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TRANSITIONS BETWEEN HYPERFINE LEVELS IN MESIC DEUTERIUM ATOMS

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The effective cross section for dμ mesic atom transitions to the lower ($F = 1/2$) hyperfine state in exchange collisions with deuterons is calculated. It is shown that practically complete depolarization of μ mesons should be observed in pure deuterium. It is also shown that the dμ transition to the $F = 1/2$ state increases the capture probability, $\mu^- + d \rightarrow 2n + \nu$ (by a factor of three for V-A), and the effect of the transition to the $F = 1/2$ state on the catalysis of the nuclear reaction $p + d \rightarrow He^3$, mentioned by L. Wolfenstein, is discussed.

It is known that the process, suggested by Ya. B. Zel'dovich, of μ-meson exchange between protons in liquid hydrogen leads to rapid transitions from the upper ($F = 1$) hyperfine state in the μ-mesonic hydrogen (pμ) atom to the lower ($F = 0$) state.[1] This effect produces complete depolarization of μ mesons in hydrogen and affects the $\mu^- + p \rightarrow n + \nu$ reaction, where it increases the transition probability by a factor of four (for V-A) and produces completely longitudinally polarized neutrons from the capture of μ mesons in the pμ atom[2] (for capture in ppμ see references 3 and 4).

In this note we investigate the transition between the hyperfine levels $F = 3/2 \rightarrow F = 1/2$ in μ-mesonic deuterium (dμ) in collisions

$$d\mu + d \rightarrow d + d\mu \tag{1}$$

and discuss its effects on capture

$$\mu^- + d \rightarrow 2n + \nu \tag{2}$$

and on the catalysis of nuclear reactions by μ mesons.

1. TRANSITION CROSS SECTION[5]

The energy difference between the hyperfine $F = 3/2$ and $F = 1/2$ levels of dμ ($\mathbf{F} = \mathbf{s} + \mathbf{i}$, where $\mathbf{s}$ is the μ-meson spin, $\mathbf{i}$ the deuteron spin) according to Fermi's formula, is

$$\Delta\varepsilon = \frac{8\pi}{3}\beta_\mu\beta_N g\,(2i+1)\,|\psi(0)|^2 = \frac{8}{3}\frac{\beta_\mu\beta_N}{a_\mu^3} g\,(2i+1)\left(1+\frac{m_\mu}{M_d}\right)^{-3} \approx 0.046\text{ ev} \tag{3}$$

(here $g = 0.8565$ is the gyromagnetic ratio of the deuteron; β_μ and β_N are the μ-meson magneton and the nuclear magneton; $a_\mu = \hbar^2/m_\mu e^2$; m_μ and M_d are the μ-meson and deuteron masses). We consider the transition between the hfs levels, due to the collision of dμ with a deuteron. Since the hfs energy can formally be obtained from the interaction

$$V = \tfrac{4}{3} g\beta_\mu\beta_N r^{-2}\,\delta(r)\,(\mathbf{si}) \tag{4}$$

(where $r = |\mathbf{r}_\mu - \mathbf{R}_d|$ is the distance between the μ meson and the deuteron), the Hamiltonian for the system consisting of a μ meson and two deuterons can be written in the form (with units $e = \hbar = m_\mu = 1$)

$$H = -\frac{1}{2M_d}\Delta_{\mathbf{R}_1} - \frac{1}{2M_d}\Delta_{\mathbf{R}_2} - \frac{1}{2}\Delta_{\mathbf{r}} - \frac{1}{r_1} - \frac{1}{r_2} + \frac{1}{R} + \frac{4}{3}g\beta_\mu\beta_N\left\{\frac{\delta(r_1)}{r_1^2}(\mathbf{si}_1) + \frac{\delta(r_2)}{r_2^2}(\mathbf{si}_2)\right\}, \tag{5}$$

where $\mathbf{R}_1$, $\mathbf{R}_2$, $\mathbf{r}$ are the positions of the deuterons and the μ meson; $r_1 = |\mathbf{r} - \mathbf{R}_1|$; $r_2 = |\mathbf{r} - \mathbf{R}_2|$; $R = |\mathbf{R}_2 - \mathbf{R}_1|$; $\mathbf{i}_1$, $\mathbf{i}_2$, $\mathbf{s}$ are the spins of the deuterons and the μ meson. The spin interaction between the deuterons is neglected in (5). Since the collisions between dμ and deuterons occur at thermal velocities, the deuterons are assumed to be in a relative S state. In this approximation the total spin of the ddμ system, which can be $J = 5/2, 3/2, 1/2$, is conserved. Obviously the $F = 3/2 \rightarrow 1/2$ transition in the collision (1) can only occur for $J = 3/2, 1/2$. We shall calculate the transition cross section separately for each of these values.

