EQUATIONS FOR PHOTOPRODUCTION OF PIONS ON NUCLEONS WITH EFFECTS DUE TO A PION-PION INTERACTION TAKEN INTO ACCOUNT

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Equations for the partial wave amplitudes for photoproduction of pions on nucleons at low energies are derived from the Mandelstam representation[11] in the Cini-Fubini approximation.[12] Nucleon recoil and pion-pion interaction are taken into account. The latter contributes only to the isotopic scalar photoproduction amplitudes. Expressions for these amplitudes, with effects of pion-pion interaction included, are obtained.

1. INTRODUCTION

RECENTLY there has been considerable interest in the study of pion-nucleon scattering including the effects due to a pion-pion interaction.[1-4] In this paper we consider the same problem for photoproduction.

Calculations based on one dimensional dispersion relations[5-7] give in general satisfactory agreement with present day experiments on photoproduction in the low energy region.[7-9] The discrepancies[9,10] may be thought of as due to an approximate treatment of the nucleon recoil[5-7] and neglect of the pion-pion interaction and of high energy contributions. In this work we obtain from the Mandelstam representation[11] in the Cini-Fubini approximation[12] a set of equations in which nucleon recoil and the pion-pion interaction are taken into account.

The pion-pion interaction enters into the equations under discussion through the amplitude for photoproduction of pions on pions. An expression for this amplitude has been obtained by one of the authors.[13] The pion-pion interaction contributes only to the isotopic scalar photoproduction amplitudes. Consequently the pion-pion resonance in the $J = I = 1$ state (if it exists) contributes only to those photoproduction amplitudes to which the pion-nucleon resonance (in the $J = I = 3/2$ state) does not.

The amplitude for photoproduction on pions depends on high-energy singularities and involves a parameter which depends on the amplitudes for the processes $\gamma\pi \rightarrow N\bar{N}$ and $\pi\pi \rightarrow N\bar{N}$ in the observable region. Consequently it is possible to write formally a system of coupled equations, which relate the amplitudes for the processes $\gamma N \rightarrow \pi N$ and $\gamma\pi \rightarrow N\bar{N}$ to the pion-nucleon and pion-pion amplitudes and contain no new parameters. At this time, however, we do not know how to take into account the contributions from the high energy regions. On the other hand, calculations of the amplitude for $\gamma\pi \rightarrow N\bar{N}$ in the observable region cannot be trusted if they utilize only the low energy singularities. For that reason it is necessary at this time to use experimental data for the process $\gamma\pi \rightarrow N\bar{N}$ (as well as $\pi\pi \rightarrow N\bar{N}$). Thus we shall be discussing only the amplitude for the process $\gamma N \rightarrow \pi N$, which involves a parameter determined by the process $\gamma\pi \rightarrow \pi\pi$ and assumed here to be known experimentally.

In order to write the above-mentioned equations it is necessary to discuss the cumbersome kinematics and unitarity condition for the processes $\gamma N \rightarrow \pi N$ and $\gamma\pi \rightarrow N\bar{N}$ (Sec. 2). The kinematics of the first process has been discussed by a number of authors,[5,6] the second process (as well as $\pi\pi \rightarrow N\bar{N}$) is best discussed using the formalism of Jacob and Wick.[14]

The Mandelstam representation[11] in the Cini-Fubini approximation[12] is written out in Sec. 3 (see also Bowcock et al.[1]). We assume the representation to be valid with no subtractions. One is encouraged in such an approach by the observation[13] that if the differential cross sections for photoproduction and forward scattering of pions on nucleons increase in the same manner at infinity then one needs one less subtraction for photoproduction than for scattering. This is a consequence of gauge invariance which leads to the appearance of an energy dependent factor in the relation between the invariant amplitude (for which one writes the Mandelstam representation) and

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the photoproduction matrix element, where no such factor appears in the scattering problem.

