ELECTRODYNAMICS OF A ZERO MASS SPINOR PARTICLE

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Submitted to JETP editor August 3, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) 40, 792-800 (March, 1961)

The electrodynamics of a massless two-component particle with charge e_1 is considered. Despite a number of peculiarities the theory turns out to be not internally inconsistent and is as complete as ordinary electrodynamics. A rough experimental estimate gives $e_1 \lesssim 10^{-8}$ e. The possibility that the known neutrinos carry such a charge e_1 is discussed.

1. INTRODUCTION

SIMPLICITY of the basic equations is an important consideration in attempts to describe the properties of elementary particles. As examples we might cite Dirac's equation, which is the simplest of all equations describing particles with half-integral spin, the two-component neutrino theories, or the universal "two-component" weak interaction.[1] It is likely that considerations of simplicity and symmetry will continue to be taken into account in the future development of the theory. In such a framework it becomes quite attractive to consider the possibility of existence of a charged, massless, two-component particle, described by the Lagrangian

$$L(x) = -u^{+}\sigma_r(p_r - e_1 A_r)u. \qquad (1)$$

Here $\sigma_r = (\sigma, 1)$, and $u(x)$ is a two-component spinor. Such a possibility is not quite obvious, however, because many of the formulas of electrodynamics contain the mass of the particle in the denominator or as the argument of a logarithm. This then raises the question of consistency of electrodynamics for a massless particle, which might be of interest for various reasons.

1. It might be useful to trace the role played by the mass in electrodynamics: are the mass and charge related in the existing theory, is there an "electromagnetic" mass?

2. For the development of the theory of particles and interactions it will, no doubt, be useful to know the properties of permissible "solutions," particularly the simplest solutions.

3. Finally, if such a theory is internally consistent one might raise the question about the existence of such particles. It is obvious that the interaction (1) is impossible with the constant $e_1^2 = e^2 = 1/137$. However it is not out of the question that a two-component massless particle (neutrino), characterized by the weakness of its interaction, has a weak interaction with photons.

2. CLASSICAL ELECTRODYNAMICS*

The Lorentz equation contains no mass; the total energy provides a "measure of inertia" and consequently the equations of motion in an external field are valid for $m = 0$ too.

The peculiarities in the interaction with radiation have to do with the equality of the speed of the particle and the speed of the radiation field. A massless particle does not fall behind the electromagnetic wave, produced by a change in the particle velocity, so that it is not possible to make a strict division of the field into the particle field and radiation field. A single spherical wave is produced, with a singularity at the point occupied by the charge. The proper field may be determined only in the vicinity of the particle, where it is legitimate to neglect the curvature of the front of the spherical wave. The electric and magnetic fields are here given by the formulas

$$\mathbf{E} = e_1 \delta(z - t)\, \boldsymbol{\rho} / \rho^2, \qquad \mathbf{H} = [\mathbf{v}\,\mathbf{E}], \qquad (2)†$$

where the z axis of the cylindrical coordinate system (z, ρ) has been chosen along the direction of the particle velocity **v**. The expression (2) represents the limiting case of the well known relativistic contraction of the proper fields as a consequence of motion.

The formulas for radiation become, for $m = 0$, logarithmically divergent for small angles θ between the directions of radiation and the particle velocity. For example in the radiation problem in

*By classical electrodynamics we mean, as is usual, the interaction with radiation of low frequency $\omega \ll \varepsilon$, where ε is the particle energy. We use units such that $h = c = 1$, and a metric such that $ab = a_r b_r = \mathbf{a}\cdot\mathbf{b} - a_0 b_0$.

†$[\mathbf{v}\mathbf{E}] = \mathbf{v} \times \mathbf{E}$.

