AN INVESTIGATION OF THE PARAMAGNETISM OF μ-MESIC ATOMS

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The nature of the paramagnetism of various mesic atoms was investigated by measuring the asymmetry of μ-e decay electrons. The experimental results indicate that in dielectrics and in normal diamagnetic and weakly paramagnetic metals the paramagnetism of mesic atoms results from the μ^--meson magnetic moment, whereas in paramagnetic transition metals, lanthanides, and actinides it is due to the magnetic moments of both the electron shell and meson. It is shown that polarized mesons can be employed to investigate the magnetic properties of the atoms and hydrides of transiton metals, actinides, and lanthanides with zero nuclear spin.

1. INTRODUCTION

THE investigation of fine structure and hyperfine structure in atomic systems and of the associated quantum electrodynamical effects is of great interest. While ordinary atoms and positronium have been studied extensively, such atomic systems as muonium and μ-mesic atoms are also attractive objects of study. An earlier paper[1] reported an investigation of the hyperfine structure of μ-mesic atoms that results from coupling of the meson and nuclear spins. The present paper is concerned mainly with the hyperfine structure due to meson-electron coupling. Some of the many reasons for practical interest in this effect will now be enumerated.

A. Polarized μ^- mesons in matter are essentially depolarized as a result of the interactions that produce fine and hyperfine structure in mesic atoms.[1] One can inquire whether it is possible to eliminate or restore meson polarization in mesic atoms. It is obvious that depolarization due to hyperfine structure resulting from meson-nucleus spin coupling can be obviated by using substances with zero nuclear spin. It would appear that depolarization due to fine structure or hyperfine structure resulting from spin coupling of the meson and electron shell could be eliminated by using a sufficiently strong magnetic field. However, the magnetic neutralization of depolarization encounters insurmountable experimental difficulties. Kinematic depolarization associated with π-μ decay in flight can also not be eliminated. The fact that the depolarization time ($\sim 10^{-12}$ sec) is many orders of magnitude shorter than the meson lifetime τ suggests that static methods of nuclear polarization be used to repolarize mesons in mesic atoms with zero nuclear spin.

For our purposes two methods are of interest, in both of which magnetic fields are applied at low temperatures. The first method uses the effect of an external magnetic field on the nuclear magnetic moment, while the second method uses the internal atomic magnetic field resulting from uncanceled electron moments. The coefficient of nuclear polarization produced directly by the external field H is given by

$$f \approx \tfrac{1}{3}\,[(I+1)/I]\,(\mu_N H/kT), \tag{1}$$

where I is the nuclear spin, μ_N is the nuclear magnetic moment, k is the Boltzmann constant, and T is the absolute temperature.

When internal atomic magnetic fields are used, nuclear polarization is given by[2]

$$f \approx \tfrac{1}{3}\, f_e\,[(I+1)/I]\,[J_e/(2J_e+1)]\,(\Delta W/kT), \quad J_e \gg 1, \tag{2a}$$

$$f \approx \tfrac{1}{3}\, f_e[(I+1)/(2I+1)]\,(\Delta W/kT), \quad J_e \ll 1. \tag{2b}$$

Here f_e is the electron polarization, J_e is the electron angular momentum, and ΔW is the hyperfine splitting energy.

It follows from (1) and (2) that the greater μ_N and ΔW, and the smaller I, the easier it is to achieve a given degree of polarization.

Since a muon has spin ½, while its magnetic moment is one order of magnitude greater than the nuclear magneton, a given degree of meson polarization in mesic atoms can be produced by using temperatures that are higher or magnetic fields that are weaker by one order of magnitude than in the case of nuclei.

It must be noted that the external field method can encounter insuperable difficulties because of the large meson spin relaxation time compared with τ. These difficulties disappear when internal atomic fields are used. Indeed, the electron moments in a sample can be polarized before irradiation with mesons. The meson spin-flip time in the field of a mesic-atom shell will then be given[1] by the "interaction time" $t' \sim \hbar/\Delta W \sim 10^{-10}$ sec, which is many orders of magnitude smaller than τ.

