DETERMINATION OF THE PARITIES OF STRANGE PARTICLES BY MEANS OF DISPERSION RELATIONS

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The set of dispersion relations of Matthews and Salam, Igi, and Amati for the scattering of K mesons by protons is regarded as an (overdetermined) system of equations for the parities and the coupling constants of the proton with the K-Y pair. The condition for consistency leads to the result that the parities of Λ and Σ hyperons are opposite. The sign of the real part of the amplitude for scattering of a K^- meson by a proton turns out to be positive, so that there are attractive forces in the Kp system.

1. INTRODUCTION

At present a number of methods are used for determining the parities of strange particles from the phenomenological analysis of reactions in which they are created or destroyed. Various difficulties are, however, inherent in all of these methods. The point is that the strong interactions conserve not only parity but also strangeness, so that the reactions in question involve the strange particles in pairs. Therefore one cannot determine the parity of one particle without knowing the properties of the nuclear interactions of the partner particle. At present these properties are as a rule unknown, and this hinders the determination of parities.

On the other hand, the phenomenological method for determining parities, which operates only with initial and final states, cannot be applied to reactions of elastic scattering, since in this case the intrinsic parity drops out of the argument. Conversely, any theory that contains the idea of intermediate states can give an answer to the question. In particular, such ideas are contained in the method of dispersion relations, and owing to this it can be used for the determination of the parities of strange particles.[1]

The basic idea of this method is that the sign of the pole term that appears on account of the transition to an intermediate state which contains one hyperon depends on the parity of the system Kp relative to this hyperon.[2] For the calculation of the pole terms one must know the dispersion integral and the real part of the scattering amplitude. The experimental data now available[3] enable us to determine these quantities.

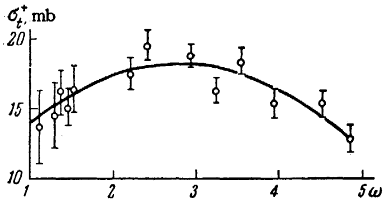

FIG 1.

Our work differs from a number of previous papers[2] in that we have avoided the simplest approximations for the energy dependence of the cross sections and have made a complete treatment of all the data by the method of least squares. This has made it possible to estimate the actual precision of the calculations, and in particular to answer a number of questions raised in the papers of Islam and Selleri.[2]

2. THE ANALYSIS OF THE EXPERIMENTAL DATA

The required experimental information on the cross sections for the interaction of K mesons

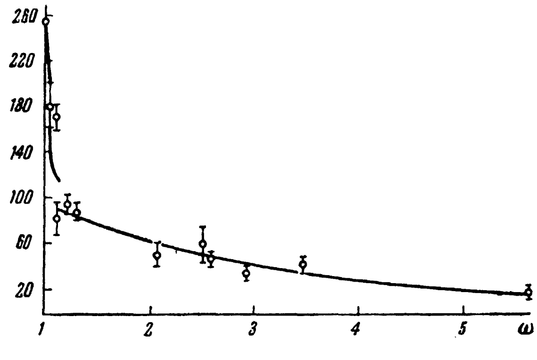

FIG. 2

with nucleons is contained in the report of Alvarez at the Kiev Conference.[3] These data are shown in Figs. 1 and 2, together with the smoothed curves obtained by treatment of the experimental data by the method of least squares.

In the obtaining of the smoothed curves it was convenient to use this method in the Chebyshev form (cf., e.g., reference 4), in which the curve is represented as an expansion in terms of polynomials

$$\sigma(\omega)=\sum_{k=1}^{n} C_k\varphi_k(\omega), \tag{2.1}$$

which are orthogonal with the weights p_i when summed over a discrete set of points ω_i:

$$\sum_{i=1}^{N} p_i\,\varphi_k(\omega_i)\,\varphi_l(\omega_i)\equiv[\varphi_k,\varphi_l]=A_k\,\delta_{kl}. \tag{2.2}$$

As the weights p_i one takes a set of quantities inversely proportional to the dispersions of the cross sections $\sigma(\omega_i)$. An advantage of this procedure is that the matrix of the errors of the coefficients C_k is diagonal. Owing to this one can avoid much computational labor in inverting the matrix of the errors, since the coefficients are automatically uncorrelated.