β decay (up to $t = 0$ the charge is at rest and thereafter moves with velocity v) the spectral energy density of radiation $I(\omega, \theta)$ is given by

$$I(\omega, \theta)\, d\omega d\theta = \frac{e_1^2}{2\pi} v^2 \frac{\sin^3\theta\, d\omega d\theta}{(1 - v\cos\theta)^2}, \tag{3}$$

so that for $v = 1$ the expression $I(\omega) = \int I(\omega, \theta)\, d\theta$ is logarithmically divergent. The divergence arises from an incorrect application of the formula, correct for the fields at a distance from the charge, at the position of the charge $\theta = 0$. If the radiation for small θ is determined as the energy emitted in that direction after the particle was deflected, then the quantity $I(\omega)$ will depend logarithmically on the time T spend by the particle in moving in the given direction:

$$I(\omega) = \frac{e_1^2}{4\pi^2} \int \frac{[\mathbf{n}\mathbf{v}]^2}{(1 - \mathbf{n}\mathbf{v})^2} |\exp\{i(\omega - \mathbf{k}\mathbf{v}) T\} - 1|^2 \approx \frac{2e_1^2}{\pi} \ln \omega T. \tag{4}$$

Consequently the particle is not absolutely stable, its energy and momentum going eventually over into radiation. A semi-classical estimate for the dependence of the particle energy on time, due to the losses in such a "forward radiation," results in the formula

$$\varepsilon(t) = \varepsilon(0) \exp\{-(e_1^2/2\pi) \ln \varepsilon(0) t\}, \qquad \varepsilon(0) t \gg 1. \tag{5}$$

This problem is considered in detail in Sec. 4. It is clear that for $e_1^2 \ll 1$ the transformation of the energy into radiation is extremely slow and the particle is metastable, immeasurably more stable than, say, the pion or neutron. It is also clear from physical considerations that any deviation of the trajectory from a straight line violates the conditions for coherent interference and the difference between the radiations of massive and massless particles disappears. Thus, the classical expressions for the radiation of an ultrarelativistic electron moving in a magnetic field[2]* or in a material medium,[3] do not contain the mass and are therefore directly applicable to our case.

3. THE DIRAC EQUATION. PRESERVATION OF MASSLESSNESS. FORBIDENNESS OF DIRECT DECAYS

The Dirac equation becomes in our case

$$\sigma(p - e_1 A)\, u(x) = 0. \tag{6}$$

The positive and negative frequency solutions of Eq. (6) go over into each other continuously. Bound states in an external field, vanishing at infinity, do not exist; quasistationary states are, of course, possible. The energy levels in a constant magnetic field $H = H_z$ are given by $\epsilon_n = [p_z^2 + 2e_1 H(n + 1)]^{1/2}$, where p_z is the z component of the momentum and n is the number of the state. In this way the field H leads to a gap between states with opposite signs of the energy, with the quantity $(2e_1 H)^{1/2}$ acting as a mass.

The quantization is carried out in the usual way. The interaction Lagrangian becomes then

$$L(x) = e_1 j_r(x) A_r(x) = e_1 N(u^+(x)\, \sigma_r\, u(x)) A_r(x). \tag{7}$$

In order to derive our case from the formulas of conventional electrodynamics in the limit $m = 0$, we observe that the operator $u(x)$ can be obtained from the full Dirac field $\psi(x)$ by simple projection: $u(x) = \frac{1}{2}(1 - \gamma_5)\psi(x)$. Since γ_5 commutes with any pair of matrices γ_r it is sufficient to insert $\frac{1}{2}(1 - \gamma_5)$ once in each electron line of a Feynman diagram.

It is clear from the formulas of electrodynamics that the electromagnetic mass δm is proportional to the bare mass m_0,[4] so that if $m_0 = 0$ then also $\delta m = 0$. Thus the masslessness is preserved and the singularity of the Green's function $G(p)$, as in the free field case, lies at the point $p^2 = 0$ [see Eq. (15)].

Unlike the electron case, the decay of the massless particle into a photon and a particle moving in the same direction is allowed by the conservation laws. Furthermore the statistical weight is not equal to zero since any division of the energy between the decay products is possible. However, as is easy to see from Eq. (7), the matrix element for the transition vanishes. This has to do with the transverse nature of the photon and conservation of the projection of the angular momentum along the direction of motion: for the photon this quantity is always $+1$ or -1, for the particle it is always $\frac{1}{2}$ and therefore the decay is forbidden. For the same reason a photon cannot produce a pair of these particles since in the final state the spin projection on the direction of motion is zero. And since the decay of one particle into three is already forbidden by statistical weight factor it follows that no direct decays occur.