1. If $J = 3/2$ the total spin of the two deuterons can be 2 or 1. The space part of the wave function of two identical particles must be symmetric for even and antisymmetric for odd total spin of the particles. Therefore, the wave function of the system with $J = 3/2$ (z projection M_J) can be written in the form

$$\Psi_{3/2,\, M_J} = G_{3/2}(R)\, \Sigma_g (r_1,\, r_2;\, R)\, S^{(2)}_{3/2,\, M_J} (1, 2; \mu) + H_{3/2}(R)\, \Sigma_u (r_1,\, r_2;\, R)\, S^{(1)}_{3/2,\, M_J} (1,\, 2;\, \mu). \qquad (6)$$

Here Σ_g and Σ_u ($1s\sigma_g$ and $2p\sigma_u$ in the usual molecular notation) are the wave functions of the muon in the field of the two deuterons with fixed separation R; Σ_g is symmetric, Σ_u is antisymmetric under interchange of the deuteron coordinates; $S^{(I)}_{J,M_J}(i, k; l)$ is the spin function of the system for total spin J and projection M_J; I is the spin of the particles whose indices are in the first two places; 1 and 2 as arguments of the spin functions denote the deuterons and μ denotes the meson. The explicit form of the spin function $S^{(I)}_{J,M_J}(i, k; l)$ can easily be written with the aid of Clebsch-Gordan coefficients. For example,

$$S^{(2)}_{3/2,\, 3/2} (1,\, 2;\, \mu) = 5^{-1/2} \{2^{-1/2} [\chi_1 (1) \chi_0 (2) + \chi_0 (1)\, \chi_1 (2)]\, \varphi_{1/2} (\mu) - 2\chi_1 (1)\, \chi_1 (2)\, \varphi_{-1/2} (\mu)\}, \qquad (7)$$

where $\chi_m (m = \pm 1,\, 0)$ and φ_ν $(\nu = \pm 1/2)$ are the spin functions of the deuteron and μ meson.

Substitution of (6) into the Schrödinger equation with the Hamiltonian (5), multiplication by the functions $\Sigma_g S^{(2)}_{3/2,M_J}$ and $\Sigma_\mu S^{(1)}_{3/2,M_J}$, and integration over the μ-meson coordinate with summation over the spin variables gives the following pair of equations for the functions $G_{3/2}(R)$ and $H_{3/2}(R)$:

$$-\frac{1}{M_d} \Delta_R G_{3/2} + U_g G_{3/2} - \Delta\varepsilon \left(\frac{1}{2} G_{3/2} - \frac{\sqrt{5}}{6} H_{3/2} \right) = \left(\varepsilon + \frac{1}{3} \Delta\varepsilon\right) G_{3/2},$$

$$-\frac{1}{M_d} \Delta_R H_{3/2} + U_u H_{3/2} + \Delta\varepsilon \left(\frac{\sqrt{5}}{6} G_{3/2} + \frac{1}{6} H_{3/2} \right) = \left(\varepsilon + \frac{1}{3} \Delta\varepsilon\right) H_{3/2}, \qquad (8)$$

where $U_g(R)$ and $U_u(R)$ are the molecular potentials in the $1s\sigma_g$ and $2p\sigma_u$ states, including dynamical corrections of first order in m_μ/M_d due to nuclear motion.[6] The energy ϵ is measured from the upper hyperfine level; $\Delta\epsilon$ is the hyperfine splitting, Eq. (3).*

Along with G(R) and H(R), it is convenient to introduce the functions L(R) and K(R), which describe the motion of a free deuteron relative to the $d\mu$ atom in its upper ($F = 3/2$) and lower ($F = 1/2$) hyperfine states, respectively. For large R, the wave function of the system (5) must be a linear combination of the functions

$$\Phi^{(3/2)}_{3/2,\, M_J} = 2^{-1/2} \{L_{3/2} (R)\, \psi\, (r_1)\, S^{(3/2)}_{3/2,\, M_J} (1,\, \mu;\, 2) + L_{3/2} (R) \psi\, (r_2)\; S^{(3/2)}_{3/2,\, M_J} (2,\, \mu;\, 1)\},$$

$$\Phi^{(1/2)}_{3/2,\, M_J} = 2^{-1/2} \{K_{3/2} (R)\, \psi\, (r_1)\, S^{(1/2)}_{3/2,\, M_J} (1, \mu;\, 2) + K_{3/2} (R)\, \psi\, (r_2)\; S^{(1/2)}_{3/2,\, M_J} (2, \mu;\, 1)\} \qquad (9)$$

[$\psi(r_1)$ and $\psi(r_2)$ are the wave functions of the meson bound to the first and second deuterons, respectively].