In the choice of the degree of the approximating polynomial one must confine oneself to the value of n for which the quantity

$$\eta=[(\sigma-\sum C_k\varphi_k),(\sigma-\sum C_k\varphi_k)]/(N-n) \tag{2.3}$$

first becomes close to unity.[5] The cross section $\sigma_+(\omega)$ is approximated by polynomials of degree no higher than the second. Table I shows the coefficients C_k, their dispersions $D(C_k)$, and the values of η.

Table I

k	C_k^+	$D(C_k^+)$	η
0	16,553	0.09	3,4
1	−0,745	0,08	3.1
2	−1.277	0.06	0.96

The data on the scattering of K^- mesons were treated in a similar way. Here, however, it was not possible to represent the entire curve by a single analytical expression; this would require polynomials of degree not lower than the seventh, which would lead to a sharp drop in accuracy. Therefore the range of interpolation was divided into two parts:

$$m\leqslant\omega\leqslant 1.1m,\quad 1.1m\leqslant\omega\leqslant 5m.$$

(Both here and in all subsequent numerical calculations energies are expressed in terms of the rest energy of the K^- meson; m = 494 Mev.) In the first interval the total cross section σ_{tot}^- was divided into two parts:

$$\sigma_{tot}^-=\sigma_{sc}^-+\sigma_{ab}^-,$$

where the cross sections for elastic scattering and for charge transfer were assumed constant,

$$\sigma_{sc}^-=90\pm17\text{ mb} \tag{2.4}$$

and the absorption cross section was approximated by the formula

$$|k|\,\sigma_{ab}^-=(7\pm1)\,m^{-1}. \tag{2.5}$$

This same expression was also used in the unphysical region $\omega\leq m$. The more exact extrapolation proposed by Dalitz and Tuan[6] is not necessary, since the entire contribution of the unphysical region is small.

Table II

k	C_k^-	$D(C_k^-)$	η
0	48.9	6.4	14.2
1	−13.8	3.1	3.4
2	3.65	1.27	1.4

In the second range the curve of the total cross section was smoothed. The data for this are shown in Table II. The approximating curve is taken as a polynomial of the second degree. In the region $\omega > 5m$ the cross sections are unknown. We have assumed that they remain constant and equal to each other:

$$\sigma_{tot}^-=\sigma_{tot}^+. \tag{2.6}$$

This contradicts the direct experimental data at $\omega = 5m$

$$\sigma_{tot}^+=13\pm1\text{mb}\,,\qquad \sigma_{tot}^-=20\pm5\text{ mb}. \tag{2.7}$$

Evidently the equalizing of the cross sections occurs at somewhat higher energies. An estimate of the error introduced by the assumption (2.6) shows that in any case it is not more than 5 percent. Of course it would be very desirable to have direct data on the cross sections in this region.

3. CHOICE OF THE DISPERSION RELATIONS

The dispersion relations for the scattering of $K^\pm$ mesons by protons are given in the paper by Matthews and Salam:[1]

$$D_\pm(\omega)=B_\pm(\omega)+\frac{1}{4\pi^2}\int_m^\infty k'd\omega'\left[\frac{\sigma_+(\omega')}{\omega'\mp\omega}+\frac{\sigma_-(\omega')}{\omega'\pm\omega}\right]$$

$$+\frac{1}{\pi}\int_{\omega_{\Lambda\pi}}^{m}\frac{A_-(\omega')\,d\omega'}{\omega'\pm\omega}, \tag{3.1}$$

where

$$D(\omega) = \mathrm{Re}\, M(\omega), \qquad A(\omega) = \mathrm{Im}\, M(\omega), \tag{3.2}$$

$$\omega_{\Lambda\pi} = [(M_\Lambda + \mu)^2 - M^2 - m^2]/2M,$$
$$\omega_Y = [M_Y^2 - M^2 - m^2]/2M, \tag{3.3}$$

$$B_\pm(\omega) = \frac{g_\Lambda^2}{4\pi}\frac{\omega_\Lambda + M + M_\Lambda p_{\Lambda K}}{2M(\omega_\Lambda \pm \omega)} + \frac{g_\Sigma^2}{4\pi}\frac{\omega_\Sigma + M + M_\Sigma p_{\Sigma K}}{2M(\omega_\Sigma \pm \omega)}. \tag{3.4}$$