Nevertheless we do not have complete stability, as is already known from classical theory. Higher order perturbation theory approximations contain logarithmic divergences at small θ, analogous to the classical Eqs. (3) and (4). The situation is very much the same as the well known "infrared catastrophe." The divergence indicates the inap-

*The classical formulas for the spectrum of radiation density are valid only for frequencies $\omega \lesssim \varepsilon$. Consequently the expressions given by Landau and Lifshitz[2] for the total radiation intensity in a magnetic field are not applicable here.

plicability of perturbation theory to ultra small angles, as well as ultra small frequencies. For small e_1^2 this divergence, like the infrared divergence, is of no practical importance since all angular integrals must be cut off at $\theta_{\min} \sim \Delta\theta$ (just like integrals over frequency are cut off at $\omega_{\min} \sim \Delta\omega$), where $\Delta\theta$ is the experimental error in the selection of photons emitted at small angles to the beam direction. However, just as in the case of the infrared catastrophe, the problem can be solved without making use of perturbation theory, and the divergence indicates in both cases the basic instability in the problem.

4. METASTABILITY OF MASSLESS PARTICLE AND PHOTON

As an example we discuss the long wavelength radiation in β decay. Our special case $v = 1$ can be obtained from known solutions of the infrared problem. A simultaneous discussion of the singularities encountered in the emission of "soft" photons and in the "forward emission" will serve to emphasize the similarity of these phenomena.

Suppose that at $t = 0$ a particle with momentum p is produced. Then in perturbation theory we get for the amplitude of the electron + photon state

$$c_{\mathbf{p}'\mathbf{k}\lambda}(t) = V_{\mathbf{p}'\mathbf{k}\lambda,\,\mathbf{p}}(\varepsilon_p - \varepsilon_{p'} - k)^{-1} \times [\exp\{i(\varepsilon_{p'} + k - \varepsilon_p)t\} - 1]. \tag{8}$$

Here $p' = p - k$ and $V_{p'k\lambda,\,p}$ is the matrix element for the transition. The exponential term in Eq. (8) describes the proper electromagnetic field of the particle,[5] whereas the other term describes the radiation. For large t, when $(\epsilon_{p'} + k - \epsilon_p)t \approx (k - \mathbf{k}\cdot\mathbf{v})t \gg 1$, these two parts of the field separate since in physical expressions, bilinear in $c_{p'k\lambda}$, the oscillating interference term drops out. Consequently the probability of radiation is given by the expression (for simplicity we restrict ourselves to the interaction with transverse photons)

$$W = \sum_{\mathbf{k}\lambda}\left|\frac{V_{\mathbf{p}'\mathbf{k}\lambda,\,\mathbf{p}}}{\varepsilon_{p'} + k - \varepsilon_p}\right|^2 = \frac{e_1^2}{4\pi^2}\int\frac{d\mathbf{k}}{k}\,\frac{v^2 - (\mathbf{n}\mathbf{v})^2}{(k - \mathbf{k}\mathbf{v})^2}. \tag{9}$$

For small k the quantity W diverges, and for $v = 1$ it also diverges for small angles between k and v. The same divergence appears in the norm of the stationary state particle + photon, as calculated in perturbation theory. At the same time the quantity $\Sigma\,|c_{p'k\lambda}(t)|^2$, as calculated from Eq. (8), is finite but grows with time (for $v < 1$ as $\ln t$, for $v = 1$ as $\ln^2 t$). Consequently the calculation of the radiation probability from Eq. (9) is valid at the instant t only if $(k - \mathbf{k}\cdot\mathbf{v})t \gg 1$, since otherwise the appropriate photons have not yet separated into "real" and "virtual" ones. The total transition probability from the state with no photons does not approach a constant for large values of t, but increases as a consequence of the continuous creation of photons with $(k - \mathbf{k}\cdot\mathbf{v})t \gg 1$. Therefore the perturbation theory Eqs. (8) and (9) are valid only for small t or $(k - \mathbf{k}\cdot\mathbf{v})^{-1}$.

