\ I\,2)$$
$$\times \sqrt{(2J+1)(2J'+1)}\ (-1)^{J+R'-I}\{\delta_{j_1 j_1'}(-1)^{j_1'-l_2-J'}$$
$$\times (j_2 \| Y^2 \| j_2')\, W(j_2\ J\ j_2'\ J';\ j_1\,2) + \delta_{j_2 j_2'}(-1)^{j_2-j_1'-J}$$
$$\times (j_1 \| Y^2 \| j_1')\, W(j_1 J j_1' J';\ j_2\ 2)\}, \qquad (10)$$

where $\varkappa = (-1)^{n_1+n_2+n_3+n_4} \times 40$ [Mev]. The reduced matrix elements $(NR \| b^2 \| N'R')$ are given in the paper by Raz,[4] and the W's are Racah coefficients.

For the calculation of the matrix elements of the pair interaction V_p we expanded the coordinate part in a series:

$$\exp\{-r_{12}^2/\rho^2\} = \sum_{k=0}^{\infty} f_k(r_1, r_2) P_k(\cos\omega_{12}) = \sum_k f_k(r_1, r_2)$$
$$\times \sum_{m=-k}^{k} \frac{4\pi}{2k+1} Y_{km}^*(\theta_1,\varphi_1) Y_{km}(\theta_2,\varphi_2) \qquad (11)$$
$$= \sum_k f_k(r_1, r_2)\,(\mathbf{T}^k(1)\cdot\mathbf{T}^k(2));$$

$\mathbf{T}^k$ is the irreducible tensor operator of the k-th rank. The radial parts of the matrix elements of the interactions have the form of Slater integrals:

$$F^k(n_1 l_1, n_2 l_2;\ n_1' l_1',\ n_2' l_2')$$
$$= \int R_{n_1 l_1}(r_1) R_{n_2 l_2}(r_2) f_k(r_1, r_2) R_{n_1' l_1'}(r_1) R_{n_2' l_2'}(r_2)\, dr_1 dr_2, \qquad (12)$$

they were computed numerically on the electronic computer "Strela." The dependence of the integrals F^k on the parameter ρ for different k is shown in Fig. 2. Finally we have

$$\langle j_1 j_2, JM \mid v_s \pi_s \sum_k f_k(r_1 r_2)\,(\mathbf{T}^k(1)\cdot\mathbf{T}^k(2) \mid j_1' j_2',\ JM\rangle$$
$$= -v_s A \begin{pmatrix} l_1\ 1/2\ j_1 \\ l_2\ 1/2\ j_2 \\ J\ 0\ J \end{pmatrix} A \begin{pmatrix} l_1'\ 1/2\ j_1' \\ l_2'\ 1/2\ j_2' \\ J\ 0\ J \end{pmatrix} F^k(n_1 l_1, n_2 l_2;\ n_1' l_1',\ n_2' l_2')$$
$$\times (-1)^{l_1+l_2'-J} \sqrt{(2l_1'+1)(2l_2'+1)}$$
$$\times C_{l_1' 0\, k\, 0}^{l_1 0}\, C_{l_2' 0\, k\, 0}^{l_2 0}\, W(l_1 l_2, l_1' l_2';\ Jk), \qquad (13)$$

where the A's are the coefficients for the transformation from jj to ls coupling,[5] and the

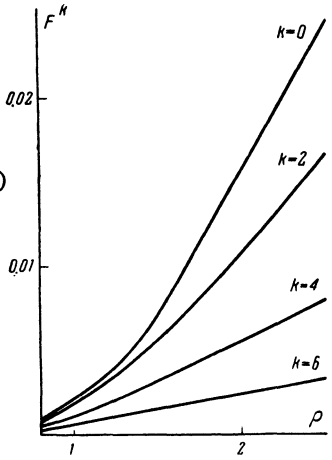

FIG. 2. Dependence of the integrals $F^k(n_1 l_1, n_2 l_2; n_1' l_1', n_2' l_2')$ on the parameter ρ for different values of k.

$C_{l_1 m_1 l_2 m_2}^{lm}$ are Clebsch-Gordan coefficients. An analogous expression holds for the triplet forces.