The term $B_\pm(\omega)$ has a pole at $\omega = \mp\omega_Y$, i.e., at the energy at which a transition to an intermediate state involving a hyperon is possible. The residue at the pole depends on the coupling constant between the system Kp and this hyperon and on the parity p_{KY} relative to the proton. The numerical values of the threshold energies are

$$\omega_{\Lambda\pi} = 0.474\, m, \qquad \omega_\Lambda = 0.129\, m, \qquad \omega_\Sigma = 0.320\, m. \tag{3.5}$$

From the relations (3.1) one can obtain other formulas which have better convergence in the region of large ω':

$$D_-(\omega) - D_+(\omega) = B_-(\omega) - B_+(\omega) + \frac{2\omega}{4\pi^2}\int_m^\infty \frac{\sigma_- - \sigma_+}{\omega'^2 - \omega^2} k'd\omega' + \frac{2\omega}{\pi}\int_{\omega_{\Lambda\pi}}^m \frac{A_-d\omega'}{\omega'^2-\omega^2} \tag{3.6}$$

(the Matthews-Salam relation[1]);

$$D_+(\omega) - \frac{\omega+m}{2m}D_+(m) + \frac{\omega-m}{2m}D_-(\omega) = \frac{k^2}{4\pi^2}\int_m^\infty \frac{d\omega'}{k'}\left[\frac{\sigma_+}{\omega'-\omega} + \frac{\sigma_-}{\omega'+\omega}\right] + \frac{k^2}{\pi}\int_{\omega_{\Lambda\pi}}^m \frac{A_-d\omega'}{k'^2(\omega'+\omega)} + 2F\frac{k^2}{\omega} \tag{3.7}$$

(the Igi relation[1]);

$$\frac{D_+(\omega) - D_+(m)}{\omega - m} = \frac{B_+(\omega) - B_+(m)}{\omega - m} + \frac{1}{4\pi^2}\int_m^\infty k'd\omega'\left[\frac{\sigma_+}{(\omega'-\omega)(\omega'-m)} - \frac{\sigma_-}{(\omega'+\omega)(\omega'+m)}\right] - \frac{1}{\pi}\int_{\omega_{\Lambda\pi}}^m \frac{A_-d\omega'}{(\omega'+\omega)(\omega'+m)} \tag{3.8}$$

(the Amati relation[2]).

We shall use Eqs. (3.6) and (3.7) at the energy m, and Eq. (3.8) at $\omega = 1.22m$ (for convenience of comparison with Selleri's paper[2]). Introducing the scattering lengths a and b by the formulas

$$D_+(\omega) = -ak/k_c, \qquad D_-(\omega) = \pm bk/k_c \tag{3.9}$$

(k_c is the momentum in the center-of-mass system), we then get

$$\pm b + a = B_1 + \frac{M}{M+m}\frac{2m}{4\pi^2}\left[\int_m^\infty \frac{\sigma_- - \sigma_+}{k'}d\omega' + \int_{\omega_{\Lambda\pi}}^m \frac{|k'|\sigma_{ab}^- d\omega'}{\omega'^2 - m^2}\right], \tag{3.10}$$

$$\pm b + a + \frac{2Mm}{M+m}D_+'(m) = B_2 + \frac{M}{M+m}\frac{m^2}{\pi^2}\left[\int_m^\infty \frac{d\omega'}{k'}\left(\frac{\sigma_+}{\omega'-m} + \frac{\sigma_-}{\omega'+m}\right) + \int_{\omega_{\Lambda\pi}}^m \frac{|k'|\sigma_{ab}^- d\omega'}{k'^2(\omega'+m)}\right], \tag{3.11}$$

$$r_+(1.22m) = B_3 + \frac{1}{\pi}\int_m^\infty k'd\omega'\left[\frac{\sigma_+}{(\omega'-1.22m)(\omega'-m)} - \frac{\sigma_-}{(\omega'+1.22m)(\omega'+m)}\right] - \frac{1}{\pi}\int_{\omega_{\Lambda\pi}}^m \frac{|k'|\sigma_{ab}^- d\omega'}{(\omega'+1.22m)(\omega'+m)}. \tag{3.12}$$

The factor $M/(M+m)$ has arisen from k/k_c after passage to the limit $\omega \to m$, and

$$r_+ = 4\pi\,[D_+(1.22\,m) - D_+(m)]/0.22\,m. \tag{3.13}$$

The explicit expressions for the pole terms B_i in terms of the coupling constants are of the following form:

$$B_i = a_{i\Lambda}g_\Lambda^2/4\pi + a_{i\Sigma}g_\Sigma^2/4\pi.$$

The coefficients $a_{i\Lambda}$ and $a_{i\Sigma}$ are given in Table III, where the upper values are for positive parity $p_{KY} > 0$.