The complete solution of the problem may be taken from the work of Glauber.[6] If the state vector is written as $\Phi(t) = S(t, t_0)\Phi(t_0)$ then in the region of soft photons under consideration the matrix $S(t, t_0)$ is given, accurate up to a phase, by*

$$S(t, t_0) = \exp\left\{i\int_{t_0}^{t}\mathbf{j}(\mathbf{x}, \tau)\,\mathbf{A}(\mathbf{x}, \tau)\,d\mathbf{x}\,d\tau\right\}, \tag{10}$$

where $\mathbf{j}(\mathbf{x}, \tau)$ is the particle current and is a prescribed function of $\mathbf{x}$ and τ, and $\mathbf{A}(\mathbf{x}, \tau)$ is the electromagnetic field operator in the interaction representation. Equation (10) corresponds to a Poisson photon distribution: the probability of emission up to the instant t of n photons in a given interval k is given by the formula

$$W_n(t) = \frac{[\bar{n}(t)]^n}{n!}\exp[-n(t)], \tag{11}$$

where, in our problem,

$$\bar{n}(t) = \frac{e_1^2}{4\pi}\int\frac{dk}{k}\,\frac{v^2 - (\mathbf{n}\mathbf{v})^2}{(k - k\mathbf{v})^2}\,|e^{i(k - \mathbf{k}\mathbf{v})t} - 1|^2. \tag{12}$$

We see that the expression $\Sigma\,|c_{p'k\lambda}(t)|^2$, with $c_{p'k\lambda}$ given by Eq. (8), indeed represents the average total number of emitted photons and may increase with t without bound. The amplitude of the photonless state $S_{00}(t) = \exp[-\bar{n}(t)]$ is damped out in time. The nonstationary formulation of the problem is here, of course, immaterial; in a stationary problem t would be replaced by R — the distance from the source or scatterer. Consequently, from the point of view of quantum mechanics, the massless particle is unstable to the same degree as the electron.

Equations (10) — (12) are valid for $k \ll \epsilon$. It is obvious that the physical picture of a logarithmic piling up of photons with time is also valid for $k \lesssim \epsilon$, although $W_n(t)$ does not for such frequencies have the Poisson form (11) since the considerable recoil causes the emissions to be correlated. For $k > \epsilon$ the singularities in the denominators in

*It is easy to verify that Glauber's result, Eq. (10), is valid for arbitrary t_0 and t, and not just for the values $t_0 = -\infty$, $t = \infty$ considered by Glauber.[6]

Eqs. (8) and (9) disappear and perturbation theory is valid. It can be seen from Eqs. (10) and (11) that as the particle is deflected the "forward emission" photons are "shaken off" and the process starts all over again. It is also seen that both the infrared and the angular singularities disappear as a result of multiple scattering, magnetic fields, or any other effect leading to a nonrectilinear trajectory, since the coherent interference length $l(\mathbf{k}) = (k - \mathbf{k}\cdot\mathbf{v})^{-1}$ is now limited by the characteristic length l_{eff} of the process: $l(\mathbf{k}) \leq l_{eff}$.

Let us indicate the manner in which the just described instability is reflected in the structure of the Green's function. The Fourier component

$$G(\mathbf{p}, \tau) = (2\pi)^{-1} \int_{-\infty}^{\infty} G(\mathbf{p}, p_0) \exp(-ip_0\tau)\, dp_0$$

is proportional to the amplitude for the probability of finding the initial state $\Phi_{0p} = a_p^+ | \mathrm{vac} \rangle$ in the physical state $\Phi_p(\tau) = \exp(-iH\tau)\Phi_{0p}$, which evolves from Φ_{0p} in the time τ.[7,8] A pole in $G(p)$ at the point $p_0 = \epsilon_p$ corresponds to a stable particle, so that for large τ one has $G(\mathbf{p}, \tau) \sim \exp(-i\epsilon_p\tau)$. In electrodynamics, however, owing to the fact that the continuous spectrum of the electron + photon states begins at the one-electron state, the pole of G becomes a branch point:

$$G(\mathbf{p}, p_0) \to \mathrm{const}\,(p_0 - \varepsilon_p)^{-1+\beta}, \qquad p_0 \to \varepsilon_p. \qquad (13)$$

This indicates a damping of the initial state with time according to $G(\mathbf{p}, \tau) \sim \exp(-i\epsilon_p\tau - \beta \ln \epsilon_p\tau)$. The quantity β can be found with the help of Eqs. (11) and (12), since $S_{00}(\tau)$ is also proportional to the amplitude of the photonless state. A covariant treatment with longitudinal and scalar photons taken into account[9,10] merely changes the quantity β in Eq. (13). For a massless particle the continuous spectrum again touches the pole so that the pole in G becomes a branch point. For purely transverse photons the singularity in $G(p, p_0)$ is, according to Eq. (12), somewhat more complex than in the electron case. When however photons of all four polarizations are included even that difference disappears: it is clear from Eq. (15), Sec. 5, that the singularity in G has the form (13).