The set of matrix elements corresponding to given spin and parity forms the energy matrix. By reducing this matrix to its diagonal form, we obtain the energy levels and the eigenfunctions of the operator $\hat{H}$ defined by (8). Our method of calculation is approximate in that we include only a limited number of levels in the matrix. The lower levels will then be determined more accurately than the higher-lying ones. Thus, if we require that the first four low-lying levels with a given I be obtained with some desired accuracy, we thereby determine at which levels the matrix should be cut off. In our paper the rank of the matrices does not exceed twenty. These matrices were diagonalized on the "Strela" computer.

DETERMINATION OF THE PARAMETERS

The pair interaction contains three parameters: v_t, v_S, and ρ. However, in the case of Pb^{206}, where we have two identical particles, all basic matrix elements contain v_S and v_t approximately in the unique combination $2v_S + v_t$. Under these circumstances we may retain only one constant, for example, v_S, which will be regarded as the effective sum of two constants.

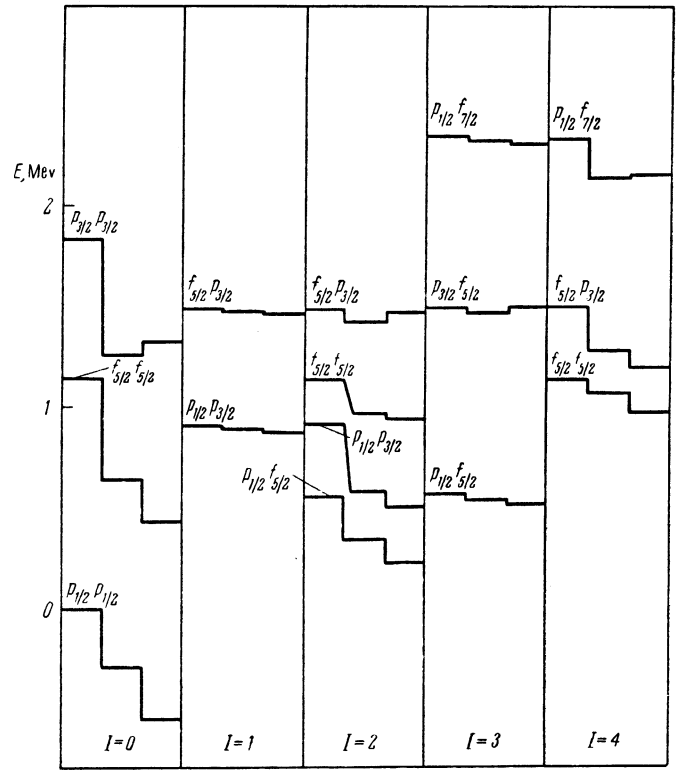

FIG. 3. Shift of the level positions as a result of the pairing forces. On the left of each column we show the single-particle level, in the center, the level after inclusion of the diagonal term of the pairing interaction only, and on the right, the level after the pair correlations have also been included. The ordinate gives the energy in Mev.

The interaction with the surface contains two parameters, the phonon energy $\hbar\omega$ and the effective surface tension C. Hence we must determine four constants: ρ, v_S, $\hbar\omega$, and C.

Not all of the required physical quantities are equally sensitive to these four constants. Thus the distance between the first 2^+ level and the ground state 0^+ depends mainly on the depth of the well v_S; small variations of the parameter ρ lead to appreciable changes in the binding energy of the two nucleons, but do not affect greatly the distance between the levels 2^+ and 0^+. The probability for the transition $2^+ \rightarrow 0^+$ is mainly determined by the ratio $\hbar\omega/C$. This allows us to make a crude determination of the possible values of the required parameters before the diagonalization.

First let us carry out the diagonalization taking only the pair interaction into account. The corresponding results are shown in Figs. 3 and 4. They lead to the following important conclusions:

1) The matrix elements of the pair interaction are large for levels with spin I = 0, approximately half as large for the levels with I = 2, even smaller for I = 4, and altogether unimportant for the other levels. The inclusion of the nondiagonal matrix elements, or in other words, of the pair correlations, leads to a considerable widening of the distance between the levels with given I. Since this shift is largest for the first level I = 0, there appears an energy gap of 0.8 to 1.0 Mev between the ground state with spin I = 0 and the other levels.