By decomposing the spin functions in (6) into the spin functions in (9) and using the fact that, as $R \rightarrow \infty$,

$$\Sigma_g \rightarrow \{\psi\, (r_1)\, + \psi\, (r_2)\} / \sqrt{2}, \quad \Sigma_u \rightarrow \{\psi\, (r_1)\, - \psi\, (r_2)\} / \sqrt{2}, \qquad (10)$$

it is not difficult to find the relations between the functions G(R), H(R), and K(R), L(R), which describe the relative motion of the nuclei:

$$L_{3/2} = \{G_{3/2} + \sqrt{5} H_{3/2}\} / \sqrt{6}, \qquad K_{3/2} = \{\sqrt{5}\, G_{3/2} - H_{3/2}\} / \sqrt{6}. \qquad (11)$$

From Eqs. (8) and (11) it is clear that, as $R \rightarrow \infty$, L(R) and K(R) satisfy the equation

$$M_d^{-1}\, \Delta_R L + \varepsilon L = 0, \quad M_d^{-1} \Delta_R K + (\varepsilon + \Delta\varepsilon)\, K = 0. \qquad (12)$$

To determine the probability for the transition $F = 3/2 \rightarrow 1/2$ in collisions of the type (1), Eq. (8) must be solved with the boundary condition that K(R) (which describes the relative nuclear motion in the lower hyperfine state) is an outgoing wave as $R \rightarrow \infty$:

$$K\, (R) \sim \gamma R^{-1} e^{ik_2 R}, \quad L\, (R) \sim (\alpha e^{ik_1 R} - e^{-ik_1 R}) / 2ik_1 R, \qquad (13)$$

where

$$k_1 = (M_d \varepsilon)^{1/2}, \quad k_2 = [M_d\, (\varepsilon + \Delta\varepsilon)]^{1/2} \approx (M_d\, \Delta\varepsilon)^{1/2} \quad (\varepsilon \ll \Delta\varepsilon).$$

The normalization of L is that corresponding to an incoming plane wave of amplitude unity at infinity. Let R_0 be of the order of magnitude of the radius of the potentials $U_g(R)$ and $U_u(R)$ in Eq. (8). The solutions of (12) in the region $R_0 \ll R \ll k_2^{-1} \ll k_1^{-1}$ can be joined to the solutions of Eqs. (8), which in this region have the form

$$G\, (R) \sim (R - \lambda_g)/R, \quad H\, (R) \sim (R - \lambda_u)/R. \qquad (14)$$

The scattering lengths λ_g and λ_u can be deter-

*Of course, the use of (4) to compute the spin interaction in (8) is not exact, and, generally speaking, is not necessary for what follows, since for small R only the first two terms in (8) are used. The use of (8), however, makes possible a direct derivation of Eq. (12), which is correct for large R.

mined by numerical integration[7] or calculated analytically by approximating the potentials by simple functions.[1,6]

In the region $k_2R \ll 1$ the solution has the form

$$L_{3/2} = \left\{R - \frac{(\lambda_g + 5\lambda_u) + 6ik_2\lambda_g\lambda_u}{6 + ik_2(5\lambda_g + \lambda_u)}\right\}\frac{1}{R},$$

$$K_{3/2} = -\frac{\sqrt{5}(\lambda_g - \lambda_u)}{6 + ik_2(5\lambda_g + \lambda_u)}\frac{1 + ik_2R}{R}. \quad (15)$$

From this, the transition cross section for the system with spin $3/2$ is obtained directly;

$$\sigma_{3/2} \approx \tfrac{5}{9}\pi(\lambda_g - \lambda_u)^2 k_2/k_1. \quad (16)$$

