GIANT QUANTUM OSCILLATIONS IN THE ACOUSTICAL ABSORPTION BY A METAL IN A MAGNETIC FIELD

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An expression is obtained for the acoustical absorption coefficient in metals in a magnetic field which satisfies the condition $\hbar\Omega \gg kT$ (Ω is the electron Larmor frequency) for the case when the acoustical wavelength is greater than the Larmor radius of the conduction electrons, but significantly less than their mean free path. It is found that under such conditions the absorption coefficient oscillates as a function of 1/H. The nature of these oscillations is the same as in the de Haas—van Alphen or Shubnikov—de Haas effects. However, the oscillations can under certain conditions be gigantic, i.e., they are not a small correction to a non-oscillatory part of the acoustical absorption coefficient, but are such that the maximum value of the absorption coefficient far exceeds the minimum.

At low temperatures, when the mean free path l of conduction electrons in a metal is sufficiently large, the absorption of sound can be considered as the direct absorption of phonons by the electrons of the metal. In the absence of a magnetic field the major role in the absorption is taken by electrons whose velocity v_κ in the direction of the acoustical wave vector $\boldsymbol{\kappa}$ is equal to the phase velocity of sound w_p.

This fact was pointed out by Akhiezer, Kaganov and Lyubarskii.[1] It is easy to satisfy oneself that, by considering the conservation laws during phonon absorption by the electron

$$\varepsilon(\mathbf{p}) + \hbar\omega_\varkappa = \varepsilon(\mathbf{p} + \hbar\boldsymbol{\varkappa}), \tag{1}$$

where $\mathbf{p}$ is the quasi-momentum of the electron, ϵ is its energy, $\omega = w_p\kappa$ is the frequency of sound, and by expanding $\epsilon(\mathbf{p} + \hbar\kappa)$ to terms of first order in the small quantity κ

$$\varepsilon(\mathbf{p} + \hbar\boldsymbol{\varkappa}) = \varepsilon(\mathbf{p}) + \hbar v_\varkappa \varkappa, \tag{2}$$

we obtain the condition

$$v_\varkappa = w_p, \tag{3}$$

which has been mentioned above. Electrons with a different velocity component in the direction of the vector $\boldsymbol{\kappa}$ can participate in the absorption only in so far as they interact with scatterers. As was shown in reference 1, if $\kappa l \gg 1$ their contribution to the absorption is of order $1/\kappa l$ compared with the contribution of electrons whose velocity satisfies condition (3).

An analogous situation also occurs in the presence of a magnetic field. We shall consider for simplicity the case of a quadratic isotropic dispersion law for electrons; the energy of an electron in the magnetic field then takes the form

$$\varepsilon_n = \hbar\Omega\,(n + 1/2) + p_z^2/2m, \tag{4}$$

where n is the Landau oscillator quantum number, $\Omega = eH/mc$ is the Larmor frequency, m is the effective mass, and p_z is the component of momentum in the direction of the magnetic field **H** which is along the z axis. Taking into account the conservation of energy and of the z component of the quasi-momentum, we have

$$\hbar\Omega\,(n + 1/2) + p_z^2/2m + \hbar\omega_\varkappa = \hbar\Omega\,(n' + 1/2) + (p_z + \hbar\varkappa_z)^2/2m. \tag{1a}$$

Putting

$$\varepsilon_{n'}(p_z + \hbar\varkappa_z) = \varepsilon_{n'}(p_z) + \hbar v_z\varkappa_z, \tag{2a}$$