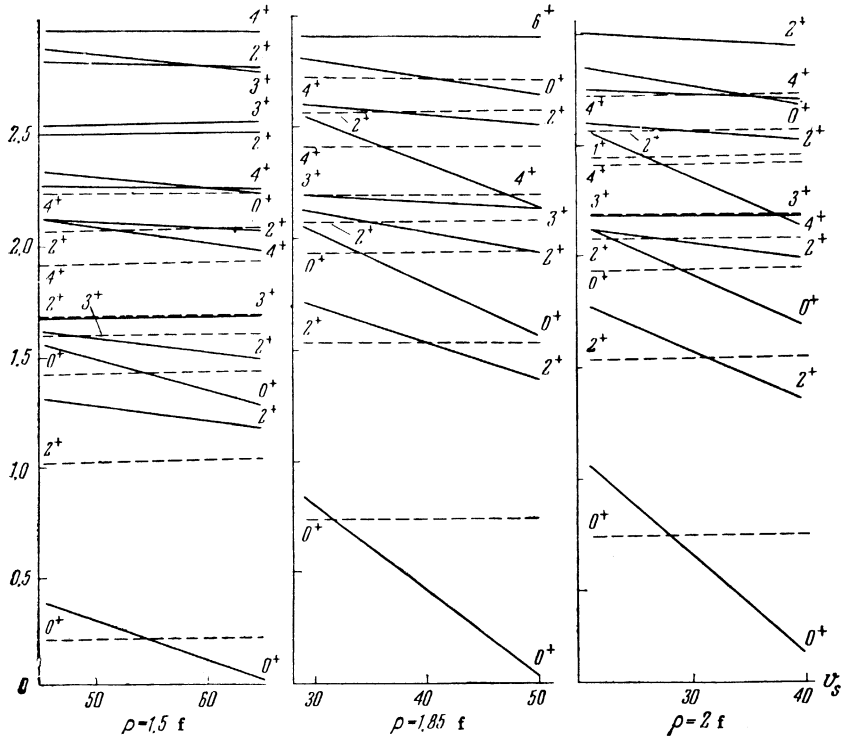

FIG. 4. Relative positions of the energy levels of the Pb^{206} nucleus for different values of the parameters ρ and v_S. The experimental levels are indicated by dotted lines.

Table I. Matrix elements of the pairing interaction using the radial wave functions of an oscillator and a diffuse potential

Configuration	Oscillator potential	Diffuse potential	Ratio
$(p_{1/2})^{20+}$	—0,427	—0.349	0.817
$(f_{5/2})^{20+}$	—0,685	—0.581	0.848
$(p_{3/2}\, f_{5/2})^{1+}$	0	0	—
$(p_{1/2}\, f_{5/2})^{2+}$	—0,329	—0.236	0.717
$(p_{1/2}\, p_{3/2})^{2+}$	—0,520	—0.399	0.767
$(p_{3/2})^{22+}$	—0,260	—0.199	0.765
$(f_{5/2})^{22+}$	—0,235	—0.195	0.830
$(p_{3/2}\, f_{5/2})^{2+}$	—0,094	—0.069	0.734
$(p_{1/2}\, f_{5/2})^{3+}$	—0.037	—0.026	0.676
$(p_{3/2}\, f_{5/2})^{3+}$	—0,046	—0,036	0.783
$(f_{5/2})^{24+}$	—0.101	—0.083	0.822
$(p_{1/2}\, f_{5/2})^{4+}$	—0.364	—0.265	0.728

Table II. The coefficients $c(j_1, j_2, J; NR)$ for the basic components of the wave functions of the levels of the Pb^{206} nucleus for I = 0