2. If the spin, J, of the system is $1/2$, the transition cross section can be calculated in a completely similar manner. The total spin of the two deuterons can be 0 or 1. Thus, the wave function of the system can be written in the form

$$\Psi_{1/2, M_J} = G_{1/2}(R)\,\Sigma_g S^{(0)}_{1/2, M_J}(1,2;\mu)$$
$$+ H_{1/2}(R)\,\Sigma_u S^{(1)}_{1/2, M_J}(1,2;\mu), \quad (17)$$

and the wave functions describing the motion of the deuteron and $d\mu$ atom in the upper and lower hyperfine states, respectively, as in (9), have the form

$$\Phi^{(3/2)}_{1/2, M_J} = 2^{-1/2}\{L_{1/2}(R)\psi(r_1)S^{(3/2)}_{1/2, M_J}(1,\mu;2)$$
$$+ L_{1/2}(R)\psi(r_2)S^{(3/2)}_{1/2, M_J}(2,\mu;1)\},$$
$$\Phi^{(1/2)}_{1/2, M_J} = 2^{-1/2}\{K_{1/2}(R)\psi(r_1)S^{(1/2)}_{1/2, M_J}(1,\mu;2)$$
$$+ K_{1/2}(R)\psi(r_2)S^{(1/2)}_{1/2, M_J}(2,\mu;1)\}. \quad (18)$$

Here, the functions $K_{1/2}(R)$ and $L_{1/2}(R)$ are connected to the functions $G_{1/2}(R)$ and $H_{1/2}(R)$ by the equations

$$K_{1/2} = \{-G_{1/2} + \sqrt{2}H_{1/2}\}/\sqrt{3}, \qquad L_{1/2} = \{\sqrt{2}G_{1/2} + H_{1/2}\}/\sqrt{3} \quad (19)$$

and satisfy boundary conditions of the form (13) as $R \to \infty$. Matching the functions gives, in the region $R_0 \ll R \ll k_2^{-1}$,

$$K_{1/2} = \frac{\sqrt{2}(\lambda_g - \lambda_u)}{3 + ik_2(\lambda_g + 2\lambda_u)}\frac{(1 + ik_2R)}{R},$$

$$L_{1/2} = \left\{R - \frac{(2\lambda_g + \lambda_u) + 3ik_2\lambda_g\lambda_u}{3 + ik_2(\lambda_g + 2\lambda_u)}\right\}\frac{1}{R}. \quad (20)$$

The transition cross section for the $dd\mu$ system with spin $J = 1/2$ is thus

$$\sigma_{1/2} \approx \tfrac{8}{9}\pi(\lambda_g - \lambda_u)^2 k_2/k_1. \quad (21)$$

With the statistical weights of the $J = 5/2, 3/2$, and $1/2$ states, we obtain for the transition cross section

$$\sigma_{F=3/2\to 1/2} = \tfrac{1}{3}\sigma_{3/2} + \tfrac{1}{6}\sigma_{1/2} = \tfrac{1}{3}\pi(\lambda_g - \lambda_u)^2 k_2/k_1. \quad (22)$$

The $F = 3/2 \to 1/2$ transition probability for $d\mu$ in a $d\mu + d$ collision, consequently, is

$$W = N_d\,\sigma_{F=3/2\to 1/2}\,v = \pi(\lambda_g - \lambda_u)^2 N_d v^*/3, \quad (23)$$

where $v^* = 2(\Delta\epsilon/M_d)^{1/2}$ is the velocity of the nucleus after the transition and N_d is the number of deuterons per cc.

It is essential to note that the $F = 3/2 \to 1/2$ transition probability in $d\mu$ is three orders of magnitude smaller than the corresponding $F = 1 \to 0$ transition probability in $p\mu$, which is

$$W_p = \tfrac{1}{4}\pi(\lambda_g^{(p)} - \lambda_u^{(p)})^2 N_p v_p^* \quad (24)$$

[where $v_p^* = 2(\Delta\epsilon_{p\mu}/M_p)^{1/2}$; N_p is the number of protons per cc, $\lambda_g^{(p)}$ and $\lambda_u^{(p)}$ are the analogs of the scattering lengths in (14) for protons]. This change in magnitude comes chiefly from the fact that the scattering lengths $\lambda_g^{(p)}$ and $\lambda_n^{(p)}$ have opposite signs, and $\lambda_g^{(p)}$ is large because of the resonance[1,6] (according to references 5 and 6, $\lambda_g^{(p)} = -17.3\,a_\mu$, $\lambda_u^{(p)} = 5.25\,a_\mu$), while for deuterons λ_g and λ_u have the same sign and are nearly equal. According to references 5 and 6, $\lambda_g = 6.67$, $\lambda_u = 5.73$; the calculations of Cohen et al.[7] give similar values (see Fig. 2 in reference 7), with $\lambda_g = -\lim_{k\to 0}(\delta^+/k)$, $\lambda_u = -\lim_{k\to 0}(\delta^-/k)$. Under the conditions in a liquid-deuterium chamber, the transition to the $F = 1/2$ state is not reversible and its probability is (for $N_d = 3.5 \times 10^{22}\ \mathrm{cm}^{-3}$)

