HIGH ENERGY ELECTRON-ELECTRON SCATTERING

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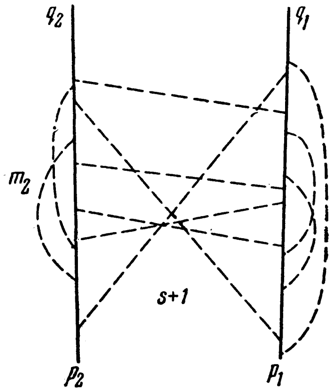
The cross section for large angle scattering of high energy electrons is calculated in the double logarithmic approximation.

INASMUCH as in the scattering of electrons by the electrons of a target at rest the energy of the colliding electrons in the center-of-mass system (c.m.s.) is equal to $E_{c.m.s.} = \frac{1}{2}\sqrt{2E_{l.s.}mc^2}$ (for maximum energy achieved at the present time $E_{l.s.} = 1.2$ Bev, $E_{c.m.s.} = 17$ Mev), in consequence of the noted smallness of the energy in the c.m.s. one could limit oneself in the calculation of the scattering cross section (with high degree of accuracy) to the lower orders of perturbation theory (see, for example, reference 1).

At the present time experiments are set up on the scattering of electrons in colliding beams with the purpose of testing quantum electrodynamics at small distances (see, for example, reference 2), wherein the energy of the colliding electrons will be several hundred million electron volts in the c.m.s. As is well known, the principal terms arising in the account of higher approximations of perturbation theory contain the square of a large logarithm, for example,[3] $\ln^2(E/m)$ for each power of e^2, by virtue of which it is not possible in the calculation of cross sections of electromagnetic processes to limit oneself to the lower orders of perturbation theory. At the same time it is necessary to have sufficiently correctly computed cross sections to establish any violation of quantum electrodynamics at small distances.

A method of calculation of the cross sections of electromagnetic processes at high energies was suggested by Abrikosov.[3] Within the framework of this method, only those contributions are retained which contain the maximum power of the large logarithm. We shall use this method to determine the scattering cross section of electrons by electrons for c.m.s. scattering angles $\theta \gg m/E$.

An arbitrary diagram of order $2(n+1)$ of perturbation theory (see the drawing) contains: $s+1$ virtual photon lines which connect the different electron lines ("staircase" lines); m_1 lines of virtual photons emitted and absorbed by the same electron ("overlapping" lines); similarly, m_2 overlapping lines for another electron line while



$s + m_1 + m_2 + 1 = n + 1$, where $n + 1$ is the total number of lines of virtual photons.

The matrix element will contain the maximum power of the logarithm $(e^2 \ln^2(E/m))^n$ under the satisfaction of two conditions: a) one of the staircase lines transfers a large momentum $l^2 \sim (q_1 - p_1)^2$, while the momenta of the remaining staircase lines $k^2 \sim 0$; b) the staircase line with the large momentum lies within all the enveloping lines. In what follows we shall consider only the class of such diagrams.

If the staircase line with the large momentum joins the r_1 and r_2 staircase vertices, then the corresponding matrix element is equal to

$$Q = (-1)^{n+r_1+r_2+s} \frac{M_0 f^n}{(r_1+m_1)!\,(r_2+m_2)!\,(s+m_1-r_1)!\,(s+m_2-r_2)!}, \quad (1)$$

where M_0 is the matrix element of second order,

$$f = \frac{e^2}{2\pi}\int \frac{du}{u}\frac{dv}{v},$$

u and v are introduced in the papers of Abrikosov.[3] Here we have used the condition that for not too small angles all the scalar problems (q_1p_1), (p_1p_2), (p_1q_2) are of the same order (the difference between them is less than the limits of accuracy of the method).

All diagrams with different topology of photon lines but with fixed s, r_1, r_2, m_1, and m_2 make the same contribution. As is easy to see, the number of such diagrams is equal to

$$N = \binom{m_1 + r_1}{m_1}\binom{m_1 + s - r_1}{m_1}\binom{m_2 + r_2}{m_2}\binom{m_2 + s - r_2}{m_2} m_1!\, m_2!\, s!. \quad (2)$$

Thus the matrix element of order $2(n+1)$ is equal to

$$M_{2(n+1)} = \sum_{\substack{s r_1 r_2 m_1 m_2 \\ s+m_1+m_2=n}} (-1)^{n+r_1+r_2+s} \times \frac{M_0 f^n N}{(r_1+m_1)!\,(r_2+m_2)!\,(s+m_1-r_2)!\,(s+m_2-r_2)!} = (-1)^n \frac{2^n f^n}{n!} M_0. \quad (3)$$

Evidently this sum differs from zero only for $s = 0$ (one staircase line); thus the contributions of the staircase photons are preserved and it would have been sufficient to consider only overlapping lines to obtain the cross section.

The scattering cross section of the electrons is then obtained by summation over n, which gives $d\sigma = d\sigma_0 \exp(-4f)$. It is of practical interest to know the total cross section of elastic and inelastic processes with given maximum energy of emitted quanta. Consideration of the radiation of real quanta, in view of the analogue character of the generalized diagram[3] relative to those considered above, can be carried out in the same fashion. To eliminate the infrared divergence, we limit ourselves to consideration of real quanta with momentum k, satisfying the condition $(p_{1,2}k) \gg (p_{1,2}\kappa) \sim m^2$. In the c.m.s., this momentum will be the same for all ends of electron lines. As a result we obtain an exponent whose index is added with the index of the exponent of the virtual photons. If the maximum energy of the radiated photons is equal to ω, then the total cross section is shown to be equal to

$$d\sigma = d\sigma_0 \exp\left[-\frac{8e^2}{\pi}\ln\frac{E}{\omega}\ln\frac{E}{m}\right], \quad (4)$$

where E is the initial energy of the electrons. If the recording apparatus records all the scattered electrons independent of their loss in energy, then $\omega = E$ and the cross section $d\sigma$ is identical with the cross section of Møller scattering.[3] The departure from Møller scattering is illustrated in the table for energies of 100 and 500 Mev.

ω, Mev	$d\sigma/d\sigma_0$	
	E = 100 Mev	E = 500 Mev
0.5	0.59	0.41
5	0.74	0.55
10	0.79	0.60
50	0.95	0.74

In view of the classical character of the radiation of real quanta, the distribution of the latter over angle and energy does not depend on their number. Taking this into account, and integrating over the angles of emission of the photons, we obtain the energy distribution of the radiated photons:

$$dn(\omega) = \frac{8e^2}{\pi}\frac{d\omega}{\omega}\ln\frac{E}{m}. \quad (5)$$

The probability of energy loss by the electron in the range ϵ to $\epsilon + d\epsilon$ is then equal to

$$dN(\varepsilon) = \frac{8e^2}{\pi}\ln\frac{E}{m}\frac{d\varepsilon}{\varepsilon}\exp\left\{-\frac{8e^2}{\pi}\ln\frac{E}{m}\ln\frac{E}{\varepsilon}\right\}. \quad (6)$$

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[3] A. A. Abrikosov, JETP **30**, 96, 386, 544 (1956), Soviet Phys. JETP **3**, 71, 474, 379 (1956).

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