Table III

i	$a_{i\Lambda}$	$a_{i\Sigma}$
1	−2.296 0.123	−2.717 0.114
2	−2.663 0.143	−2.697 0.113
3	−9.305 0.498	−7.540 0.316

4. CALCULATION OF THE POLE TERMS

By means of the dispersion equations (3.10) – (3.12) the pole terms can be expressed in terms of the scattering lengths and the dispersion integrals. The problem is reduced to the calculation of these quantities and of their standard errors.

1. The scattering lengths. By definition [cf. Eq. (3.9)]

$$a = [\sigma_{sc}^+(m)/4\pi]^{1/2}, \qquad b = [\sigma_{sc}^-(m)/4\pi - [k_c\sigma_{ab}^-(m)/4\pi]^2]^{1/2}, \tag{4.1}$$

with the positive sign of $D(\omega)$ corresponding to attraction of the K meson to the proton. At the present time only the sign of $D_+(m)$ has been determined; it is negative,[3] i.e., the K^+ meson is repelled from the proton. The sign of $D_-(m)$ has not yet been determined, but there are certain indications (interference between Coulomb and nuclear scattering) that this sign is positive.[3] We shall investigate both possibilities.

Let us show how the scattering length a was calculated. According to Eq. (4.1) it can be expressed in terms of the coefficients C_k of the polynomial which approximates $\sigma_+(\omega)$. These coefficients are independent random variables distributed according to the normal law. Since their dispersions are small, we can use the simple differential formula

$$a = \bar{a} + \sum \frac{\partial a}{\partial C_k} \Delta C_k. \tag{4.2}$$

Substituting all the necessary data and making the calculation, we find

$$a = (0.8389 \pm 0.0268)\, m^{-1}. \tag{4.3}$$

The quantity b is calculated in a similar way, but is expressed in terms of different random quantities (σ^-_{sc}, σ^-_{ab}):

$$b = (2.046 \pm 0.330)\, m^{-1}. \tag{4.4}$$

The other quantities associated with the scattering lengths are found in just the same way. Thus the derivative $D'_+(m)$, which appears in the Igi relation (3.11), is given by

$$D'_+(m) = \frac{Mm}{M+m} a^3 - \frac{M+m}{M}\frac{\partial a}{\partial \omega} - \frac{a}{M+m}. \tag{4.5}$$

After substitution of the numerical data this formula gives

$$D'_+(m) = (-0.26 \pm 1.65)\, m^{-2}. \tag{4.5a}$$

Finally, the Amati relation (3.12) involves the effective radius r_+ [see Eq. (3.13)]. Its value is

$$r_+ = (-0.22 \pm 1.04)\, m^{-2}. \tag{4.6}$$

2. The dispersion integrals. All of the integrals that it is necessary to calculate can be divided into four types, which differ from each other by the ranges of integration:

$$\omega_{\Lambda\pi} \leqslant \omega' \leqslant m, \qquad m \leqslant \omega' \leqslant 1.1 m,$$

$$1.1\, m \leqslant \omega' \leqslant 5m, \qquad 5m \leqslant \omega' < \infty.$$

The only contribution in the first interval is that from the absorption cross section σ^-_{ab}, which is taken from Eq. (2.5). In the second interval σ^-_{ab}, σ^+_{tot}, and σ^-_{sc} come in. The value of σ^-_{sc} is taken from Eq. (2.4), and for σ^+_{tot} we use the approximating polynomial (2.1). In the next interval we need data on the total cross sections σ^-_{tot} and σ^+_{tot}, which we have in the form of smoothed polynomials. In the last interval we use the equality of the cross sections [cf. Eq. (2.6)]

$$\sigma^-_{tot} = \sigma^+_{tot} = 13 \pm 1 \text{ mb},$$

which is necessary for the convergence of the integrals in the relations of Salam and Amati. In the Igi relations, where the integrals converge even without this condition, we have taken the direct experimental values of the cross sections [cf. Eq. (2.7)]. We recall that in this region all of the cross sections have been assumed independent of the energy.