$$W \approx 6\cdot 10^6\ \mathrm{sec}^{-1} \quad (25)$$

We mention that in real deuterium, the $d\mu$ atoms collide not with atoms, but with D_2 molecules, and the effective cross section may be different for ortho and para deuterium. The calculation for this case can be performed by a method like Fermi's pseudopotential method, which was proposed for the scattering of slow neutrons on molecules. Since, however, the consideration of molecular structure in $p\mu + H_2$ scattering gives an insignificant effect,[8] one may suppose that it is also not essential for $d\mu + D_2$ scattering (see note added in proof).

2. DEPOLARIZATION OF μ MESONS AND CROSS SECTION FOR ELASTIC SCATTERING OF THE $d\mu$ ATOM IN ITS LOWER HYPERFINE STATE

The collision of $d\mu$ atoms with deuterons proceeding with μ exchange gives rise to a supplementary mechanism for the depolarization of μ mesons in the K shell of deuterium.

Let Φ_{F,M_F} be the spin function of a $d\mu$ atom with spin F and projection M_F along the direction of the original polarization of the μ meson, and let χ_m be the spin function of the free deuteron. Then, by decomposing $\Phi_{F,M_F}\chi_m$ into the spin functions $S^{(F)}_{J,M_J}(1,\mu;2)$ of the combined system and using the scattering amplitude in each of the spin states [see (15) and (20); the scattering length in the $J = 5/2$ state is obviously λ_g], it is easy to obtain the scattering amplitude for the state $\Phi_{F,M_F}\chi_m$ and thus calculate the change of polarization of the μ meson in the scattering (1).

As an example, we give the scattered wave amplitude for the state $\Phi_{3/2,3/2}\chi_{-1}$

$$[\Phi_{3/2,3/2}\chi_{-1}] \to -\frac{1}{R}\left\{\frac{1}{2}(\lambda_g+\lambda_u)\Phi_{3/2,3/2}\chi_{-1} + \frac{1}{2\sqrt{3}}(\lambda_g-\lambda_u)\Phi_{3/2,-1/2}\chi_1\right\} - \frac{1}{\sqrt{6}}\frac{1}{R}e^{ik_2R}(\lambda_g-\lambda_u)\Phi_{1/2,-1/2}\chi_1. \qquad (26)$$

From Eq. (26) it is clear that depolarization occurs both in the transition to the lower hyperfine state and also in the elastic channel. In both cases, the depolarization is zero for $\lambda_g = \lambda_u$. Since the polarization of the μ meson in the $\Phi_{1/2,-1/2}$ state is $+1/3$, the transition to $F = 1/2$ does not give a complete loss of polarization. Consideration, as in (26), of the different hyperfine states of the $d\mu$ atom shows that if only $F = 3/2 \to 1/2$ transitions are taken into account the μ meson could have a polarization of 1 or 2 per cent (if one assumes that the polarization is of the order of 10 or 20 per cent after the μ meson cascades down to the K shell). However, a simple calculation shows that further collisions of the $d\mu$ atom in its lower hyperfine state lead to practically complete depolarization.

To calculate the scattering cross section for

$$d\mu\,(F=1/2)+d \to d\mu\,(F=1/2)+d \qquad (1')$$

at energies significantly less than the hyperfine splitting $\Delta\epsilon$, one must replace the boundary condition (13) by the condition that there be no exponentially increasing term in the solution for L(R) [see (12); $\epsilon \approx -\Delta\epsilon$] as $R \to \infty$:

$$K \approx (R-\lambda)/R, \qquad L \approx \gamma R^{-1}e^{-\varkappa R} \approx \gamma\,(1-\varkappa R)/R. \qquad (27)$$

(27), along with (11), (19), and (14), gives for the scattering in the states with total spin $J = 3/2$ and $J = 1/2$

$$K_{3/2} \approx \left\{R - \frac{(5\lambda_g+\lambda_u)+4\lambda_g\lambda_u\varkappa}{6-\varkappa(\lambda_g-5\lambda_u)}\right\}\frac{1}{R},$$

$$K_{1/2} \approx \left\{R - \frac{(\lambda_g+2\lambda_u)-3\lambda_g\lambda_u\varkappa}{3-\varkappa(2\lambda_g+\lambda_u)}\right\}\frac{1}{R}. \qquad (28)$$