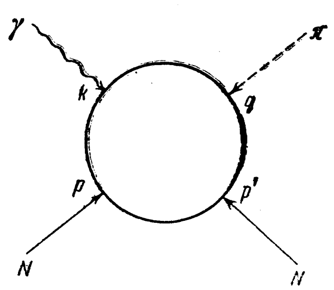
By making use of the kinematic relations of Sec. 2 the integral equations for the partial amplitudes are derived from the Cini-Fubini representation by simple integrations. Here the nucleon recoil is important only in the terms involving the direct pion-photon interaction and the magnetic dipole transitions in the $I = 3/2$ state that do not depend on the pion-pion interaction. This interaction contributes to the isotopic scalar amplitudes, and for those the nucleon recoil may be taken into account approximately. The expressions derived for the isotopic scalar amplitudes contain the effects of a pion-pion interaction and are a generalization of previously obtained results.[5-7]

2. KINEMATICS AND UNITARITY CONDITION

The matrix elements for the processes

$$\begin{aligned} &\text{I. } \gamma + N \to \pi + N', \\ &\text{II. } \pi + N \to \gamma + N', \\ &\text{III. } \gamma + \pi \to N + \overline{N}, \end{aligned}$$

described by the diagram shown (where k stands for the photon momentum, q for the pion momentum, and p and p′ for the momenta of the initial



and final nucleons in reactions I and II or the nucleon and antinucleon in reaction III) are given by

$$S_{fi} = -i(2\pi)^4\delta(k+q+p+p')\, m(4k^0q^0p^0p'^0)^{-1/2}F, \quad (1)$$

$$F = \left\langle f \left| \left(\delta_{\rho 3}F^{(1)} + \tau_\rho F^{(2)} + \tfrac{1}{2}[\tau_\rho, \tau_3]F^{(3)}\right) \right| i \right\rangle, \quad (2)$$

$$F^{(\alpha)} = \sum_{i=1}^{4} R_i H_i^{(\alpha)}(s, \bar{s}, t); \quad (3)$$

$$\begin{aligned} R_1 &= \bar{u}\gamma^5[(pk)(p'e) - (pe)(p'k)]u, \\ R_2 &= \bar{u}\gamma^5[((p'-p)k)(\gamma e) - ((p'-p)e)(\gamma k)]u, \\ R_3 &= \bar{u}\gamma^5[(qk)(\gamma e) - (qe)(\gamma k)]u, \\ R_4 &= \bar{u}\gamma^5[(\gamma k)(\gamma e) - (\gamma e)(\gamma k)]u. \end{aligned} \quad (4)$$

Here $|i\rangle$, $|f\rangle$ stand for the isotopic functions of the initial and final states, $\bar{u}$ is the final nucleon spinor, u is the initial nucleon spinor (processes I and II) or the final antinucleon spinor (process III) and e is the photon polarization vector.* The twelve invariant functions $H_i^{(\alpha)}$, which characterize the processes under discussion, depend on the arguments

$$s = (k+p)^2, \quad \bar{s} = (q+p)^2, \quad t = (p+p')^2; \quad s + \bar{s} + t = 2m^2 + \mu^2 \quad (5)$$

(m is the nucleon mass, μ is the pion mass). From crossing symmetry of F (for processes I and II), or from invariance under charge conjugation (for process III), we can conclude that

$$H_i^{(\alpha)}(s, \bar{s}, t) = \pm H_i^{(\alpha)}(\bar{s}, s, t). \quad (6)$$

Here and in the following the upper sign goes with the values $\alpha = 1, 2$, $i = 1, 2, 4$ and $\alpha = 3$, $i = 3$. The lower sign corresponds to $\alpha = 3$, $i = 1, 2, 4$ and $\alpha = 1, 2$, $i = 3$.

