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ON THE E2 TRANSITION PROBABILITY FROM THE FIRST 2^+ *LEVEL IN SPHERICAL NUCLEI*

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The probability for a E2 transition from the first excited 2^+ state to the ground state as a function of the number of particles is derived on the basis of a microscopic description of the first excited 2^+ state in spherical nuclei.

IT is well known that in spherical nuclei the first excited state with spin 2 has collective properties. In particular, the radiative transition probability to the ground state is on the average an order of magnitude larger than a single particle transition probability.

Recently there have appeared several papers[1-3] in which the first 2^+ level was treated assuming pairing and quadrupole forces. Then, for example, for the case of N particles in a level with angular momentum j the Hamiltonian has the form[3]

$$H = \frac{f_0}{2}\sum_\mu A^+_{2\mu}A_{2\mu} - \frac{F_2}{2}(uv)^2\sum_\mu [A^+_{2\mu} - (-1)^\mu A_{2-\mu}][A_{2\mu} - (-1)^\mu A^+_{2-\mu}],$$

$$A^+_{2\mu} = \sum_{m,m'} (jjm - m'|2\mu)\, \alpha^+_m \alpha^+_{-m'}, \tag{1}$$

where f_0 and F_2 are constants characterizing the strength of the pairing and the quadrupole force, $A^+_{2\mu}$ is the creation operator of a "phonon" of angular momentum 2 and projection of the angular momentum μ, and $(jjm - m'|2\mu)$ are Clebsch-Gordan coefficients.

The particle operators a_m are connected with the quasi-particle operators α_μ by the relation

$$a_m = u\alpha_m + v(-1)^{j+m}\alpha^+_{-m}, \tag{2}$$

where

$$v = (N/2\Omega)^{1/2}, \qquad u = (1 - N/2\Omega)^{1/2}, \qquad \Omega = j + 1/2.$$

The Hamiltonian (1) can be diagonalized by introducing new operators

$$B^+_{2\mu} = \chi A^+_{2\mu} - (-1)^\mu \varphi A_{2-\mu}, \tag{3}$$

$$\chi = (f_0 + 2E_2)/2\sqrt{2f_0E_2}, \qquad \varphi = -(f_0 - 2E_2)/2\sqrt{2f_0E_2}, \tag{4}$$

$$E_2 = \frac{f_0}{2}\sqrt{1 - \frac{2N}{\Omega}\left(1 - \frac{N}{2\Omega}\right)\frac{E_2}{f_0}}. \tag{5}$$

E_2 is the energy of the 2^+ level; its wave function can be written as

$$\Phi_{2^+} = B^+_{2\mu}|\Phi_0\rangle, \tag{6}$$

where $|\Phi_0\rangle$ — ground state wave function.

Let us investigate the probability for the ground state transition for the simple case where N particles are in one level with an angular momentum $j \gg 1$. Then the operator for the quadrupole moment has the form

$$\hat{Q}_{2\mu} = -\langle j|r^2|j\rangle \sum_{m,m'} (-1)^{j-m}(jjm - m'|2 - \mu)\sqrt{\frac{2j+1}{5}}\, a^+_{m'}a_m, \tag{7}$$

where $\langle j|r^2|j\rangle$ — radial matrix element. The reduced E2 transition probability has the form

$$B_{2\to0}(E2) = \sum_\mu |\langle\Phi_0|\sqrt{\frac{5}{16\pi}}\hat{Q}_{2\mu}|\Phi_{2^+}\rangle|^2. \tag{8}$$

Using (2), (3), (6) and (7) one finds easily

$$B_{2\to0}(E2) = (2j/16\pi)\langle j|r^2|j\rangle^2 (uv)^2(\chi - \varphi)^2. \tag{9}$$

Inserting in (9) the expressions for u, v, χ and φ we obtain

$$B_{2\to0}(E2) = \frac{\langle j|r^2|j\rangle^2}{16\pi} N\left(1 - \frac{N}{2\Omega}\right)\left[1 - \frac{2N}{\Omega}\left(1 - \frac{N}{2\Omega}\right)\frac{F_2}{f_0}\right]^{-1/2} \tag{10}$$

or

$$B_{2\to0}(E2) = \frac{\langle j|r^2|j\rangle^2}{16\pi} N\left(1 - \frac{N}{2\Omega}\right)\frac{f_0}{2E_2}. \tag{11}$$

Equation (10) gives the dependence of $B_{2\to0}$ on the number of particles in the shell.

Equation (11) shows more clearly the origin of the collective nature of the transition. The transition probability is proportional, first, to the number N of the particles in the shell and, second, to the ratio $f_0/2E_2$ which increases monotonically as the shell is being filled. Indeed, f/2 corresponds to the energy required to create two quasi-particles. It has a constant magnitude. On the other hand one sees from (5) that the energy E_2 decreases as the shell is being filled, up to the point where the shell is half full. It can be several times smaller than $f_0/2$. The last factor is associated with the quadrupole force. For $F_2 = 0$ the quantity $f_0/2E_2$ takes on its smallest value which equals unity.

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