In all cases the integration can be done by elementary methods, and the result is that all of the dispersion integrals are expressed in terms of the 10 independent random quantities

$$C^+_0,\ C^+_1,\ C^+_2,\ C^-_0,\ C^-_1,\ C^-_2,\ \sigma^-_{sc},\ \sigma^-_{ab},\ \sigma^-(\infty),\ \sigma^+(\infty). \tag{4.7}$$

Some of them have already been encountered above in the calculation of the scattering lengths.

3. The pole terms. Equations (3.10) – (3.12) serve for the determination of the pole terms B_i in terms of the dispersion integrals, the scattering lengths, and related quantities. Substituting everything in Eqs. (3.10) – (3.12), we get

$$B_i = \bar{B}_i + \sum_{k=1}^{10} b_{ik} \Delta C_k, \tag{4.8}$$

where

$$\overline{\Delta C_k} = 0, \qquad \overline{\Delta C_k \Delta C_l} = \delta_{kl} D(C_k), \tag{4.9}$$

$$b_{ik} = \begin{pmatrix} 0.073 & -0.105 & 0.132 & -0.039 & 0.025 & -0.006 & \substack{0.010 \\ -0.029} & \substack{0.030 \\ 0.036} & & 0 \\ 1.773 & -4.034 & 6.362 & -0.025 & 0.024 & -0.017 & \substack{0.010 \\ -0.029} & \substack{0.037 \\ 0.043} & -0.008 & -0.009 \\ 0.662 & -3.235 & 7.213 & 0.123 & -0.008 & -0.104 & -0.002 & 0.057 & -0.001 & -0.031 \end{pmatrix}. \tag{4.10}$$

The columns of the matrix b_{ik} are numbered in the order of the quantities in the list (4.7), and the rows correspond to i = 1, 2, 3. In the rows in which two values are given, the upper corresponds to attraction of K^- to the proton (i.e., to the positive sign of b).

Since the number of terms in Eq. (4.8) is rather large, we can apply the limit theorem of probability theory and assume that the value of a pole term is distributed around its average value $\bar{B}$ according to the normal law, the dispersion of this distribution being a linear function of the $D(C_k)$. By means of the formula (4.8) we can convince ourselves that the three pole terms B_i are statistically independent quantities. Substituting the numerical values, we get the data shown in Table IV, which gives the values of the pole terms with their standard errors (the diagonal elements of the error matrix).

Table IV

i	B_i
1	0.799 ± 0.211 -3.293 ± 0.503
2	0.531 ± 2.055 -3.561 ± 2.108
3	1.410 ± 2.097

The largest dispersion is obtained by the use of the Igi relations (3.7). In doing the calculation one finds that the main contribution (90 percent) to the dispersion comes from the uncertainty in the quantity $D'_+(m)$. Therefore although the subtraction method used by Igi gives convergent quantities, it at the same time leads to large ambiguities. In fact, it has been shown in a paper by Islam[2] that the different ways of interpolating the experimental data on K^+p scattering in the energy range up to 100 Mev can lead to the negative sign for the pole term, independently of the sign of the potential of the K^-p interaction.

In our case, owing to the use of the method of least squares, there are no ambiguities in the interpolation. It can be seen from Table IV, however, that although the average value $\overline{B}_2$ is indeed positive (for $b > 0$), there is a considerable probability (40 percent) that $B_2 < 0$. Thus in this case we cannot reliably indicate the sign of the pole term.

In their paper Karplus and others[2] used a dispersion relation of the same type as Eq. (3.7), but with a variable point of subtraction. In the light of what we have found, it is clear why the standard deviations in their paper were as large as 150 percent and the sign of B_2 remained completely uncertain.