we obtain the condition

$$\Omega\,(n' - n) + w_p\,\varkappa = v_z\varkappa_z. \tag{5}$$

If the magnetic field is so large that

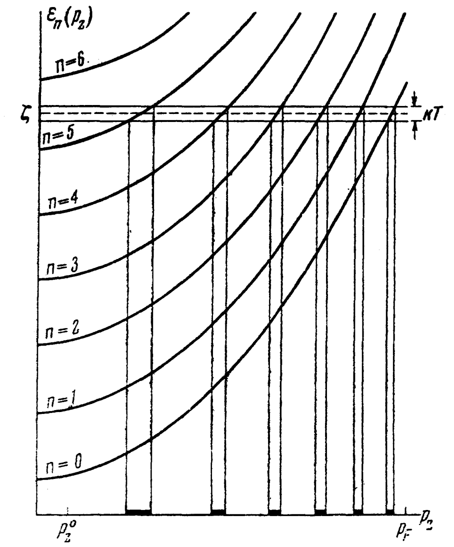
$$\Omega > \varkappa_z v_F \tag{6}$$

(v_F is the Fermi velocity), then condition (5) can only be satisfied for $n = n'$. Consequently, the quasi-momentum of the electrons which participate in the acoustical absorption in a magnetic field is equal to

$$p_z^0 = mw_p / \cos\vartheta, \tag{7}$$

where θ is the angle between the vectors κ and H. But $w_p \ll v_F$, therefore, if this angle differs sufficiently from a right angle, most of the absorption is due to electrons with small values of p_z, far smaller than the limiting Fermi momentum p_F.

On the other hand, it is obvious that only electrons with energies lying in the interval where the Fermi distribution is changing can participate in absorption. In order to see more clearly the values of p_z which correspond to these energy values, we turn to the figure. In it are shown the parabolae which give the variation of the electron



energy with p_z for the first few values of n. The parabolae intersect the strip of width kT, the center of which is at the chemical potential ζ. The width of the strip is smaller than the distance between the parabolae, which corresponds to the condition

$$\hbar\Omega \gg kT. \tag{8}$$

By projecting onto the abscissa axis the portions of the parabolae cut off by the strip, we find that, when condition (8) is satisfied where the Fermi distribution is changing, intervals of allowed and forbidden values of p_z exist. The former intervals are indicated in the Figure by thick lines; for the corresponding values of p_z the derivative with respect to ζ/kT of the Fermi function is of the order of unity. The latter intervals are indicated by thin lines on the abscissa axis, and for the corresponding values of p_z the derivative with respect to ζ/kT of the Fermi function is exponentially small. The position of these intervals depends on the magnetic field H, because the distance between the parabolae in the figure changes with changing H. If the value of H is such that the momentum p_z^0 is in the range of allowed values of p_z, strong acoustical absorption by free electrons occurs. For a different value of H the momentum p_z^0 can lie in a range of forbidden values of p_z, and then acoustical absorption occurs only because of the scattering of conduction electrons, i.e., it is much smaller than in the first case. The general situation is thus one in which giant oscillations in the acoustical absorption coefficient occur provided that

$$\zeta \gg \hbar\Omega \gg kT. \tag{9}$$

We note that the other kinetic coefficients, such as the conductivity, also oscillate when (9) is satisfied. The occurrence of these quantum oscillations is also connected with the presence of allowed and forbidden intervals of p_z on the Fermi surface. However, these oscillations are a small correction — usually of order $(\hbar\Omega/\zeta)^{1/2}$ — to a major effect, and can be calculated using classical theory.

In our case giant quantum oscillations occur. Therefore, when calculating the energy absorbed, which we now proceed to do, it is necessary to use quantum theory. We neglect the contribution to the absorption from transverse electric fields arising during deformation of the conductor by the sound. (The question of whether it is permissible to neglect this has been considered previously.[2,3]) Then the energy operator for the interaction of the electron with the sound wave has the form

$$V = \tfrac{1}{2}(Ue^{-i\omega t} + U^+ e^{i\omega t}), \tag{10}$$

where $U = \Lambda_{ik}u_{ik}e^{i\kappa\cdot\mathbf{r}}$. Here u_{ik} is the tensor amplitude of the deformation in the sound wave, Λ_{ik} is the tensor calculated in reference 1, the components of which in general can depend on the components of the quasi-momentum operator. Strictly, the quantity $\Lambda_{ik}u_{ik}$, being the difference between the deformation and electrostatic potentials, should itself oscillate in the magnetic field. However, it is not difficult to see that the relative size of these oscillations is small; therefore they will be ignored when considering the giant oscillations in the absorption.