As in the derivation of (26), the amplitude of the scattered wave for the $d\mu$ atom in the $F = 1/2$ state can be obtained:

$$[\Phi_{1/2,1/2}\chi_{-1}] \to -\frac{1}{R}\left\{\frac{1}{2}(\lambda_g+\lambda_u)\Phi_{1/2,1/2}\chi_{-1} + \frac{\sqrt{2}}{6}(\lambda_g-\lambda_u)\Phi_{1/2,-1/2}\chi_0\right\},$$

$$[\Phi_{1/2,1/2}\chi_0] \to -\frac{1}{R}\left\{\frac{1}{3}(2\lambda_g+\lambda_u)\Phi_{1/2,1/2}\chi_0 + \frac{\sqrt{2}}{6}(\lambda_g-\lambda_u)\Phi_{1/2,-1/2}\chi_1\right\},$$

$$[\Phi_{1/2,1/2}\chi_1] \to -\frac{1}{R}\left\{\frac{1}{6}(5\lambda_g+\lambda_u)\Phi_{1/2,1/2}\chi_1\right\}. \qquad (29)$$

Thus, the cross section for (1′) with reoriented $d\mu$ atom spin is

$$\sigma_e = \tfrac{4}{27}\pi(\lambda_g-\lambda_u)^2. \qquad (30)$$

Since the reorientation probability under liquid-deuterium conditions ($N = 3.5\times10^{22}$ cm^{-3}; $v \approx 5\times10^4$ cm/sec, with λ_g and λ_u from reference 6)

$$W_e = Nv\sigma_e \approx 5\cdot10^5\ \text{sec}^{-1} \qquad (31)$$

is of the order of the decay probability, $\mu \to e + \nu + \bar{\nu}$, one can conclude that in liquid deuterium, as in liquid hydrogen, the μ mesons are practically completely depolarized.

The effective cross section for the scattering (1) for the $F = 1/2$ $d\mu$ atom is, according to (28),

$$\sigma^{(e)}_{(F=1/2)} = \frac{2}{3}\left(\frac{5\lambda_g+\lambda_u}{6}\right)^2 + \frac{1}{3}\left(\frac{\lambda_g+2\lambda_u}{3}\right)^2. \qquad (32)$$

Note that, since λ_g and λ_u are nearly equal and have the same sign, the inclusion of hyperfine effects in $d\mu + d$ scattering does not lead to a significant change in the scattering cross section, while in $p\mu + p$ scattering the interference leads to a considerable reduction in the cross section.

3. EFFECT OF $F = 3/2 \to 1/2$ TRANSITIONS ON CAPTURE IN DEUTERIUM

First it must be mentioned that μ capture in deuterium must take place from the $d\mu$ atomic state and not from the $dd\mu$ molecular state, since the formation of the molecule (which, by the way, has a small probability) would lead to practically instantaneous catalysis of the nuclear d+d reaction.[10] Thus, the interpretation of the data on μ capture in deuterium is in a certain sense simpler than for capture in hydrogen, where the formation of the molecule complicates the picture.[3,4] The experimental study of these processes in a bubble chamber will become possible, obviously, when a μ-meson beam with a very low π-meson contamination can be obtained.

The capture of μ mesons in deuterium has been discussed previously.[11-13] As Bukhvostov and Shmushkevich[13] have shown, the capture probability depends strongly on the distribution of mesons between the hyperfine states of the $d\mu$ atom. In particular, for the V-A interaction there is no capture from the $F = 3/2$ state. Therefore, the completeness of the $F = 3/2 \rightarrow 1/2$ transition increases the capture probability by a factor of three over the value which would be obtained with a statistical distribution in the $F = 3/2$ and $F = 1/2$ states.

If the μ-capture interaction is not V-A, then capture is possible from the $F = 3/2$ state. However, since the final state neutrons are in the triplet state (as can be seen from the work of Überall and Wolfenstein[12]), the $F = 3/2$ capture is always suppressed in comparison with $F = 1/2$ capture. Thus, in this case the $F = 3/2 \rightarrow 1/2$ transition leads to a considerable increase in the capture probability. (The exact value of the capture probability can be obtained from Bukhvostov and Shmushkevich's formula[13] by replacing the $F = 1/2$ probability (p_-) by unity.)