B. At the present time one of the most effective methods of investigating the electron configurations of paramagnetic atoms is the study of the emission asymmetry of polarized nuclei. This method is obviously feasible only for elements with nonzero nuclear spin. By investigating the asymmetry of the decay electrons of polarized muons in mesic atoms it is possible to study the electronic structure of atoms in paramagnetic substances having zero nuclear spin.

C. The measurement of the gyromagnetic ratio of mesic atoms having zero nuclear spin and exhibiting electron spin paramagnetism due to a single unpaired electron makes it possible to determine μ^- spins directly.

D. The paramagnetism of mesic atoms must be investigated before the depolarization mechanism of $\mu^\pm$ meşons can be determined.

2. BASIC THEORETICAL POSTULATES

The formation of mesic atoms is associated with the destruction of the electron-shell state of the original atom. The lifetime t_0 of an excited shell state of free mesic atoms depends only on the electron configuration and degree of excitation. When mesic atoms are formed in a substance the lifetime t_0 depends greatly on the nature of the atomic bonds. In a metal the electron shell of a mesic atom returns to the ground state in a very short time ($< 10^{-12}$ sec) compared with t' and τ. On the other hand, in ionic crystals or dielectrics t_0 is greater than t' and τ. Consequently, at the instant of decay the electron state of a mesic atom depends on the kind of compound that contains the given atom and on the state of aggregation.

In the case of isolated mesic atoms with zero nuclear spin, when the electron shell has a nonzero moment the paramagnetic moment will consist of three parts:

1) the magnetic moment of the muon,

$$\mu_\mu = \sqrt{3}\, M_B/207, \tag{3}$$

2) the electron orbital magnetic moment

$$\mu_L = M_B \sqrt{L(L+1)}, \tag{4}$$

3) the electron spin magnetic moment

$$\mu_S = 2M_B \sqrt{S(S+1)}. \tag{5}$$

When mesic atoms are formed in a medium their paramagnetism will be influenced by neighboring atoms, and some magnetic moments will be canceled, depending upon the electron shells that are responsible for the magnetic moment. By analogy with the properties of the ionic magnetic moments of paramagnetic substances we can expect that when mesic atoms are formed from lanthanide or actinide atoms, where the magnetic moment is due to deep-lying electrons that are least subject to external influences, the paramagnetism of such mesic atoms in a substance will result from all magnetic moments — μ_L, μ_S, and μ_μ. In the case of the transition elements, where the electrons responsible for the atomic magnetic moment are not deep-lying and are thus more subject to external influence, mesic atoms in a substance can possess only the spin moments μ_S and μ_μ, the orbital moments being canceled, as a rule. Finally, mesic atoms of diamagnetic substances, or normal weakly paramagnetic metals can possess only μ_μ moments on the average.

In substances with uncanceled moments the electron shell can affect muon polarization only in the mesic-atom ground state. This is easily seen by comparing the lifetime t of a meson on lower levels with the time t' required for meson spin flip in the field of the shell. The inequality $t' \ll t$ is fulfilled only for the 1S level, as indicated in Sec. 1.

It must be noted that in substances with angular momentum $J_e \neq 0$ of the electron shell, mesic atoms will be formed in two hyperfine states with total spins $F = J_e \pm 1/2$. In first approximation the hyperfine interaction energy ΔW in mesic atoms will be of the same order as in muonium, that is,

$$\Delta W = -32\, \mu_\mu p_{\mathrm{eff}} / 3a_{e\mu}^3.$$

Here μ_μ is the muon magnetic moment, p_{eff} is the effective magnetic moment of the electron shell, and $a_{e\mu}$ is the Bohr radius of the hydrogen atom. The hyperfine splitting of a mesic-atom ground state is much larger than $\hbar/\tau$. Therefore the states with $F = J_e + 1/2$ and $J_e - 1/2$ are incoherent in a mesic atom; each state is characterized by its own value of the gyromagnetic ratio. Thus the expressions for g are[4]

$$g_+ = (\mu_\mu + p_{\mathrm{eff}})/(J_e + 1/2), \tag{6}$$

$$g_- = -\left[\mu_\mu - \frac{(J_e + 1)}{J_e} p_{\text{eff}}\right] / (J_e - {}^1/_2). \quad (7)$$