$j_1\, j_2,\ J;\ NR$	E, Mev			
	—1,06 7	0,26	1.00	2.4
$p_{1/2}\ p_{1/2},\ 0;\ 00$	0.836	—0.355	—0,267	—0,026
$f_{5/2}\ f_{5/2},\ 0;\ 00$	0.283	0.838	—0.089	—0.113
$p_{3/2}\ p_{3/2},\ 0;\ 00$	0.264	0,014	0,898	0.053
$i_{13/2}\ i_{13/2},\ 0;\ 00$	—0.051	—0.077	—0,047	0,859
$f_{7/2}\ f_{7/2},\ 0;\ 00$	0.073	0,110	0,038	—0.148
$p_{1/2}\ p_{1/2},\ 0;\ 20$	0,045	0,002	—0,003	0.033
$f_{5/2}\ f_{5/2},\ 0;\ 20$	0,023	0,040	—0,013	0,070
$p_{1/2}\ f_{5/2},\ 2;\ 12$	0.271	0,067	—0.139	0,077
$p_{1/1}\ p_{3/2},\ 2;\ 12$	—0,216	0,076	—0.151	—0,089
$f_{5/2}\ f_{5/2},\ 2;\ 12$	0,099	0.368	—0,071	0.441
$f_{5/2}\ f_{5/2},\ 2;\ 12$	—0.049	—0.069	—0,102	—0,010
$p_{3/2}\ p_{3/2},\ 2,\ 12$	0.056	0.004	0.226	0,046
$f_{5/2}\ f_{5/2},\ 4;\ 24$	0.029	0,037	—0.030	0,073
$f_{5/2}\ p_{3/2},\ 4;\ 24$	—0.051	—0,009	—0,024	—0,041

2) The inclusion of the pair interaction alone is not sufficient to obtain the correct energy levels, let alone the correct transition probabilities.

3) For $\rho < 1.5$ f and $\rho < 2.5$ f the level ordering does not agree with experiment. This discrepancy cannot be removed by taking the interaction with the surface into account. It follows, in particular, that, for reasons of simplicity, δ forces can be used to obtain various qualitative results; but they are inadequate for a quantitative investigation.

It is interesting to compare our results with the calculations with oscillator functions. For this purpose we list in Table I the values of some matrix elements of the pair interaction from the paper of True and Ford[3] together with our values, after they have been reduced to the same ρ and v_s. It is seen that the use of oscillator functions gives too large matrix elements.

We must now include the interaction with the surface before diagonalizing the matrix. Regarding the choice of the parameters of the surface interaction $\hbar\omega$ and C we note that the number of possible single-particle levels with spin $I = 2^+$ is equal to the number of the known experimental levels with that spin up to excitation energies of 2 Mev. Hence $\hbar\omega$ cannot be smaller than 2 Mev. We have investigated all values from 2 to 10 Mev