4. EFFECT OF THE $F = 3/2 \rightarrow 1/2$ TRANSITION ON μ CATALYSIS

L. Wolfenstein (private communication) has pointed out a very curious possible consequence of the $F = 3/2 \rightarrow 1/2$ transition in the $d\mu$ atom for the catalysis of nuclear reactions in mixtures of hygrogen and deuterium.

The fact is that the nuclear $p + d$ reaction in the $pd\mu$ molecule depends strongly on the molecular spin state. The $pd\mu$ molecule has states with total spin 2, 1 (two states), and 0 (see, e.g., reference 6); since the spacing of the molecular levels with different spins is significantly greater than the level widths, the nuclear reaction proceeds independently in each of these states. In the spin-2 state, the total spin of the proton and deuteron is $3/2$; in the spin-0 state it is $1/2$. The spin-1 states are superpositions of states in which the total spin of the proton and deuteron is $3/2$ and $1/2$. Since the decisive role in the transition $p + d \rightarrow He^3$ with μ-meson conversion is played by the E0 transition which occurs when the total spin of the proton and deuteron is $1/2$,[14-16] it is clear that the $d\mu$ transition ($F = 3/2 \rightarrow 1/2$) occurring before formation of the molecule raises the statistical weight of such states and thus increases the number of regenerated μ mesons.

For a more accurate estimate, one must find the weights of the states with proton deuteron spins $3/2$ and $1/2$ in the $pd\mu$ molecular states with total spin 1. The spin functions for the $pd\mu$ states with total spin 1 can be represented in the form

$$\chi_{1,M_J} = -C_1 S^{(1/2)}_{1,M_J}(p, d;, \mu) + C_2 S^{(3/2)}_{1,M_J}(p, d; \mu),$$

$$\chi'_{1,M_J} = C_2 S^{(1/2)}_{1,M_J}(p, d; \mu) + C_1 S^{(3/2)}_{1,M_J}(p, d\ \mu) \qquad (33)$$

(where, according to a rough estimate,[6] $C_1 = 0.41$, and $C_2 = 0.91$). If the transition of the $d\mu$ atom to the $F = 1/2$ state occurs before formation of the $pd\mu$ molecule, then at the instant of formation of the molecule the following spin functions can occur with equal probabilities

$$\Phi_{1/2,1/2}(d, \mu)\,\varphi_{1/2}(p), \quad \Phi_{1/2,-1/2}(d, \mu)\,\varphi_{-1/2}(p),$$

$$\Phi_{1/2,1/2}(d, \mu)\,\varphi_{-1/2}(p), \quad \Phi_{1/2,-1/2}(d, \mu)\,\varphi_{1/2}(p).$$

It is not hard to verify that the spin functions of the system which correspond to the above initial functions at $t = 0$ are the following

$$\Sigma_1 = \Theta_{1,1}(t) \underset{t=0}{\rightarrow} \Phi_{1/2,1/2}(d, \mu)\,\varphi_{1/2}(p),$$

$$\Sigma_2 = -\Theta_{1,-1}(t) \underset{t=0}{\rightarrow} \Phi_{1/2,-1/2}(d, \mu)\,\varphi_{-1/2}(p),$$

$$\Sigma_3 = \frac{1}{\sqrt{2}}\{\Theta_{1,0}(t) + \chi_{0,0}e^{-iE_0t}\} \underset{t=0}{\rightarrow} \Phi_{1/2,\,1/2}\ \ \varphi_{-1/2},$$

$$\Sigma_4 = \frac{1}{\sqrt{2}}\{\Theta_{1,0}(t) - \chi_{0,0}e^{-iE_0t}\} \underset{t=0}{\rightarrow} \Phi_{1/2,-1/2}\ (d, \mu)\,\varphi_{1/2}(p), \qquad (34)$$

where

$$\Theta_{1,M_J}(t) = \left(-\tfrac{1}{3}C_1 + \tfrac{2\sqrt{2}}{3}C_2\right)\chi_{1,M_J}e^{-iE_1t} + \left(\tfrac{2\sqrt{2}}{3}C_1 + \tfrac{1}{3}C_2\right)\chi'_{1,M_J}e^{-iE'_1t}$$

E_1, E'_1, and E_0 are the energies of the states χ_{1,M_J}, χ'_{1,M_J}, and $\chi_{0,0}$). From (34) it follows that the probabilities that the system is in the states χ_1, χ'_1 and χ_0 are

$$W_{\chi_1} = \tfrac{1}{12}(-C_1 + 2\sqrt{2}C_2)^2 \approx 0.39,$$

$$W_{\chi'_1} = \tfrac{1}{12}(2\sqrt{2}\,C_1 + C_2)^2 \approx 0.36, \qquad W_{\chi_0} = \tfrac{1}{4}. \qquad (35)$$