If mesic-atom paramagnetism is studied by measuring the asymmetry of decay electrons,[5] the experimentally observed precession curve will represent the superposition of the precession curves of mesons decaying from both hyperfine states. The larger the value of J_e, the more difficult it will obviously be to interpret this curve. In actuality we denote by P_0 the degree of polarization of the muon beam. According to the literature,[4,6-8] the degree of polarization P of a meson in the K shell, averaged over the two hyperfine states, will be

$$P = \tfrac{1}{6} P_0 \cdot \tfrac{1}{3} [1 + 2/(2J_e + 1)^2]. \quad (8)$$

It follows from this formula that the polarization depends on the total moment J_e of the electron shell. Thus, for $J_e = {}^1/_2$ polarization will be reduced to one-half, while for mesic atoms with $J_e \gg 1$ it will be reduced to one-third. All of the foregoing discussion obviously pertains to isolated mesic atoms. The presence of a surrounding medium can complicate the picture; for example, meson transitions between hyperfine levels can occur.

3. EXPERIMENTAL PROCEDURE

The following materials were investigated: 1) dielectrics (paraffin, polythene, water, and sulfur); 2) diamagnetic and normal weakly paramagnetic metals (graphite, magnesium, zinc, cadmium, and lead); and 3) paramagnetic transition metals (chromium, molybdenum, palladium, and tungsten). The results obtained for water, magnesium, sulfur, zinc, cadmium, and lead have been published in reference 9; the remaining substances were investigated in the present work.

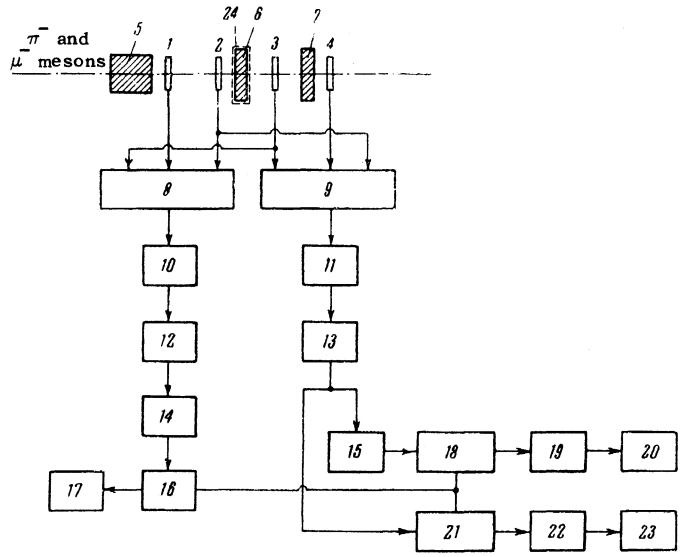
Chromium, molybdenum, tungsten, and palladium were studied in metallic form. The carbonaceous media were graphite, polythene, and paraffin, which were selected for the following reasons. The direction of the μ^--meson spin has so far been an open question,[10] the answer to which has been sought, in one way, by investigating the asymmetry of B^{12} decay electrons in the reaction $\mu^- + C^{12} \rightarrow B^{12} + \nu$.[11] The experiments reported in reference 10 revealed an absence of asymmetry that evidently resulted from the depolarization of B^{12} nuclei in the investigated samples of hexane and pentane. According to Jackson, Treiman, and Wyld[11] there are two possible causes of B^{12} depolarization: 1) the hyperfine structure due to the spin coupling of the B^{12} nucleus and electron shell, and 2) the short relaxation time of B^{12} nuclear spin in the given materials compared with the B^{12} lifetime. We proposed to check the presence of a B^{12} depolarization mechanism due to hyperfine structure by using different carbon compounds, with the expectation that mesic carbon atoms in these substances would have the electron configuration of the boron atom.[11]