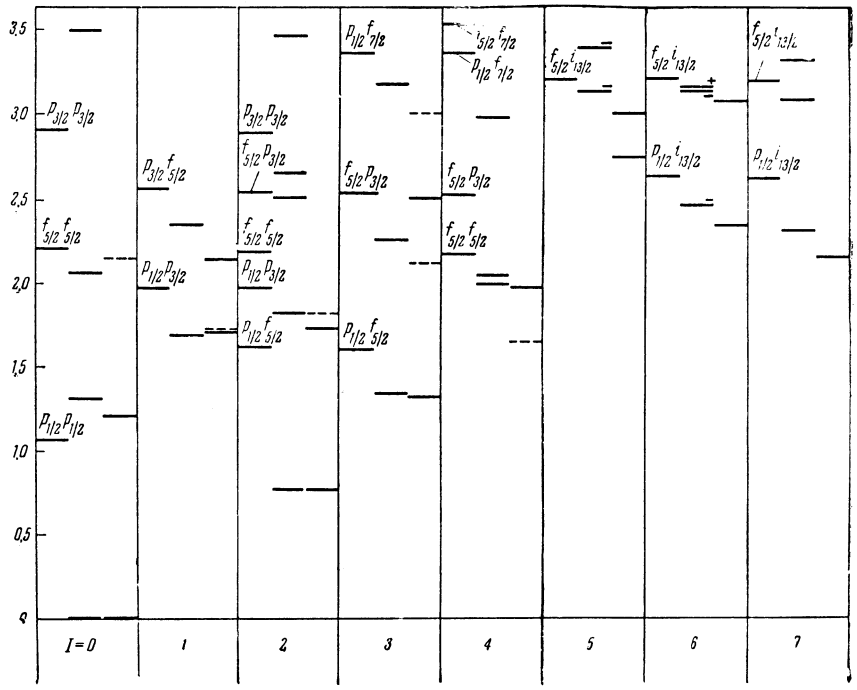

FIG. 5. Comparison of the results obtained for the energy levels of Pb^{206} with the experimental data. On the left of each column we give the position and configuration of the single-particle levels; in the center, the final positions of the levels after the inclusion of the pairing forces and the interaction with the surface of the core; and on the right, the experimental levels. Dotted lines designate levels which have not been determined with sufficient reliability. A minus sign above a level denotes negative parity of the level.

Table III. The coefficients $c(j_1, j_2, J; NR)$ for the basic components of the wave functions of the levels of the Pb^{206} nucleus for I = 2

$j_1 j_2, J; NR$	E, Mev					
	−0.30	0.38	0.76	1.48	1.60	2.71
$p_{1/2} p_{1/2}$, 0; 12	0.200	−0.043	−0.064	0.071	−0.056	0.921
$f_{5/2} f_{5/2}$, 0; 12	0.094	0.044	0.109	−0.041	−0.014	0.147
$p_{3/2} p_{3/2}$, 0; 12	0.067	−0.085	−0.007	−0.056	0.081	0.235
$p_{1/2} f_{5/2}$, 2; 00	0.766	0.508	−0.240	0.105	0.016	−0.141
$p_{1/2} p_{3/2}$, 2; 00	−0.440	0.803	0.055	−0.196	0.255	0.154
$f_{5/2} f_{5/2}$, 2; 00	0.209	0.064	0.935	0.033	−0.003	0.012
$f_{5/2} p_{3/2}$, 2; 00	−0.143	0.041	0.003	0.933	0.252	−0.017
$p_{3/2} p_{3/2}$, 2; 00	0.144	−0.222	−0.005	−0.184	−0.912	0.013
$p_{1/2} f_{5/2}$, 2; 12	0.152	0.046	0.154	−0.052	−0.038	−0.099
$p_{1/2} p_{3/2}$, 2; 12	−0.102	0.109	0.018	0.106	−0.162	0.080
$f_{5/2} f_{5/2}$, 2; 12	0.097	0.072	−0.124	−0.023	−0.005	−0.057
$p_{1/2} f_{5/2}$, 2; 20	0.036	0.004	0.003	−0.000	−0.002	0.059
$f_{5/2} f_{5/2}$, 4; 12	0.103	0.077	0.097	0.122	0.032	−0.039
$f_{5/2} p_{3/2}$, 4; 12	−0.166	0.074	0.020	0.072	0.007	0.085

in the three-phonon approximation and found that the best results are obtained for $\hbar\omega$ = 3 Mev and C = 1000 Mev.

In Fig. 5 and Tables II to V we give the main results of the diagonalization of the energy matrix, where the interaction parameters have the following values: v_S = 25 Mev, ρ = 2 f, $\hbar\omega$ = 3 Mev, and C = 1000 Mev.*

The results show that our choice of forces reproduces all experimentally known excited levels up to an excitation energy of 3 Mev. The only exception is the 4^+ level at 1.66 Mev. One should, therefore, first re-examine the experimental evidence for the existence of this level. The theoretical spectrum contains many more levels than the experimental one. This is plausible, since the transition probabilities from certain levels are very small and difficult to observe.