For purposes of exposition we have used above noncovariant expressions [for example Eq. (10)]. It is easy to pass to a covariant description. In the infrared catastrophe problem this is accomplished by introducing a small "photon mass" λ.[11] After addition of the probabilities of production of the particle with and without photons the quantity λ drops out of the answer, which is simply a result of unitarity — the total probability that the particle will be in one state or the other is conserved. These methods may be generalized so as to be applicable in our case and the quantity λ will disappear from the final answer after summing over the frequency interval $\omega < \omega_{min}$ and the angular interval $\theta < \theta_{min}$.

Up to now we were concerned with the change in time of the state of the particle. It follows from the considerations on the spectra and also simply from perturbation theory that a photon will also be unstable and will dissociate itself in time into a pair of massless particles moving in the same direction. In the photon Green's function too the pole changes into a branch point, again indicating the damping of the "bare" photon state and the appearance of the probability of pair formation. Under scattering by an external field one of the particles of the pair is deflected, and perturbation theory gives a probability for the emission of the other particle in the direction of motion of the photon that is logarithmically divergent with angle. In the absence of other physical reasons this divergence can be cut off by making use of the relations

$$(\varepsilon_p + \varepsilon_{\mathbf{k}-\mathbf{p}} - k)t \sim \frac{kp}{k-p}\frac{\theta^2}{2} t \gtrsim 1, \qquad \theta_{min} \gtrsim \left(\frac{\lambda\!\!\!^-}{R}\right)^{1/2}, \qquad (14)$$

i.e., by making use of the finiteness of the time available for coherent interference in pair formation.

This then is the general picture of instability and mutual transformations if a massless particle exists. Let us emphasize once more that this lack of stationary states is only of importance in principle. For $e_1^2 < e^2$, and for reasonable values of $\lambda\!\!\!^-$, the distances R, at which the probability for "decay into emptiness", which is of order $e_1^2 \ln(R/\lambda\!\!\!^-)$, becomes comparable with unity are many times in excess of any acceptable dimensions. At the same time the presence of any material medium will lead to multiple scattering and the divergence will disappear.

5. GREEN'S FUNCTION, VERTEX PART, RENORMALIZATIONS

The basic functions of electrodynamics may be obtained by taking the ultraviolet asymptote $p^2 \gg m^2$ of conventional electrodynamics.[12,13] Taking for simplicity the longitudinal part d_l of the photon Green's function to be constant we get for the Green's function $G(p)$ of the particle[13]

$$iG(p) = \frac{1-\gamma_5}{2}\frac{1}{\hat{p}} \exp\left(-\frac{e_{1\Lambda}^2}{4\pi} d_l \ln\frac{\Lambda^2}{p^2}\right). \qquad (15)$$

Here $e_{1\Lambda}$ is the bare charge and Λ is the cut-off momentum. The vertex part, for $p_1 \sim p_2 \sim k \sim p$, has the form

$$\Gamma_n(p_1, p_2, k) = \gamma_n \frac{1-\gamma_5}{2} \exp\left(\frac{e_{1\Lambda}^2}{4\pi} d_t \ln \frac{\Lambda^2}{p^2}\right). \quad (16)$$

Finally, for the transverse part d_t of the photon Green's function we get

$$d_{t\Lambda}(k^2) = \left(1 + \frac{e_{1\Lambda}^2}{6\pi} \ln \frac{\Lambda^2}{k^2}\right)^{-1}. \quad (17)$$

The task of the renormalization program consists in eliminating Λ from Eq. (17), since the renormalization constants in G and Γ cancel each other. The function $d_{t\Lambda}(k^2)$ is given in conventional electrodynamics by

$$d_{t\Lambda}(k^2)_{\rm el} = \left(1 + \frac{e_\Lambda^2}{3\pi} \ln \frac{\Lambda^2}{k^2 + 4m^2}\right)^{-1}. \quad (18)$$

