DISPERSION RELATIONS FOR VERTEX PARTS

Yu. M. MALYUTA

Physics Institute, Academy of Sciences, Ukrainian S.S.R.

Submitted to JETP editor October 26, 1960

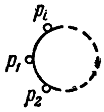
J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 1128-1133 (April, 1960)

The primitive diagrams are found and the location of the nearest singularities for the $\Lambda\Lambda\pi$, $\Lambda\Sigma\pi$ and $\Sigma\Sigma\pi$ vertex parts are determined by making use of the Nambu-Symanzik majorization method. Our investigation differs from Nambu's study of the hyperon form factor[1] in that we do not restrict ourselves to the consideration of a simplified model but study the situation that arises when all strongly interacting particles are allowed to interact.

1. INTRODUCTION

RECENTLY in the theory of dispersion relations the method of majorization[1,2] for the determination of the location of the nearest singularities of scattering amplitudes was developed. The idea of the method consists of the following.

Consider a connected diagram of arbitrary order*

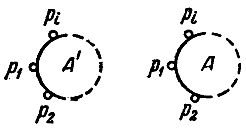


(1)

where p_i is the sum of the momenta acting on the i-th external vertex. The matrix element corresponding to diagram (1) is a function of the invariants constructed from the vectors p_i. We are interested in the analytic properties of the matrix element as a function of one of the invariants with the remaining ones held constant.

Let us formulate the basic theorem of the majorization method.†[1,2]

Let A′ and A be two connected diagrams with equal numbers of external vertices:



(2)

Let diagram A′ reduce to diagram A according to the rules of majorization given in Fig. 1,‡ and

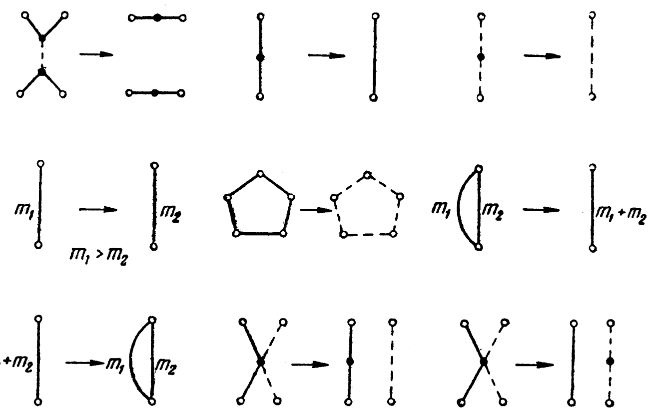


Fig. 1

in accordance with selection rules (see below). Then the nearest singular point of diagram A will lie no further than the nearest singular point of diagram A′, provided that the set of vectors p_i is Euclidean.*

Let us list the selection rules.

1) In the majorization process the baryon number and strangeness must be conserved;†

2) a strongly connected diagram should be majorized by a strongly connected diagram;‡

3) internal pion lines attached to external vertices should not be removed if as a result the structure of the diagram is broken;

4) it is desirable to keep cubic type vertices (this simplifies the investigation);

*Without loss of generality we discuss the scalar version of the theory. Divergent diagrams are regularized by the auxiliary masses method.[3]

†The formulation is given in graphical language, in a form convenient for subsequent applications.

‡In Fig. 1 are shown the majorization rules for a pion-nucleon theory. These rules may be expanded by replacing the nucleon line by a kaon or hyperon line, or by a line denoting an arbitrary nucleus or hyperfragment (for the notation for the lines see Sec. 2)

*The set of vectors p_i is Euclidean if the Gram determinants composed of these vectors are nonnegative.

†To simplify the discussion we do not take into account other conservation laws, connected with inversions and gauge invariance.

‡The majorization of a strongly connected diagram by a weakly connected diagram is forbidden because this would result in a loss of information about the analyticity properties of the scattering amplitude.