As we have obtained the eigenfunctions for the levels of the nucleus, we can compute the transition probabilities. The transition probability for a γ ray of given multipolarity λ is equal to

$$T(\lambda) = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2}\frac{1}{\hbar}\left(\frac{\Delta E}{\hbar c}\right)^{2\lambda+1} B(\lambda), \qquad (14)$$

$$B(\lambda) = (2I+1)^{-1} |\langle \alpha I \| \hat{M}_\lambda \| \beta I' \rangle|^2,$$

where I′ and I are the nuclear spins in the initial and final states, respectively; α and β are additional quantum numbers characterizing the states; $\hat{M}_\lambda$ is the operator of the multipole transition; ΔE is the transition energy. The complete expressions for the operator $\hat{M}_\lambda$ and the quantity $B(\lambda)$ for the electric and magnetic transitions with various values of the initial and final quantum numbers j_1, j_2, N, R, and I have already been given.[2-4] We shall, therefore, not write down the formulas by which the probabilities were computed, but only give the final results (see Table VI).

Table IV. The coefficients $c(j_1, j_2, J; NR)$ for the basic components of the wave functions of the levels of the Pb^{206} nucleus for I = 3

$j_1 j_2, J; NR$	E, MeV		
	0.320	1.20	2.14
$p_{1/2} f_{5/2}$, 3; 00	0.969	0.057	0.022
$p_{3/2} f_{5/2}$, 3; 00	−0.037	0.955	0.116
$p_{1/2} f_{7/2}$, 3; 00	−0.012	−0.091	0.945
$f_{5/2} f_{7/2}$, 3; 00	−0.007	0.024	−0.032
$p_{3/2} f_{7/2}$, 3; 00	0.013	0.023	−0.017
$p_{1/2} p_{3/2}$, 1; 12	0.025	−0.109	0.227
$f_{5/2} p_{3/2}$, 1; 12	−0.067	0.099	0.013
$f_{5/2} f_{7/2}$, 1; 12	0.001	−0.013	0.073
$p_{1/2} f_{5/2}$, 2; 12	−0.061	0.107	0.153
$p_{1/2} p_{3/2}$, 2; 12	0.062	0.148	−0.100
$p_{1/2} f_{5/2}$, 3; 12	−0.134	0.132	−0.021
$p_{3/2} f_{5/2}$, 3; 12	0.081	−0.044	−0.001
$f_{5/2} f_{5/2}$, 4; 12	0.149	0.043	0.010

For an understanding of the above results we recall that the functions of the initial and final states are mixtures of various single-particle and collective levels. The transition probability will therefore be of the form of the square of the modulus of a sum of transition matrix elements

*The single-particle level energies for Pb^{207} used by us were obtained without corrections for the interaction with the surface. These corrections can be easily included. They have significant effect on the computations; possibly the parameter v_S is somewhat increased while ρ becomes a little smaller. The levels with large I are lowered somewhat if the single-particle level in Pb^{206} lies lower than in Pb^{207}.

between different components of the mixture. The terms in this sum will have either sign, and it is therefore not surprising that two levels of the same type can have transition probabilities which differ by two orders of magnitude. The contribution of a given level to the mixture is determined by the force parameters and the position of the level. The E2 transition probabilities are particularly sensitive to the parameters of the interaction with the surface $\hbar\omega$ and C; here the E2 transition from the first level 2_1^+ is less sensitive to these parameters than that from the other levels. Thus one can reproduce the experimentally observed value of the probability for the E2 transition with the parameters $\hbar\omega$ = 3 Mev, C = 1000 Mev as well as with $\hbar\omega$ = 10 Mev, C = 1000 Mev.* However, for the second choice of parameters the probability for an E2 transition from the second level 2_2^+ to 0^+ will be only one fifth of the true value. The pair correlations, on the other hand, lead to a decrease of the probability of E2 transitions, especially for the high-lying levels.

Table V. The coefficients c (j_1, j_2, J; NR) for the basic components of the wave functions of the levels of the Pb^{206} nucleus for I = 4