A. Reaction I. For photoproduction processes (reaction I) the isotopic functions in Eq. (2) are given by†

$$|i\rangle = N, \quad |f\rangle = \pi N. \quad (7)$$

Here N is the proton or neutron function:

$$p = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (8)$$

and π is the pion function:

$$\pi^{\pm} = \mp \frac{1}{\sqrt{2}}(\delta_{\rho 1} \pm i\delta_{\rho 3}), \quad \pi^0 = \delta_{\rho 3}. \quad (9)$$

The amplitudes

$$F^{(1/2)} = F^{(1)} + 2F^{(3)}, \quad F^{(3/2)} = F^{(1)} - F^{(3)} \quad (10)$$

describe isotopic vector transitions into states with total isotopic spin 1/2 and 3/2 respectively. $F^{(2)}$ describes the isotopic scalar transition into the state with isotopic spin 1/2.

In the barycentric frame of reaction I

$$s = (k+E_1)^2 = (\omega + E_2)^2, \quad \bar{s} = m^2 - 2k(E_2 + qx), \quad t = \mu^2 - 2k(\omega - qx). \quad (11)$$

Here (and in the following) k and q are the magnitudes of the three dimensional photon and pion momenta, x is the cosine of the angle between these momenta, $\omega = \sqrt{\mu^2 + q^2}$ is the pion energy, and $E_1 = (m^2 + k^2)^{1/2}$ and $E_2 = (m^2 + q^2)^{1/2}$ are the energies of the initial and final nucleon.

The spin structure of F in the barycentric frame is

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*Our metric is such that $ab = a^0b^0 - \mathbf{a}\cdot\mathbf{b}$, and our Feynman matrices are such that $\gamma^m\gamma^n + \gamma^n\gamma^m = 2g^{mn}$.

†With the usual normalization for the Clebsch-Gordan coefficients.

$$\mathcal{F} = \frac{m}{4\pi W} F = \chi^*_f \Big(i\sigma e \mathcal{F}_1 + \frac{(\sigma q)(\sigma [ke])}{qk} \mathcal{F}_2 + i \frac{(\sigma k)(qe)}{qk} \mathcal{F}_3 + i \frac{(\sigma q)(qe)}{q^2} \mathcal{F}_4 \Big) \chi_i, \tag{12*}$$

where χ are two-component nucleon spinors. The $\mathcal{F}_i$ are functions of the total energy W and of x, and are related to the invariant functions H_i as follows:

$$\mathcal{F}_1 = C \frac{W+m}{2W} \Big[(W + E_2 + qx) H_2 + (\omega - qx) H_3 + \frac{4W}{W+m} H_4 \Big],$$

$$\mathcal{F}_2 = C \frac{q (W-m)}{2W (E_2 + m)} \Big[(W + E_2 + qx) H_2 + (\omega - qx) H_3 - \frac{4W}{W-m} H_4 \Big], \tag{13}$$

$$\mathcal{F}_3 = -Cq \Big(\frac{W-m}{2} H_1 + H_2 - H_3 \Big),$$

$$\mathcal{F}_4 = C \frac{q^2}{E_2 + m} \Big(\frac{W+m}{2} H_1 - H_2 + H_3 \Big);$$

$$C = [(W-m)/8\pi W] \, [(E_1 + m)(E_2 + m)]^{1/2}. \tag{14}$$

Their partial wave expansion is given by

$$\mathcal{F}_1 = \sum_{l=0}^{\infty} \{ [l M_{l+} + E_{l+}] P'_{l+1}(x) + [(l+1) M_{l-} + E_{l-}] P'_{l-1}(x) \},$$

$$\mathcal{F}_2 = \sum_{l=1}^{\infty} [(l+1) M_{l+} + l M_{l-}] P'_l(x),$$

$$\mathcal{F}_3 = \sum_{l=1}^{\infty} \{ [E_{l+} - M_{l+}] P''_{l+1}(x) + [M_{l-} + E_{l-}] P''_{l-1}(x) \},$$

$$\mathcal{F}_4 = \sum_{l=2}^{\infty} [M_{l+} - M_{l-} - E_{l+} - E_{l-}] P''_l(x). \tag{15}$$

Here $E_{l\pm}(M_{l\pm})$ correspond to pions produced with angular momentum l by an electric (magnetic) photon of angular momentum $l \pm 1$ in a state of total angular momentum $l \pm 1/2$ and parity $-(-1)^l$.