The paramagnetism of mesic atoms of carbon, chromium, molybdenum, tungsten, and palladium in the given substances was investigated by the precession technique,[5] where the nature of the paramagnetism can be determined by measuring the asymmetry of μ-e decay electrons in the following manner. The electronic circuit is tuned to register the precession frequency of the free meson spin. It follows from Eqs. (6) and (7) that in a given magnetic field H the spin precession frequency of a mesic atom possessing both an electron moment and a meson moment will be several orders of magnitude greater than the spin precession frequency of mesic atoms in which paramagnetism is due to meson spin alone. In virtue of the large frequency difference the nature of the paramagnetism can be determined by measuring the numbers of μ-e decay electrons, N_{max}, and N_{min} at two field strengths $\pm$H, using the formula

$$t_1 + \Delta t = T/2 = \pi mc/eH, \quad (9)$$

where t_1 is the delay time, Δt is the gate width, and T is the spin precession period of a "free" muon. For mesic atoms possessing an electron moment the ratio $\xi = N_{max}/N_{min}$ will be unity, whereas for mesic atoms in which paramagnetism results from meson spin alone ξ will differ from unity. A control experiment that confirmed directly the existence of electronic paramagnetism by this technique was the measurement of ξ in the hydrides of paramagnetic metals having the hydrogen concentration at which the paramagnetism of the compound, such as $PdH_{0.6}$, decreases to zero. Indeed, the palladium atoms in a $PdH_{0.6}$ solution have no magnetic moment, and the hydrogen will not participate in the formation of mesic atoms.

The experiments were performed with the same apparatus, except for the electronic equipment, and under the same experimental conditions as in our earlier work.[9,12] A block diagram of the apparatus is shown in the figure. Negative muons stopped in the target 6 were registered by the anticoincidence scheme 8 (1 + 2 − 3). Pulses from 8 were amplified, shaped, and delayed for the time t_1. The delayed pulses operated the trigger 16, which opened simultaneously two identical gates 18 and 21 for



Block diagram of apparatus. 1, 2, 3, 4 – scintillation counters; 5 – copper filter; 6 – target; 7 – paraffin filter; 8, 9 – anticoincidence circuit; 10, 11 – amplifiers; 12, 13 – pulse shapers; 14 – delay t_1; 15 – delay t_2; 16 – trigger; 18, 21 – gating circuits; 19, 22 – discriminators; 17, 20, 23 – scalars; 24 – magnetizing coil.

the time Δt. The meson decay electrons and the background were registered by the anticoincidence circuit 9 (3 + 4 – 2). Pulses from 9, after passing through the amplifier 11 and the pulse shaper 13, were split, and were then fed to the gating circuit 18 after a delay $t_2 > t_1 + \Delta t$ and to the gating circuit 21 without delay. The use of two identical gating circuits and the delay t_2 made it possible for the scaler 23 to register simultaneously the number of pulses produced by decay electrons and by the background, while the scaler 20 registered only the background pulses. The count difference between scalers 23 and 20 represents the number of decay electrons. Pulses from the trigger 16 were fed through a separate output to the scaler 17, which registered the number of gate openings, and served as a monitor.

In each experiment the ratio $t_1/\Delta t$ was ~ 0.2. The axis of the electron detector and the target formed the angles 90° and 45°, respectively, with the meson beam axis. The target area was 15×15 cm^2, and the target thickness was 4 – 6 g/cm^2. In the experiments with graphite, polythene, and paraffin the paraffin filter 7 between counters 3 and 4 was 4 g/cm^2 thick. In the experiments with chromium, molybdenum, tungsten, and palladium, the paraffin filter 7 was replaced by a 4 g/cm^2 aluminum filter, which reduced to under 10^{-3} the registration efficiency for γ rays under 10 Mev emitted from the target as a result of μ^- absorption.[13]

Substance	$\xi = N_{max}/N_{min}$
Graphite	1.10 ± 0.02
Paraffin	1.09 ± 0.02
Polythene	1.10 ± 0.02
Palladium hydride	1.09 ± 0.02
Palladium	1.00 ± 0.02
Chromium	1.00 ± 0.02
Molybdenum	0.99 ± 0.02
Tungsten	0.99 ± 0.02

The table gives the values obtained for $\xi = N_{max}/N_{min}$, including corrections for t_1 and Δt, meson decay and the solid angle of the electron detector. The statistical errors are indicated.