Here the mass is introduced in a manner described by Bogolyubov, Logunov and Shirkov.[14] In the limit $k^2 \to 0$ the value of $e_\Lambda^2 d_{t\Lambda}(k^2)_{\rm el}$ equals the renormalized charge e^2, and it is natural to reexpress this quantity in terms of the renormalized charge:

$$e_\Lambda^2 d_{t\Lambda}(k^2)_{\rm el} = e^2\left(1 - \frac{e^2}{3\pi} \ln \frac{k^2 + 4m^2}{4m^2}\right)^{-1},$$

$$e^2 = e_\Lambda^2\left(1 + \frac{e_\Lambda^2}{3\pi} \ln \frac{\Lambda^2}{m^2}\right)^{-1} \quad (19)$$

In the case of Eq. (17), however, the point $k^2 = 0$ cannot be used for normalization since the function (17) vanishes there. It is therefore necessary to express the function $e_{1\Lambda}^2 d_{t\Lambda}(k^2)$ in terms of its value e_1^2 at some other point $k^2 = k_0^2$:

$$e_{1\Lambda}^2 d_{t\Lambda}(k^2) = e_1^2\left(1 - \frac{e_1^2}{6\pi} \ln \frac{k^2}{k_0^2}\right)^{-1},$$

$$e_1^2 = e_{1\Lambda}^2\left(1 + \frac{e_{1\Lambda}^2}{6\pi} \ln \frac{\Lambda^2}{k_0^2}\right)^{-1}. \quad (20)$$

The renormalized function $d_t(k^2)$ is given by

$$d_t(k^2) = \left(1 - \frac{e_1^2}{6\pi} \ln \frac{k^2}{k_0^2}\right)^{-1}. \quad (21)$$

The renormalization invariant quantity $6\pi e_1^{-2} - \ln k_0^2 = 6\pi e_{1\Lambda}^{-2} - \ln \Lambda t$ represents the only (dimensional) constant of the theory. We shall assume that $e_1^2 \ll 1$ for values of k_0 of the order of magnitude of energies likely to appear in experiments, and that therefore perturbation theory is applicable and d_t may be replaced by unity. The behavior of $d_t(k^2)$ for "superlarge" $k^2 \sim k_0^2 \exp(6\pi e_1^{-2})$ will not be considered here, since the difficulties that appear in this region should be removed by the same methods as are used in conventional electrodynamics.

Our results are easily generalized to the case of several charges. For example in the case of two bare charges e_Λ and $e_{1\Lambda}$, with e_Λ corresponding to a particle of mass m, we have instead of Eq. (18) the following expression for $d_{t\Lambda}$

$$d_{t\Lambda}(k^2) = \left(1 + \frac{e_\Lambda^2}{3\pi} \ln \frac{\Lambda^2}{k^2 + 4m^2} + \frac{e_{1\Lambda}^2}{6\pi} \ln \frac{\Lambda_1^2}{k^2}\right)^{-1}. \quad (22)$$

One then eliminates e_Λ, Λ, $e_{1\Lambda}$ and Λ_1 by expressing d_t in terms of the two experimental values $e_0^2 = e_\Lambda^2 d_{t\Lambda}(k_0^2)$ and $e_1^2 = e_{1\Lambda}^2 d_{t\Lambda}(k_1^2)$. For example, for $k_1^2 = k_0^2$ we obtain for d_t instead of Eq. (21)

$$d_t(k^2) = \left(1 + \frac{e^2}{3\pi} \ln \frac{k_0^2 + 4m^2}{k^2 + 4m^2} + \frac{e_1^2}{6\pi} \ln \frac{k_0^2}{k^2}\right)^{-1}. \quad (23)$$

For brevity we use below Eq. (21).

6. VACUUM POLARIZATION, SPECIFIC PROCESSES

For not ultrasmall distances $r \gg k_0^{-1} \exp(6\pi e_1^{-2})$ the field due to a point charge Ze is of the form

$$\varphi(r) = \frac{Ze}{2\pi^2} \int k^{-2} \left(1 + \frac{e_1^2}{6\pi} \ln \frac{k_0^2}{k^2}\right)^{-1} e^{i\mathbf{kr}} d\mathbf{k}$$

$$\approx \frac{Ze}{r}\left[1 - \frac{e_1^2}{3\pi}(\ln k_0 r + \text{const})\right] \quad (24)$$

in correspondence with the usual expression[15] for $rm \ll 1$. In the general case of an external field $\sim V$, whose extent in space is $\sim a$, the induced charge density ρ_1 is of the order of magnitude $\rho_1 \sim -e_1^2 V a^{-2}$.