It follows from the unitarity condition that below threshold for production of two pions we have

$$E_{l\pm} = |E_{l\pm}| \exp \{ i\delta_{l\pm} \} \tag{16}$$

and analogously for $M_{l\pm}$, where $E_{l\pm}$, $M_{l\pm}$ are amplitudes for transitions with well defined isotopic spin (10), and $\delta_{l\pm}$ are the pion-nucleon scattering phase shifts in the corresponding states.

In what follows we shall need the connection between the partial amplitudes and the invariant functions H_i:

$$M_{l\pm} = C \int_{-1}^{1} dx \frac{1}{2} A_{l\pm} \Big[\frac{q(W-m)}{2} \frac{(1-x^2) P'_l(x)}{l(l+1)} H_1 + \Big\{ \frac{W+m}{2W} (W + E_2 + qx) \Big[P_l(x) - \frac{q(W-m) P_{l\pm 1}(x)}{(E_2+m)(W+m)} \Big] + q \frac{(1-x^2) P'_l(x)}{l(l+1)} \Big\} H_2 + \Big\{ \frac{W+m}{2W} (\omega - qx) \times \Big[P_l(x) - \frac{q(W-m) P_{l\pm 1}(x)}{(E_2+m)(W+m)} \Big] - q \frac{(1-x^2) P'_l(x)}{l(l+1)} \Big\} H_3 + 2 \Big\{ P_l(x) + \frac{q P_{l\pm 1}(x)}{E_2 + m} \Big\} H_4 \Big],$$

$$E_{l\pm} = C \int_{-1}^{1} dx \frac{1}{2} B_{l\pm} \Big[\frac{q(1-x^2)}{2} \Big\{ \frac{q(W+m)}{E_2+m} D_{l\pm} P'_{l\pm 1}(x) - (W-m) A_{l\pm} P'_l(x) \Big\} H_1 + \Big\{ \frac{W+m}{2W} (W + E_2 + qx) \Big[P_l(x) - \frac{q(W-m) P_{l\pm 1}(x)}{(E_2+m)(W+m)} \Big] - q(1-x^2) \Big[A_{l\pm} P'_l(x) + D_{l\pm} \frac{q P'_{l\pm 1}(x)}{E_2+m} \Big] \Big\} H_2 + \Big\{ \frac{W+m}{2W} (\omega - qx) \Big[P_l(x) - \frac{q(W-m) P_{l\pm 1}(x)}{(E_2+m)(W+m)} \Big] + q(1-x^2) \Big[A_{l\pm} P'_l(x) + D_{l\pm} \frac{q P'_{l\pm 1}(x)}{E_2+m} \Big] \Big\} H_3 + 2 \Big\{ P_l(x) + \frac{q P_{l\pm 1}(x)}{E_2+m} \Big\} H_4 \Big], \tag{17}$$

where

$$A_{l\pm} = \begin{cases} (l+1)^{-1} \\ -l^{-1} \end{cases}, \quad B_{l\pm} = \begin{cases} (l+1)^{-1} \\ l^{-1} \end{cases}, \quad D_{l\pm} = \begin{cases} (l+2)^{-1} \\ (1-l)^{-1} \end{cases}.$$

The differential photoproduction cross section is given by

$$d\sigma/d\Omega = (q/k) |\mathcal{F}|^2. \tag{18}$$

B. <u>Reaction III.</u> For nucleon-antinucleon pair production processes (reaction III) the isotopic functions in Eq. (2) are given by

$$|i\rangle = \pi \bar{N}, \qquad |f\rangle = N, \tag{19}$$

where $\bar{N}$ is the antiproton or antineutron function:

$$\bar{p} = -\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \qquad \bar{n} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{20}$$

The amplitudes $F^{(1)}$ and $F^{(3)}$ correspond to isotopic vector transitions into states with total isotopic spin 0 and 1 respectively, $F^{(2)}$ corresponds to isotopic scalar transition into state with isotopic spin 1.