Thus the subtraction used by Igi depresses the pole term too much and does not allow us to determine the necessary quantities with enough accuracy. On the other hand, the relation used by Amati (the effective-radius approximation) is less sensitive to experimental errors. This is because in this relation the main part is played by the dispersion integrals, which are determined with high accuracy. The ambiguities that arise here turn out to be very slight. Thus the probability that the sign of the pole term is opposite to that shown in the table is only 20 percent.

Of all the relations we have considered, the one that gives the most reliable data on the pole terms is the Matthews-Salam relation. The reason is that no subtraction procedure was used in the derivation of this relation, and owing to this the effects of experimental errors on the value of the pole term are reduced to a minimum. The accuracy obtained in this case is in fact the best. At any rate the sign of the pole term is determined with a probability very close to unity.

5. THE PARITIES AND COUPLING CONSTANTS OF THE STRANGE PARTICLES

The results of the treatment of the experimental data are presented in Table IV, which shows the values of the pole terms. In Table III these same terms are expressed in terms of the (unknown) parities and coupling constants of the strange particles. Combining the data of the two tables, we get an equation of the form

$$a_\Lambda g_\Lambda^2/4\pi + a_\Sigma g_\Sigma^2/4\pi = B, \qquad (5.1)$$

in which the coefficients a_Y are taken from Table III and depend on the parities of the strange particles, and the quantity B is from Table IV and depends on the sign of the potential of the K^-p interaction.

In all previous papers[1,2] devoted to this problem attempts have been made to determine both parities from a single equation (5.1). For example, if we use the Amati relation, in which $B > 0$, it is clear that the parities of the Λ and Σ hyperons cannot both be positive at the same time, since then $a_\Lambda < 0$, $a_\Sigma < 0$ and Eq. (5.1) cannot be satisfied by positive values of $g_Y^2/4\pi$. If, on the other hand, both hyperons have negative parities, the equation can be satisfied. This still leads to no conclusion, however, since it remains completely obscure what the situation is if the parities of the hyperons are opposite to each other.

The point is, of course, that one equation is not enough for the determination of the parities. Therefore one must make additional assumptions to make up for the lack of information. Such an approach naturally cannot be accepted as correct. We prefer to consider not a single dispersion relation, but the entire set, and to treat it as a system of equations for the unknown coupling constants. We have

$$a_{1\Lambda} g_\Lambda^2/4\pi + a_{1\Sigma} g_\Sigma^2/4\pi = B_1, \qquad a_{2\Lambda} g_\Lambda^2/4\pi + a_{2\Sigma} g_\Sigma^2/4\pi = B_2,$$
$$a_{3\Lambda} g_\Lambda^2/4\pi + a_{3\Sigma} g_\Sigma^2/4\pi = B_3. \qquad (5.2)$$

Positive definite solutions of this system do not exist in all cases, but only with a definite choice of the parities. This enables us to find them. Moreover, since we have three equations for two unknowns, the additional condition for their consistency

$$\begin{vmatrix} a_{1\Lambda} & a_{1\Sigma} & B_1 \\ a_{2\Lambda} & a_{2\Sigma} & B_2 \\ a_{3\Lambda} & a_{3\Sigma} & B_3 \end{vmatrix} = 0 \qquad (5.3)$$

is a very severe one.

If we assume that attractive forces act between the K^- meson and the proton at small energies, the system (5.2) is consistent, and its solutions are positive only with the following choice of the parities:

$$p\,(K^+\Lambda_0) > 0, \qquad p\,(K^+\Sigma_0) < 0. \qquad (5.4)$$

The most probable values of the coupling constants are then

$$g_\Lambda^2/4\pi = 0.28 \pm 0.67, \quad g_\Sigma^2/4\pi = 12.7 \pm 13.6. \quad (5.5)$$

Let us now consider the solutions that are obtained with a choice of the parities different from that of Eq. (5.4), for example for $p_{K\Lambda} > 0$, $p_{K\Sigma} > 0$. There is then just a change of the coefficients $a_{i\Sigma}$ in the system (5.2), with the new values proportional to the old ones. The proportionality constant can be found from Eq. (3.4), and is

$$k = (\omega_\Sigma + M + M_\Sigma)/(\omega_\Sigma + M - M_\Sigma) = -23.9. \quad (5.6)$$