We consider the scattering of electrons to be elastic, which is permissible in most cases. Then the energy density absorbed by the electrons in unit time is

$$q = -\frac{1}{V_0}\frac{\pi}{2\hbar} \times \sum_{aa'}(F_a - F_{a'})\,\hbar\omega_{aa'}\,|\langle a|U|a'\rangle|^2\,\delta(\hbar\omega_{aa'} + \hbar\omega). \tag{11}$$

Here a and a′ denote the states of the electron in the field of the scatterers in the presence of the external magnetic field, $\hbar\omega_{aa'} = \epsilon_a - \epsilon_{a'}$, F_a is the Fermi function, $\langle a | v | a' \rangle$ is the matrix element of the operator U, V_0 is the volume of the crystal.

If $\hbar\omega \ll kT$, we can put

$$F_a - F_{a'} = (\partial F_a / \partial\zeta)\, \hbar\omega_{a'a} = (\partial F_a / \partial\zeta)\, \hbar\omega. \quad (12)$$

The absorption coefficient Γ is obtained by dividing equation (11) by the flux density of acoustical energy equal to

$$\rho\omega^2 u_0^2 w / 2, \quad (13)$$

where ρ is the density of the crystal, u_0 is the amplitude of the oscillations in the sound wave, w is the group velocity of sound. In order to obtain the zero-order approximation we will neglect the general effect of the scatterers on the absorption of sound. To do this it suffices to replace the indices a and a′ in (10) by α and α', respectively, which characterize the free electron states in a magnetic field. The quantity α is the totality of the quantum numbers n, p_z, X, and s_z, where $\hbar s_z/2$ is the spin component in the direction of the magnetic field, X is the coordinate of the center of the Landau oscillator. After this the following expression is obtained for the acoustical absorption coefficient:

$$\Gamma = \frac{\pi}{V_0 \rho u_0^2 w} \sum_{\alpha\alpha'} \frac{\partial F_\alpha}{\partial \zeta} \langle \alpha | U | \alpha' \rangle |^2 \delta(\omega_{\alpha\alpha'} + \omega). \quad (14)$$

For simplicity we now consider the idealized case where the components of the tensor Λ_{ik} are essentially constant, the electron spectrum is quadratic and isotropic, and the vectors $\boldsymbol{\kappa}$ and $\mathbf{H}$ are parallel to one another. Then

$$\langle \alpha' | U | \alpha \rangle = \Lambda_{ik} u_{ik} \delta_{s_z s_z'} \delta_{nn'} \delta_{XX'} \delta_{p_z + \hbar\kappa_z, p_z'} \quad (15)$$

and we have for the coefficient Γ

$$\Gamma = \Gamma_0 \frac{\hbar\Omega}{8kT} \frac{\kappa}{m} \sum_{n,\, s_z} \int dp_z\, \delta\left(\frac{\kappa p_z}{m} + \frac{\hbar\kappa^2}{2m} - \omega\right) \times \mathrm{ch}^{-2}\left[\frac{\zeta - \hbar\Omega(n + 1/2) - \mu_0 s_z H - p_z^2/2m}{2kT}\right]. \quad (16)^*$$

Here μ_0 is the Bohr magneton,

$$\Gamma_0 = m^2 |\Lambda_{ik} u_{ik}|^2 / 2\pi\hbar^2 \rho u_0^2 \kappa w \quad (17)$$

is the acoustical absorption coefficient when H = 0, obtained by Akhiezer et al.[1]

After integrating over p_z, we obtain

$$\Gamma = \Gamma_0 \frac{\hbar\Omega}{8kT} \sum_{n,\, s_z} \mathrm{ch}^{-2}\left[\frac{\hbar\Omega(n + 1/2) + s_z \mu_0 H - \zeta}{2kT}\right]. \quad (18)$$

In deriving (18) we have neglected in the argument of the hyperbolic cosine the small quantity $(p_z^0)^2/2m$, which can only cause an insignificant overall shift of the oscillatory picture.