In the barycentric frame of reaction III we have

$$s = m^2 - 2\varkappa (E - py), \qquad \bar{s} = m^2 - 2\varkappa (E + py), \qquad t = (\varkappa + \omega_\varkappa)^2 = 4E^2. \tag{21}$$

Here κ and p are the magnitudes of the photon and nucleon momenta, $y = \cos \alpha$ is the cosine of the angle between these momenta, and E is the nucleon energy.

The spin structure of F in the barycentric frame is

*$[ke] = k \times e$; $\sigma k = \sigma \cdot k$.

$$F = \chi_N^*\left(i\frac{\mathbf{pe}}{p}\Phi_1 + \frac{[\mathbf{p}[\sigma\mathbf{p}]][\varkappa\mathbf{e}]}{p^2\varkappa}\Phi_2 + \frac{\sigma[\mathbf{pe}]}{p}\Phi_3 + \frac{\sigma[\varkappa\mathbf{e}]}{\varkappa}\Phi_4\right)\chi_{\bar{N}}. \quad (22)$$

The Φ_i are functions of E and y and are related to the H_i by:

$$\Phi_1 = -\frac{2p\varkappa}{m}(E^2H_1 + mH_2 + H_4),$$
$$\Phi_2 = \frac{2p^2\varkappa}{m}\left(H_2 + \frac{H_4}{E+m}\right),$$
$$\Phi_3 = (2p\varkappa E/m)\,H_3, \qquad \Phi_4 = -(2\varkappa E/m)\,H_4. \quad (23)$$

The amplitudes of Eq. (22) and the amplitudes $\Phi^{\nu}_{\lambda\bar{\lambda}}$ (the superscript specifies the helicity of the photon, the subscripts the helicities of the nucleon and antinucleon) for transitions between states with well defined helicities are related by:

$$\Phi^+_{\pm\pm} = \Phi^-_{\mp\mp} = \pm 2^{-1/2}(\Phi_1 \mp \Phi_4)\sin\alpha,$$
$$\Phi^+_{\pm\mp} = -\Phi^-_{\mp\pm} = 2^{-1/2}(\Phi_2 \pm \Phi_3 + \Phi_4)(1 \pm y). \quad (24)$$

According to Jacob and Wick the amplitudes (24) may be expanded as follows:

$$\Phi^+_{\pm\pm} = -\sum_J \sqrt{2}\Phi^J_{\pm\pm}\sin\alpha P'_J(y),$$
$$\Phi^+_{\pm\mp} = \sum_J\left[\frac{2}{J(J+1)}\right]^{1/2}\Phi^J_{\pm\mp}[(1\mp y)P'_J(y) \pm J(J+1)P_J(y)]. \quad (25)$$

Therefore

$$\Phi_1 = \sum_J(\Phi^J_{--} - \Phi^J_{++})P'_J,$$
$$\Phi_2 = \sum_J\{[J(J+1)]^{-1/2}[(\Phi^J_{-+} + \Phi^J_{+-})(P'_J + yP''_J) + (\Phi^J_{-+} - \Phi^J_{+-})P''_J - (\Phi^J_{++} + \Phi^J_{--})P'_J\},$$
$$\Phi_3 = -\sum_J[J(J+1)]^{-1/2}[(\Phi^J_{-+} + \Phi^J_{+-})P''_J + (\Phi^J_{-+} - \Phi^J_{+-})(P'_J + yP''_J)],$$
$$\Phi_4 = \sum_J(\Phi^J_{--} + \Phi^J_{++})P'_J. \quad (26)$$

From Eqs. (6) and (23) it follows that for $\alpha = 1, 2$ the amplitudes $\Phi^J_{\pm\pm}$ and $\Phi^J_{-+} + \Phi^J_{+-}$ are different from zero only for odd values of J, and for $\alpha = 3$ only for even values; whereas the amplitude $\Phi^J_{-+} - \Phi^J_{+-}$ does not vanish for $\alpha = 1, 2$ for even J, and for $\alpha = 3$ for odd J.