Since the coefficients $a_{i\Sigma}$ are proportional to their previous values, the condition of consistency, Eq. (5.3), is not disturbed. The new solution will differ from the old one only in the quantity $g_\Sigma^2/4\pi$; it is now negative and smaller by a factor 23.9. We have an analogous situation if we change the assumption about the parity of the Λ hyperon, but in this case k = − 18.7. Thus among the four choices for the parities only one is physically possible. With each of the other three choices at least one constant is negative. Of course, owing to the presence of the dispersions there is a nonvanishing probability of getting positive values of $g_Y^2/4\pi$ in these cases also, but this probability is not larger than 20 percent, whereas for the choice (5.4) it is 65 percent. These figures show the degree of uniqueness of the result expressed by the equations (5.4) and (5.5).

If we assume that the K^- meson is repelled from the proton (b < 0), the right members of the equations (5.2) take different values (see Table IV). In this case a direct calculation shows that the condition of consistency (5.3) is violated. Therefore there is no choice of the parities for which one can satisfy the system of equations with positive coupling constants. Naturally this result also is only a probable one. Knowing the dispersions and correlations of the right members, we have calculated the probability that the condition (5.3) can be satisfied. It is 3 percent, whereas in the previous case (b > 0) we had ~ 65 percent.

Thus owing to our use of the system of dispersion relations we are able to exclude the a priori possible case of K^-p repulsion. This means that the sign of this scattering length is determined by the very structure of the dispersion equations. We note that the conclusion that the K^- meson and proton attract each other does not contradict the data on the interference of the Coulomb and nuclear scatterings of the K^- meson.[3]

6. CONCLUSION

Our analysis of the dispersion relations for the scattering of $K^\pm$ mesons by protons leads to the following conclusions:

1. In the determination of the parities one must not use only the sign of the pole term, as has been done in all previous papers,[1,2] but must regard the dispersion relations as equations in the unknowns g_Y^2.

2. The most probable result is that the hyperons have opposite parities, which has the consequence that the constants g_Y^2 differ by a factor of about 40.

3. There is no need to make any assumptions about the sign of the scattering length of the K^- meson, since it is determined by the very structure of the dispersion relations.

4. The Igi dispersion relation leads to a broad distribution of the quantities being studied, which makes it hard to get an unambiguous answer.

5. The contribution of the absorption to the dispersion integral is small, and owing to this we can use the simplest extrapolations in the unphysical region.

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Note added in proof (January 16, 1961). The linear combinations $g_\Lambda^2/4\pi = 0.028 \pm 0.71$ and $g_\Sigma^2/4\pi - 20.6\, g_\Lambda^2/4\pi = 6.93 \pm 1.84$ have no correlation between them, and their dispersions give an optimal characterization of the accuracy attained in the present paper. Inclusion of new data[7] leads to the values $g_\Lambda^2/4\pi = 0.89$, $g_\Sigma^2/4\pi = 25.0$, with the same relative accuracy as before.

[1] P. T. Matthews and A. Salam, Phys. Rev. **110**, 565 (1958). K. Igi, Progr. Theoret. Phys. **19**, 238 (1958). C. Goebel, Phys. Rev. **110**, 572 (1958).

[2] Karplus, Kerth, and Kycia, Phys. Rev. Letters **2**, 510 (1959). M. M. Islam, Nuovo cimento **13**, 224 (1959). E. Galzenati and B. Vitale, Phys. Rev. **113**, 1365 (1959). D. Amati, Phys. Rev. **113**, 1692 (1959). F. Selleri, Nuovo cimento **15**, 986 (1960).

[3] L. Alvarez, Report at the International Conference on High-energy Particle Physics, Kiev, 1959.

[4] Yu. V. Linnik, Метод наименьших квадратов и основы теории обработки наблюдений (The Method of Least Squares and the Foundations of the Theory of the Treatment of Observations), Fizmatgiz, 1958.

[5] M. E. Rose, Phys. Rev. **91**, 610 (1953).

[6] S. F. Tuan, Phys. Rev. **113**, 1375 (1959). R. Dalitz and S. F. Tuan, Ann. Phys. **8**, 100 (1959).

[7] von Dardel, Frisch, Mermod, Milburn, Piroué, Vivargent, Weber, and Winter, Phys. Rev. Letters **5**, 333 (1960).

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