For $\hbar\Omega \ll kT$ the sum over n in (18) can be replaced by an integral which is easily evaluated and gives

$$\Gamma = (\Gamma_0/2) \int_0^\infty \frac{dy}{\mathrm{ch}^2(y - \zeta/2kT)} \approx \Gamma_0.$$

This expression for Γ agrees with that obtained previously.[3]

When $\hbar\Omega \gg kT$ the graph of the function $\Gamma(1/H)$ shows a series of sharp oscillatory maxima, the distance between which is

$$\Delta H^{-1} = e\hbar/mc\zeta, \quad (19)$$

and the height of which is proportional to H, separated by wide, gently sloping minima, the heights of which are exponentially small.

Equation (19) can only describe the behavior of the absorption coefficient close to the maxima. In order to obtain the value of the absorption coefficient for the minima, and at the same time to estimate the limits of applicability of (19), it is necessary to obtain the correction to it due to scattering. To do this we replace the δ function in (16) by the expression

$$\frac{1}{\pi} \frac{\nu}{\nu^2 + (\hbar\kappa p_z/m + \hbar\kappa^2/2m - \omega)^2}. \quad (20)$$

Such a replacement corresponds to the assumption that a relaxation time $t_0 = 1/\nu$ exists, and yields the order of magnitude of the necessary correction. In the classical theory (see reference 2) when $\kappa l \gg 1$ a relaxation time exists for most scattering mechanisms. The proof that a relaxation time exists when $\kappa l \gg 1$ in the quantum region presents well-known difficulties, and will be given for some cases in a separate paper.

We consider the case of greatest interest as regards possible experiments: $\hbar\kappa^2/2m \ll \nu$ and $\omega \ll \nu$. The expression (20) can then be taken as equal to

$$\nu / \pi\, [\nu^2 + (\hbar\kappa p_z/m)^2]. \quad (21)$$

We replace the δ function in (16) by (21) and transform to the dimensionless integration variable $y = p_z(2mkT)^{-1/2}$. Then (16) becomes

$$\Gamma = \Gamma_0 \frac{\hbar\Omega}{2kT} \int dy \frac{1}{\pi} \frac{B}{1 + B^2 y^2} \sum_{n,\, s_z} \frac{1}{4} \mathrm{ch}^{-2}\left(\frac{y - A_n}{2}\right). \quad (22)$$

Here $B = (2kT/m)^{1/2}\kappa/\nu$; $A_n = [\zeta - \hbar\Omega(n + 1/2) - s_z\mu_0 H]/kT$.

The expression under the integral in (22) contains the product of two rapidly-changing functions. The maximum of the first is at y = 0 and has a

*ch = cosh.

width $1/B$; far from the maximum it falls off as $1/\pi B y^2$. The second function consists of a system of peaks, the width of each one being of order unity. The position of the peaks changes as the magnetic field changes, and the distance between them far exceeds their width when $\hbar\Omega/kT \gg 1$. Far from the peaks the second function decreases exponentially: $\exp(-|A_n - y^2|/2)$.

We shall treat the case when the distance between the two peaks of the second function closest to the maximum of the first function greatly exceeds the width of this maximum. This condition is equivalent to the inequality

$$B(\hbar\Omega/kT)^{1/2} \gg 1, \text{ or } \varkappa l(\hbar\Omega/\zeta)^{1/2} \gg 1. \tag{23}$$

Now, if the value of the magnetic field is such that the position of one of the peaks of the second function coincides with the maximum of the first, then a giant oscillation occurs. If the width of the maximum of the first function is much smaller than the width of the peak of the second, i.e.,

$$B \sim \varkappa l(kT/\zeta)^{1/2} \gg 1, \tag{24}$$

the first function can be replaced by a δ function. Thus, under condition (24), formula (18) accurately describes the shape of the absorption line close to a maximum. In the case $B \lesssim 1$ the shape of the absorption line is different.