The relation between the amplitudes $\Phi^J_{\lambda\bar{\lambda}}$ and the amplitudes for transitions between states with well defined parity is

$$\Phi^J_{--} - \Phi^J_{+-} = \mathscr{E}_{J-}, \quad \Phi^J_{--} + \Phi^J_{++} = J\mathscr{M}_{J-1,+} - (J+1)\mathscr{M}_{J+1,-},$$
$$\Phi^J_{-+} - \Phi^J_{+-} = [J(J+1)]^{1/2}\mathscr{E}_{J+},$$
$$\Phi^J_{-+} + \Phi^J_{+-} = [J(J+1)]^{1/2}(\mathscr{M}_{J-1,+} + \mathscr{M}_{J+1,-}), \quad (27)$$

where $\mathscr{E}_{l\pm}$ ($\mathscr{M}_{l\pm}$) describe the production by an electric (magnetic) photon of a nucleon-antinucleon pair with orbital angular momentum l, spin $\frac{1}{2} \pm \frac{1}{2}(1)$, and total angular momentum l ($l \pm 1$). The expansion (26) becomes therefore

$$\Phi_1 = \sum_l \mathscr{E}_{l-}P'_l, \quad \Phi_2 = \sum_l[\mathscr{M}_{l+} + \mathscr{M}_{l-} + \mathscr{E}_{l+}]P''_l,$$
$$\Phi_3 = -\sum_l[\mathscr{M}_{l+}P''_{l+1} + \mathscr{M}_{l-}P''_{l-1} + \mathscr{E}_{l+}(P'_l + yP''_l)],$$
$$\Phi_4 = \sum_l[(l+1)\mathscr{M}_{l+}P'_{l+1} - l\mathscr{M}_{l-}P'_{l-1}]. \quad (28)$$

For $\alpha = 1, 2$ the amplitudes $\mathscr{M}_{l\pm}$, $\mathscr{E}_{l+}$ have l even and $\mathscr{E}_{l-}$ has l odd; whereas for $\alpha = 3$ $\mathscr{E}_{l-}$ has l even and $\mathscr{M}_{l\pm}$, $\mathscr{E}_{l+}$ have l odd.

Let us consider the unitarity condition for reaction III with only the two-pion intermediate state taken into account. For the isotopic scalar amplitudes with odd total angular momentum J we find

$$\mathrm{Im}\,\Phi^{(2)J}_{\lambda\bar{\lambda}} = (q_1\varkappa/16\pi)\,T^{(-)\,J}_{\lambda\bar{\lambda}}f_J^*, \quad (29)$$

where $q_1 = (E^2 - \mu^2)^{1/2}$ is the pion momentum in the intermediate state,

$$T^{(-)J}_{\pm\pm} = T^{(-)J}_{+}, \qquad T^{(-)J}_{\pm\mp} = T^{(-)J}_{-} \quad (30)$$

represent the amplitudes for the process $\pi\pi \to N\bar{N}$, determined by Eqs. (3.13) – (3.17) of Frazer and Fulco,[15] and f_J is the amplitude for the process $\gamma\pi \to \pi\pi$, determined by Eq. (6) of Solov'ev, who gives an explicit formula for f_1.[13]

The two-pion contribution vanishes for the isotopic vector amplitudes $\Phi^{(1,3)}$. It follows from Eqs. (27), (29), and (30) that the two-pion intermediate state contributes only to the isotopic scalar magnetic transitions of reaction III:

$$\mathrm{Im}\,(\Phi^{(2)J}_{\substack{--\\-+}} + \Phi^{(2)J}_{\substack{++\\+-}}) = \frac{q_1\varkappa}{8\pi}T^{(-)J}_{\pm}f_J^*. \quad (31)$$