4. DISCUSSION OF RESULTS

The table shows that for graphite, polythene, paraffin, and palladium hydride ($PdH_{0.6}$) identical values of ξ were obtained within the limits of statistical errors. Equal values of ξ were also obtained for chromium, molybdenum, tungsten, and palladium. However, the absolute values of ξ for graphite, polythene, paraffin and $PdH_{0.6}$ differ from those for chromium, molybdenum, tungsten, and palladium. It should be noted that for graphite a_0 – the asymmetry coefficient in the electron angular distribution $1 + a \cos \theta$, integrated over the energies – obtained from ξ agrees within error limits with the value of a_0 obtained in earlier work[1,9,12] from a large number of measured points on the precession curve.

The departure of ξ from unity for graphite, paraffin, polythene, and the agreement of the values among themselves, indicate that the paramagnetism of mesic carbon atoms in these substances is due to the meson magnetic moment alone. Let us first consider the formation of mesic atoms in metals, which, together with graphite, can be regarded in first approximation as ion aggregates immersed in an electron gas. Diamagnetic metals and normal weakly paramagnetic metals have ions with zero magnetic moment. It is therefore probable that when mesic atoms are formed in these metals, since $t_0 \ll t'$ the electron state of an ion is not actually destroyed and the ionization of atoms is accompanied only by the emission of collective conduction electrons. In view of the constant measured value of a_0 previously obtained[9] for metals in this group such as magnesium, zinc, cadmium, and lead, and also the equality of this value to a_0 for graphite, it can be asserted that electronic paramagnetism does not arise when mesic atoms are formed in diamagnetic or normal weakly paramagnetic metals.

The formation of mesic atoms in dielectrics, where $t_0 \gg t'$, presents a different picture. Mesic carbon atoms in paraffin and polythene, mesic oxygen atoms in water, and mesic sulfur atoms[9] can lack an electron magnetic moment for two possible reasons. First, the mesic atoms can be negative ions with the electron configuration of the original atoms. Secondly, the formation of mesic atoms can possibly be associated with disturbance of the original atomic electron shell. In this case, according to Dzhrbashyan,[6] the electron moment can be canceled completely by neighboring atoms.

Further investigation is required before a choice can be made between these two hypotheses. The experimental results will determine whether a B^{12} depolarization mechanism exists due to hyperfine structure arising from the spin coupling of the nucleus to the electron shell of boron atoms produced in the reaction $\mu^- + C^{12} \rightarrow B^{12} + \nu$. It would appear that mesic carbon atoms in graphite should have the electron configuration of boron atoms,[11] but experimental results indicate the absence of any electron moment. It is obvious that electronic paramagnetism cannot arise in boron atoms produced by the given reaction in graphite. Therefore a B^{12} depolarization mechanism due to hyperfine structure in graphite cannot exist, whereas for the aforementioned reasons this mechanism can exist when boron atoms are formed in dielectrics.

The experimental results obtained for palladium in $PdH_{0.6}$ furnish direct evidence that the paramagnetism of mesic palladium atoms is due to the magnetic moments of the electron shell and meson. The electrons of the magnetically active inner 4d shell are responsible for the magnetic moment of ions of the transition metal palladium. The measurements of ξ for carbon, paraffin, and polythene show that in the compound $PdH_{0.6}$ hydrogen does not affect meson depolarization.

We wish to emphasize especially that, as the experiments with Pd and $PdH_{0.6}$ have shown, polarized μ^- mesons can possibly be used to investigate the magnetic properties of atoms and hydrides, with zero nuclear spin, of the transition metals, lanthanides, and actinides. Unfortunately, no control experiment can be performed to confirm directly the existence of electronic paramagnetism in chromium, molybdenum, or tungsten, since hydrides are not formed when hydrogen is dissolved in these metals.