The absorption coefficient has a minimum when two closest peaks of the second function are symmetrically situated with respect to the maximum of the first. In this case the first function changes slowly in the region of both peaks; therefore the peaks themselves can be approximated by δ functions. The evaluation of the coefficient Γ at a minimum then presents no difficulty, and gives the following ratio of the maximum value of the absorption coefficient to the minimum:

$$\Gamma_{max}/\Gamma_{min} \sim \varkappa l(\hbar\Omega/\zeta)^{1/2}\hbar\Omega/kT \gg 1, \tag{25}$$

i.e., the oscillations are, in fact, gigantic. The estimate (25) retains its significance also when $B \sim 1$, and gives

$$\Gamma_{max}/\Gamma_{min} \sim (\hbar\Omega/kT)^{3/2}. \tag{26}$$

The case $B \ll 1$ is not of great interest, because it is difficult to satisfy simultaneously in an experiment this inequality and (23).

Giant oscillations of the acoustical absorption coefficient can also occur for an arbitrary shape of the Fermi surface and for arbitrary form of the deformation potential, provided that the motion of electrons with those values of p_z which make $\langle\alpha|v_z|\alpha\rangle$ tend to zero is finite. In fact, in this case the matrix elements of the deformation potential (neglecting spin-orbit interaction) are

$$\langle\alpha|U|\alpha\rangle = \langle n_\alpha|\Lambda_{ik}u_{ik}|n_\alpha\rangle\,\delta_{s_z s_z'}\,\delta_{p_z+\hbar\kappa_z,p_z'}\,\delta_{XX'}. \tag{27}$$

But if $\Omega > \kappa_z v_F$, then $n = n'$, and we conclude that all the previous estimates of the coefficient Γ at a maximum and a minimum remain valid. However, the whole oscillatory picture can be very complicated, because there can exist several values of p_{zm} for which $\langle\alpha|v_z|\alpha\rangle$ is close to zero. The number of these values can be determined from the number of different periods on the curve of $\Gamma(1/H)$.

As in other well-known cases (see, for example, the work of Lifshitz and Kosevich[4]), the period of the quantum oscillations in our case is related to the area $S(\epsilon, p_z)$ cut from the constant energy surface by the plane $p_z = p_{zm_0}$ by the formula

$$\Delta H_m^{-1} = 2\pi e\hbar/cS(\varepsilon, p_{zm}). \tag{28}$$

However, the essential difference is that the periods of the oscillations in our case are determined not by the extremal sections, but by the sections involving those $p_z = p_{zm}$ which make $\langle\alpha|v_z|\alpha\rangle$ tend to zero.

Clearly the giant oscillations can be smeared out if the specimen has an even relatively weak polycrystallinity. If this is to be avoided, the angle φ describing the spread in orientation of the crystallites which constitute the specimen must satisfy the condition

$$\varphi \ll \hbar\Omega/\zeta. \tag{29}$$

This inequality is satisfied more easily, the greater the magnetic field and the smaller the value of ζ. The giant oscillation effect is, therefore, most easily observed in semimetals and semiconductors with Fermi degeneracy.

Thus, the classical theory of acoustical absorption in a magnetic field previously developed[2,3] is, generally speaking, only applicable when $\hbar\Omega \ll kT$. If the reverse inequality applies, quantum theory must be used.

[1] Akhiezer, Kaganov, and Lyubarskii, JETP **32**, 837 (1957), Soviet Phys. JETP **5**, 000 (1957).

[2] V. L. Gurevich, JETP **37**, 71 (1959), Soviet Phys. JETP **10**, 51 (1960).

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[4] I. M. Lifshitz and A. M. Kosevich, JETP **24**, 730 (1955).

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