3. SPECTRAL REPRESENTATIONS AND PHOTOPRODUCTION EQUATIONS

As remarked in Sec. 1 we shall assume that spectral representations without subtractions are valid for the invariant photoproduction amplitudes. The Mandelstam representation[11] takes the form:

$$H_i^{(\alpha)}(s, \bar{s}, t) = \left(B_i^{(\alpha)} - \frac{2eg\delta_{i1}}{t-\mu^2}\right)\left(\frac{1}{s-m^2} \pm \frac{1}{\bar{s}-m^2}\right) + \frac{1}{\pi^2}\int_{(m+\mu)^2}^{\infty}dx\int_{4\mu^2}^{\infty}dy\left(\frac{1}{x-s} \pm \frac{1}{x-\bar{s}}\right)\frac{h_{i1}^{(\alpha)}(x, y)}{y-t} + \frac{1}{\pi^2}\iint_{(m+\mu)^2}^{\infty}dx\,dy\,\frac{h_{i2}^{(\alpha)}(x, y)}{(x-s)(y-\bar{s})}, \quad (32$$

$$h_{i2}^{(\alpha)}(x, y) = \pm h_{i2}^{(\alpha)}(y, x). \quad (33)$$

The signs in Eqs. (32) and (33) correspond to the signs in Eq. (6):

$$B_1^{(\alpha)} = 0, \quad B_2^{(\alpha)} = B_3^{(\alpha)} = \frac{g}{2}\begin{cases} \mu_p' - \mu_n, & \alpha = 1 \\ \mu_p' + \mu_n, & \alpha = 2, \\ \mu_p' - \mu_n, & \alpha = 3 \end{cases}$$

$$B_4^{(\alpha)} = -\tfrac{1}{4} eg - mB_2^{(\alpha)} \tag{34}$$

(rational units). In contrast to pion-nucleon scattering in photoproduction one obtains a contribution from the single pion state, related by gauge invariance to the single nucleon contribution.

We restrict ourselves to low energies and consider the approximate representation of Cini and Fubini.[2,3] If the amplitudes F, H, etc., for photoproduction of pions on pions are ignored then the representation takes the form

$$H_i^{(\alpha)}(s, \bar{s}, t) = \text{Born term}$$
$$+ \frac{1}{\pi}\int_{(m+\mu)^2}^{\infty} dx \left(\frac{1}{x-s} \pm \frac{1}{x-\bar{s}}\right) a_i^{(\alpha)}(x, t)$$
$$+ \delta_{\alpha 2}\frac{1 \pm 1}{2\pi}\int_{4\mu^2}^{\infty} \frac{b_i(x)\, dx}{x - t}, \tag{35}$$

where

$$a_i^{(\alpha)} = \operatorname{Im} H_i^{(\alpha)} \text{ for reaction I}, \quad \delta_{\alpha 2} b_i = \operatorname{Im} H_i^{(\alpha)} \text{ for reaction III}. \tag{36}$$

The $a_i^{(\alpha)}(s,t)$ correspond to only the pion-nucleon intermediate state in the unitarity condition. Consequently their singularities in t do not start until $16\mu^2$ and they may therefore be expanded in a Taylor series. If the D, F, etc., pion-nucleon scattering phase shifts are ignored then this expansion takes the form

$$a_i^{(\alpha)}(s, t) = a_{i0}^{(\alpha)}(s) + (t - t_0)\, a_{i1}^{(\alpha)}(s). \tag{37}$$

To make sure that the functions $a_i^{(\alpha)}(s,t)$ do not contain unobservable angles it is necessary to take for t_0 the threshold value of t:

$$t_0 = \mu^2 - 2\mu k_{th}, \qquad k_{th} = \mu(2m + \mu)/2(m + \mu). \tag{38}$$

With the help of Eqs. (36), (11), (13) and (15) the functions $a_{i0}^{(\alpha)}$ and $a_{i1}^{(\alpha)}$ are easily expressed in terms of the imaginary parts of the partial photoproduction amplitudes. If we then substitute Eq. (35) into Eq. (17) we obtain the integral equation for photoproduction.