Estimates of the nuclear reaction probability based on nuclear models are highly uncertain. However, if we accept Cohen, Judd, and Riddell's result,[7] we can conclude that the M1 transition occurring in the spin $1/2$ $p + d$ state is the most probable γ transition in the $p + d \to He^3 + \gamma$ reaction. If we let R be the probability for this transition, and A be the probability of the E0 transition with μ-meson conversion, then we obtain for the reaction yields in the $pd\mu$ molecule (formed by $d\mu$ atoms in the $F = 1/2$ state)

$$Y_\gamma = \left\{ 0.39 \frac{C_1^2 R}{\lambda_0 + C_1^2 (R + A)} + 0.36 \frac{C_2^2 R}{\lambda_0 + C_2^2 (R + A)} + \frac{1}{4} \frac{R}{\lambda_0 + R + A} \right\},$$

$$Y_\mu = \left\{ 0.39 \frac{C_1^2 A}{\lambda_0 + C_1^2 (R + A)} + 0.36 \frac{C_2^2 A}{\lambda_0 + C_2^2 (R + A)} + \frac{1}{4} \frac{A}{\lambda_0 + R + A} \right\}, \qquad (36)$$

when Y_μ and Y_γ are the yields of the $p + d \to He^3$ reaction with μ-meson conversion and γ-ray emission, respectively; $\lambda_0 = 0.45 \times 10^6$ sec^{-1} is the decay probability of the μ meson. In Eqs. (36) the M1 and E2 transitions from the spin $3/2$ state and the conversion of the μ meson in the M1 transition are neglected. According to (36), the reaction yield exceeds that from the $pd\mu$ molecule formed by $d\mu$ atoms with a statistical distribution in the $F = 3/2$ and $F = 1/2$ states. In the latter case (with the assumptions made above) the reaction yield is

$$Y_\mu = \left\{ \frac{1}{4} \frac{C_1^2 A}{\lambda_0 + C_1^2 (R + A)} + \frac{1}{4} \frac{C_2^2 A}{\lambda_0 + C_2^2 (R + A)} + \frac{1}{12} \frac{A}{\lambda_0 + R + A} \right\}. \qquad (37)$$

The numerical value of the $F = 3/2 \to 1/2$ transition probability in the $d\mu$ atom, apparently is consistent with Wolfenstein's conjecture that the $3/2 \to 1/2$ transition does not take place in hydrogen-deuterium mixtures with a deuterium concentration of the order of one per cent, while for larger deuterium concentrations the $3/2 \to 1/2$ transition takes place before the formation of the $pd\mu$ molecule. This effect accounts for some of the increase in the $p + d \to He^3$ reaction yield at large deuterium concentrations, but is still insufficient to explain the results of Fetkovich et al.[17] if the expressions for the probability of formation of the $pd\mu$ molecule calculated in references 7 and 18 are used.

It should be noted that if the nuclear reaction probability from the state with spin $3/2$ for p+d is significantly less than that from the state with spin $1/2$, then the transition of the $d\mu$ atom to its lower hyperfine state increases the reaction yield equally for μ-meson conversion and for γ-ray emission, so that the conversion coefficient remains constant. If the $p + d \to He^3 + \gamma$ reaction were to go mainly from the state with spin $3/2$ for p+d, then the transition of $d\mu$ to its lower $F = 1/2$ state would lead to an increase in the E0 transition probability and a decrease in the radiative transition probability. In principle, these effects could be examined experimentally by studying the yield of the $p + d \to He^3 + \gamma$ reaction in mixtures of hydrogen and deuterium, as in the experiments of Ashmore et al.,[19] but with larger concentrations of deuterium.

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Note added in proof (January 12, 1961). A. M. Moskalenko has calculated the $F = 3/2 \to 1/2$ transition probability for the $d\mu$ atom in collisions with D_2 molecules. The transition cross section in collisions with orthodeuterium molecules is $\sigma \approx 0.76\pi(\lambda_g - \lambda_u)^2$ and in collisions with paradeuterium $\sigma \approx 0.93\pi(\lambda_g - \lambda_u)^2$. Thus, the consideration of molecular binding does not lead to a significant change in the transition cross section.

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108