ξ can equal unity for these transition metals and they can agree among themselves if 1) mesons are completely depolarized, or 2) the paramagnetism of mesic atoms is due to the magnetic moments of the electron shell and muon.

We shall now consider the depolarization of μ^- mesons in different mesic atoms. An identical value of $\sim 17\%$ within experimental error has been obtained for the polarization of μ^- mesons in the mesic atoms of diamagnetic metals with zero nuclear spin. Identical degrees of meson polarization in graphite, polythene, and paraffin have been obtained from measurements of ξ, the absolute values being equal to that for the aforementioned metals. The absolute degree of polarization and the fact that the measured polarization for $Z \geq 6$ is independent of Z are in good agreement with theoretical calculations[6,7] considering only the spin-orbit interaction mechanism.

It is difficult to understand how chromium and molybdenum can differ essentially from the substances already mentioned with respect to μ^- depolarization. Ford and Mullin[14] have shown that there is practically no depolarization of mesons slowed down and captured into the higher levels of mesic atoms. The probability of mesic atom formation is known to be unity. The elements Cr and Mo consist 80 — 90% of atoms with zero nuclear spin. The nuclei of these atoms possess no properties that could result in complete meson depolarization. It is therefore unlikely that depolarization mechanisms other than spin-orbit interaction exist in mesic atoms with the corresponding values of Z. Chromium and molybdenum differ from the previously mentioned substances only in their possession of unfilled inner electron shells. Therefore the results of the experiments with palladium and $PdH_{0.6}$ show that the hyperfine structure is very probably involved in the cases of chromium and molybdenum, whose ions possess nonzero magnetic moments due to the magnetically active inner 3d and 4d shells, respectively.

The experiments with tungsten merit special attention, since tungsten, unlike palladium, chromium, and molybdenum, has mesic atoms with deformed nuclei. It has been shown by Zaretskii and Novikov[15] that the interaction of a meson with the quadrupole deformation of a nucleus can result in considerable depolarization of μ^- mesons. We shall now compare the measured values ξ_e with theoretical predictions ξ_T. If the hypothesis of the relationship between the observed depolarization of μ^- mesons and the quadrupole deformation of the nucleus is correct, then we should theoretically[15] have the relationship $a_0(W) \approx 0.4\, a_0(C)$ between the values of a_0 for carbon and tungsten. The spin precession frequency of mesic tungsten atoms in a magnetic field should agree with the spin precession frequency of a free meson.

It can be shown with the aid of the normal error distribution that the case $\xi_e = \xi_T$ is rejected, since $\xi_e < \xi_T$ with 70% probability. This provides evidence that the experimentally observed "total" depolarization of mesons can hardly be accounted for only by meson interaction with the quadrupole deformation of the nucleus. In this connection it must be pointed out that experiments with palladium, chromium, and molybdenum indicate with high probability that in the case of tungsten we are also dealing with hyperfine structure due to spin coupling between the meson and electrons of the incomplete inner 5d shell.

Although tungsten atoms have a very small effective magnetic moment compared with chromium atoms, for example, the experimental technique is sufficiently sensitive to detect this magnetic moment. Muons have a magnetic moment one order of magnitude larger than the nuclear magneton, while the spin-flip time ($\sim 10^{-10}$ sec) of a meson in the field of the mesic-atom shell is many orders of magnitude shorter than the meson lifetime. In order to arrive at more definite conclusions it is obviously necessary to observe directly the spin precession curves of mesic Cr, Mo and W atoms.

The present and earlier investigations[1,9,12] lead to the following conclusions regarding the process of μ^- depolarization:

1) The interaction between the magnetic moments of the meson and electron shell affects meson polarization only in mesic atoms of the transition elements, lanthanides, and actinides.

2) Spin-orbit interaction reduces the polarization of μ^- mesons to one-sixth. This result agrees well with the theoretical calculations of Dzhrbashyan[6] and Shmushkevich.[7]

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