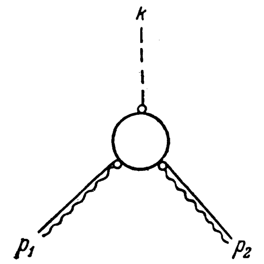


Fig. 2

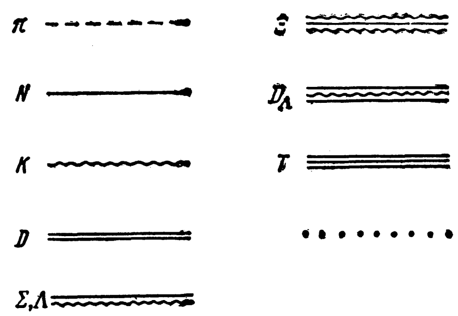


Fig. 3

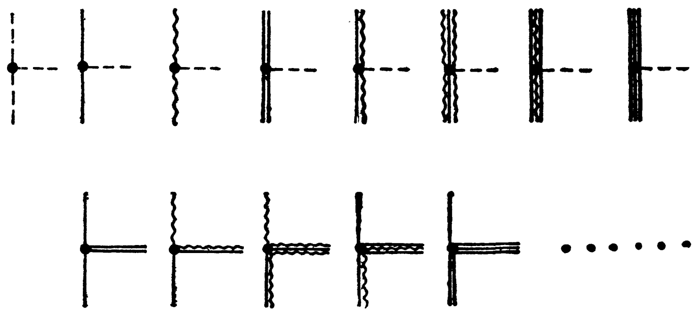


Fig. 4

5) stability conditions should not be violated.[4,5]

With the help of the majorization rules depicted in Fig. 1 and the selection rules any arbitrarily complicated diagram may be reduced to simplest form. The final diagrams to which the scattering amplitude is reduced are called primitive diagrams. In order to determine the position of the nearest singularities of the scattering amplitude it is sufficient to determine the position of the nearest singularities of the primitive diagrams.

In this work concrete applications of the majorization of the vertex part shown in Fig. 2 are presented.

2. MAJORIZATION OF THE VERTEX PART

We introduce graphic notation for the strongly interacting particles (see Fig. 3). Other nuclei and hypernuclei may be analogously denoted by an appropriate number of straight and wavy lines, representing the baryon number and strangeness.

Strong interactions of the cubic type are shown in Fig. 4.[6]*

We consider the vertex part shown in Fig. 2 and find its primitive diagrams. The vertex part depends on the following invariants:†

$$p_1^2 = \lambda^2, \qquad p_2^2 = \sigma^2, \qquad k^2 = (p_1 + p_2)^2,$$

where λ and σ are hyperon masses ($\lambda \le \sigma$). We are interested in the analytic properties as a function of k^2. It is easy to verify that the set of vectors p_1, p_2 and k is Euclidean provided that k^2 lies in the interval

$$(\lambda - \sigma)^2 \leqslant k^2 \leqslant (\lambda + \sigma)^2. \tag{3}$$

Therefore if the singular points of the primitive diagrams fall within the interval (3), then the nearest of these will be the nearest singular point of the vertex part.

Let us enumerate the elements of which the vertex part is composed:

a) the baryon line, joining the external hyperon lines;

b) the strangeness line, joining the external hyperon lines;

c) baryon closed loops and strangeness closed loops;

d) lines a), b), and c) may coincide with each other forming various virtual particles;

e) pion lines act as connecting links between all lines, including pion lines;

f) an external pion line may be attached to any line.

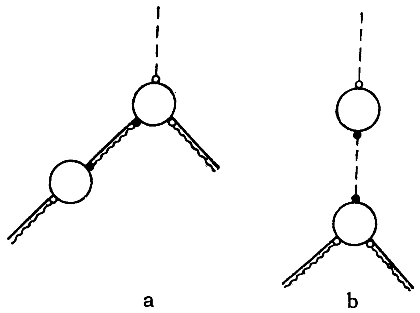
We first replace the closed loops c) by pion closed loops in accordance with Fig. 1 and the selection rules,* and lower the masses of the virtual Σ hyperons to the masses of the Λ hyperons (in accordance with Fig. 1). Following this operation the vertex part will contain only the elements a), b), e), f) and coincidences only between a) and b).