Equation (21) for d_t represents in the case of vacuum polarization the first term in an expansion in the external field. The field produces a pair and thereafter the produced particles are considered to be free and described by plane waves. Such an approximation is legitimate only for virtual particles with energies $\epsilon \gg e_1 V$. Since for electrons $\epsilon > 2m$ this condition is in their case usually fulfilled; in our case it is violated for momenta $p \lesssim e_1 V$. For such particles the external field is strong and they are produced in vacuum with a probability ~ 1. The resultant density ρ_2 may be estimated by filling a Fermi sphere with limiting momentum $p_0 \sim e_1 V$, so that $\rho_2 \sim -e_1(e_1 V)^3$. For $e_1^2 (Va)^2 \ll 1$ the quantity ρ_2 may be ignored in comparison with ρ_1.

In the case of the Coulomb field $V \approx Zea^{-1}$ and the above condition reduces to $Zee_1 \ll 1$, i.e., the usual condition for applicability of perturbation theory. However for macroscopic systems ρ_2

may exceed ρ_1. In that case the screening charge q is $\rho_2 a^3$ and this screening will be small only if the condition

$$q/aV \sim e_1^4 (Va)^2 \lesssim 1 \quad (25)$$

is satisfied.

The formulas for specific processes may be obtained from conventional electrodynamics by replacing m by 0 and by an appropriate selection of polarization. We remark that the cross sections for bremsstrahlung and pair production in the field of an atom will now be proportional to the atomic cross section a_0^2 instead of the usual m^{-2} obtained for electrons. The logarithmic angular divergence will be, in the absence of other causes, cut off by multiple scattering. In the high energy region $\epsilon > (Z^2 e^2 e_1^2 n a_0^4)^{-1}$ (where n is the density) multiple scattering becomes the determining factor and the conventional formulas for radiation and pair production in a condensed medium hold.[16,17]

7. EXPERIMENTAL ESTIMATE OF e_1

The severest restrictions on the size of e_1 apparently come from macroscopic physics. The absence of a mass makes the pair production process thresholdless and therefore electromagnetic radiation will produce pairs in any external field. For small values of e_1 simple pair production in the Earth magnetic field is likely to be the most intense process. If the Larmor radius of the resultant particles is less than the radius of the Earth, the particles will be trapped by the Earth magnetic field. As a result they will accumulate up to a density when annihilation becomes probable. If there is time for equilibrium to be established the particle density in a unit of volume will be of the order of photon density. Thus the Earth and the adjacent magnetic field region represent a plasma of such particles. The presence of this plasma should be detectable by various macroscopic effects.

It is likely that the strongest inequality on e_1 is obtained from considerations on the propagation of long radio waves. In a magnetized plasma electromagnetic waves are attenuated. The absence of this effect leads to a gross estimate

$$e_1 \lesssim 10^{-8} e, \quad (26)$$

accurate to within one or two orders of magnitude.

Most likely the estimate (26) means that $e_1 = 0$; one might, however, ask what would be the consequences of the existence of a weak charge e_1 for the known neutrinos. For the β-decay neutrino such a possibility is unlikely. If it is assumed that e_1 is conserved then it follows from the experiments on measurements of the charge of the neutron and neutral molecules[18] that $e_1 \lesssim 10^{-15}$ e. Nonconservation of e_1 on the other hand, although it could be the reason for the weakness of the interaction, just like strangeness nonconservation is the reason for the weakness of hyperon decay interactions, leads to difficulties in connection with gauge invariance.

If, however, the two-neutrino hypothesis[19] is accepted then the introduction of a weak charge e_1 for the muon neutrino is not impossible. Assuming that e_1 is conserved we conclude that the charge of the muon differs from that of the electron by the amount e_1. This also explains the absence of the decays $\mu \to e + \gamma$, $\mu \to 3e$ to any order in the interaction,[20] and lepton conservation in the reactions $\pi \to \mu$, $K \to \mu$, $\mu \to e$ reduces simply to charge conservation.

The author is deeply grateful to A. I. Larkin for help and advice, and to A. B. Migdal, V. M. Galitskii, I. Ya. Pomeranchuk and B. M. Pontecorvo for interest in this work and valuable remarks.

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Translated by A. M. Bincer
130