A. Isotopic vector amplitudes. The last integral in Eq. (35) is absent for the isotopic vector amplitudes ($\alpha = 1, 3$), and we obtain relations previously discussed[9-11] by expanding in powers of ω/m and ω'/m and retaining only the leading terms. The nucleon recoil must be taken into account more accurately for the resonant magnetic dipole amplitude, and this is easy to do with the help of Eqs. (17) and (35).

B. Isotopic scalar amplitudes. For the isotopic scalar amplitudes ($\alpha = 2$) reaction III contributes to Eq. (35). From Eqs. (36), (23), (26) and (31) we obtain the following expressions for $b_i(t)$:

$$b_1(t) = \frac{mq_1}{16\pi E p^2}\left[T_+^{(-)1} - \frac{m}{\sqrt{2}E} T_-^{(-)1}\right] f_1^*,$$
$$b_2(t) = \frac{mq_1}{16\pi p^2}\left[-\frac{m}{E} T_+^{(-)1} + \frac{1}{\sqrt{2}} T_-^{(-)1}\right] f_1^*,$$
$$b_3(t) = 0, \quad b_4(t) = -(mq_1/16\pi E)\, T_+^{(-)1} f_1^*, \tag{39}$$

where, as noted above, the amplitudes $T_\pm^{(-)1}$ have been discussed by Frazer and Fulco[15] and f_1 by Solov'ev.[13]

For the amplitude for photoproduction of pions we have[13]

$$f_1(t) = \Lambda e^{i\delta(t)}\varphi(t), \tag{40}$$

where $\delta \equiv \delta_1^1$ is the pion-pion scattering phase shift in the J = I = 1 state, and $\varphi(t)$ is a known integral containing this phase shift [Eqs. (36), (38) of Solov'ev[13]]. The parameter Λ is given by[13]

$$\Lambda = \frac{2}{\pi}\int_{4m^2}^{\infty}\frac{ds'}{s'}\left(\operatorname{Im}\mathcal{F}(s', \cos\theta = 1)\right)_{N\bar{N}}, \tag{41}$$

where the function appearing under the integral sign is related to the amplitudes for the processes $\gamma\pi \rightarrow N\bar{N}$ and $\pi\pi \rightarrow N\bar{N}$ as follows:

$$(\operatorname{Im}\mathcal{F}(s', \cos\theta))_{N\bar{N}} = \sum_{J\text{ odd}} (\operatorname{Im} f_J(s'))_{N\bar{N}} P_J'(\cos\theta), \tag{42}$$

$$(\operatorname{Im} f_J)_{N\bar{N}} = \frac{m^2}{\pi s'^2}\frac{p'}{q_1'}$$
$$\times\left[T_+^{(-)J*}\left(\Phi_{++}^{(2)J} + \Phi_{--}^{(2)J}\right) + T_-^{(-)J*}\left(\Phi_{-+}^{(2)J} + \Phi_{+-}^{(2)J}\right)\right];$$
$$s'^2 = 2(p'^2 + m^2) = 2(q_1'^2 + \mu^2). \tag{43}$$

The amplitudes that enter here correspond to high energies and cannot be calculated at present. Therefore the parameter Λ must be taken from experiment.

If it is assumed that the phase shift δ_1^1 has a resonance then the function φ in Eq. (40) also has a resonant character.[13] If one also ignores the nonresonant pion-nucleon scattering phase shifts, then one obtains from Eq. (35) the following expressions for the isotopic scalar photoproduction amplitudes:

$$H_i^{(2)}(s, \bar{s}, t) = \left(B_i^{(2)} - \frac{2eg\delta_{i1}}{t - \mu^2}\right)\left(\frac{1}{s - m^2} \pm \frac{1}{\bar{s} - m^2}\right)$$
$$+ \frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{b_i(t')\, dt'}{t' - t}, \tag{44}$$

where the b_i are given by Eq. (39). The π^+ to π^- photoproduction cross sections ratio at threshold should depend sensitively on the magnitude of expression (44).

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