$j_1 j_2$, J; NR	E, Mev			
	0.965	1.02	1.95	2.69
$f_{5/2}\, f_{5/2}$, 4; 00	0.977	—0.087	0.071	0.066
$f_{5/2}\, p_{3/2}$, 4; 00	0.080	0.941	—0.226	0.004
$p_{1/2}\, f_{7/2}$, 4; 00	—0.038	0.229	0.903	—0.119
$f_{5/2}\, f_{7/2}$, 4; 00	—0.063	0.049	0.115	0.959
$p_{3/2}\, f_{7/2}$, 4; 00	0.011	—0.107	—0.052	0.028
$f_{5/2}\, f_{5/2}$, 4; 12	0.100	0.063	—0.001	0.135
$f_{5/2}\, p_{3/2}$, 4; 12	0.088	0.186	0.076	—0.107
$p_{1/2}\, f_{7/2}$, 4; 12	0.011	—0.010	—0.134	—0.110
$p_{1/2}\, p_{5/2}$, 2; 12	0.002	—0.005	—0.393	0.022
$p_{1/2}\, p_{5/2}$, 2; 12	0.005	—0.046	—0.291	0.086
$f_{5/2}\, f_{5/2}$, 2; 12	0.120	0.007	0.007	—0.103
$p_{1/2}\, p_{1/2}$, 0; 24	—0.001	0.006	0.049	—0.021
$f_{7/2}\, f_{5/2}$, 0; 24	0.012	0.001	0.006	—0.018
$p_{1/2}\, f_{5/2}$, 2; 22	0.005	—0.001	—0.004	0.010

Table VI. Transition probabilities * between the levels of the Pb^{206} nucleus

Initial state	Final state	Type of transition	ΔE, Mev	B	T	Initial state	Final state	Type of transition	ΔE, Mev	B	T
2_1	0_1	E2	0.815	0.254	0.092	2_4	0_1	E2	2.400	0.003	0.252
2_2	0_1	E2	1.460	0.008	0.053	2_5	0_1	E2	2.650	0.002	0.327
2_2	2_1	M1	0.650	2.125	0.570	3_1	2_1	E2	0.560	0.010	0.001
0_2	2_1	E2	0.360	0.033	0.0002	3_2	2_1	E2	1.430	0.001	0.004
0_3	2_1	E2	1.250	0.001	0.002	4_1	2_1	E2	1.200	0.015	0.037
2_3	0_1	E2	1.800	0.001	0.010						

*The transition probability $T_{E2}(2_1 \to 0_1) = 0.103 \times 10^{12}$ sec^{-1} was found experimentally. For the transitions from the level 2_2 the following relation holds: $T_{M1}(2_2 \to 2_1) : T_{E2}(2_2 \to 0_1) = 4 : 1$. The relative intensities for the other levels are given in the paper of Alburger and Pryce.[6] In the table $B = T(\lambda)/(\Delta E)^{2\lambda+1}$, where ΔE is in Mev.

It is of interest to determine the mean radius of a pair. With the functions obtained above we find $(|\mathbf{r}_1 - \mathbf{r}_2|^2)^{1/2}$ cm, which is approximately equal to the nuclear radius.

CONCLUSIONS

The account of the surface and pair forces leads not only to a change in the energies of the single-particle states. A more important effect is the change of the character of the states as expressed in the structure of the wave functions. In the first place, all states, for all values of the spin I, contain a considerable admixture of collective states which complicate the properties of the levels. In the second place, the pair correlations lead to a mixing of single-particle states, which becomes more pronounced as the level density increases.

The probability for transitions from the second, third, etc. levels with a given I are very sensitive to the interaction parameters. Thus the E2 transition $2_2 \to 2_1$ neglecting the surface interaction is smaller by three orders of magnitude than the value found with our choice of parameters (if the surface interaction is included in a perturbation treatment, we find a value which is smaller by one order of magnitude than the value corresponding to our choice of parameters). The mixing of states sometimes decreases the transition probability by several orders of magni-

*For $\hbar\omega$ = 10 Mev and C = 1000 Mev the results are close to those obtained by perturbation methods.

tude. For example, the E2 transitions $2_3 \rightarrow 0_1$ and $0_3 \rightarrow 2_1$ have, despite the admixture of collective states, a vanishingly small transition probability. In particular, the mixing of states has as a consequence that not all high E2 transitions will be enhanced.

All this shows that a correct account of the forces acting on the nucleons in the nucleus leads to a considerably more detailed picture of the properties of the nucleus than is provided by the existing models. On the other hand, the picture is not too complicated to permit a theoretical description.

In future papers we shall give the results for other nuclei with two and more particles outside closed shells.

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