We extract from the vertex part under discussion the weakly connected diagrams shown in Fig. 5.

The diagram of Fig. 5a need not be considered since it contains a self-energy part on the free hyperon line and, consequently, the corresponding matrix element vanishes. The diagram of Fig. 5b reduces to a strongly connected diagram since the self-energy part on the virtual pion line may be included in the external source.

*To simplify the discussion we assume the pion to be scalar.
†Our metric is 1, −1, −1, −1.

*It is possible in this way to obtain higher degree vertices involving pion lines. However they can be reduced to the cubic type by the removal of the relevant pion lines.



a b

Fig. 5

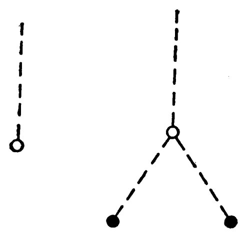
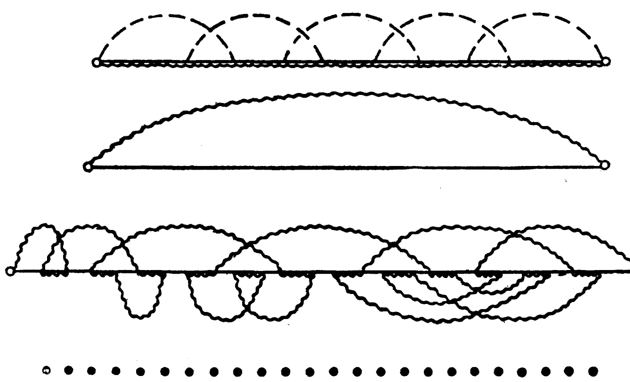


Fig. 6



a)

b)

c)

d)

Fig. 7

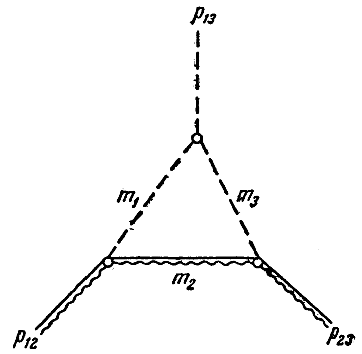


Fig. 8

The strongly connected diagrams may be majorized (in accordance with Fig. 1 and the selection rules) by diagrams obtained by attaching in all possible ways the elements depicted in Fig. 6 to the chains shown in Fig. 7 (the dots in Fig. 7 stand for all possible strongly connected combinations and variations of the chains indicated above). The diagrams resulting from the chains of Fig. 7b, c, d may in turn be majorized (in accordance with Fig. 1 and the selection rules) by the diagrams resulting from Fig. 7a.* We finally obtain the single primitive diagram shown in Fig. 8 by majorizing the diagrams resulting from the chain in Fig. 7a (in accordance with Fig. 1 and the selection rules), and making use of the method of induction.

3. DETERMINATION OF THE POSITION OF THE SINGULARITIES

Taking into account the results obtained by Karplus, Sommerfield and Wichmann,[4] and by Tarski,[5] we determine the position of the nearest lying singularities for the vertex parts $\Lambda\Lambda\pi$, $\Lambda\Sigma\pi$, and $\Sigma\Sigma\pi$.

The matrix element corresponding to the primitive diagram of Fig. 8 has the form

$$G = \text{const}\cdot\int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \int_0^1 d\alpha_3 \frac{\delta(1-\alpha_1-\alpha_2-\alpha_3)}{(\alpha_1 m_2 m_3+\alpha_2 m_1 m_3+\alpha_3 m_1 m_2)\, V(p\alpha)},$$

where

$$V(p\alpha) = \alpha_1^2+\alpha_2^2+\alpha_3^2+2\alpha_1\alpha_2 y_{12}+2\alpha_1\alpha_3 y_{13}+2\alpha_2\alpha_3 y_{23},$$

$$p_{ij}^2 = m_i^2+m_j^2-2m_i m_j y_{ij}, \qquad y_{ij} = \cos\theta_{ij}, \qquad 0 \leqslant \theta_{ij} \leqslant \pi.$$

It follows from the work of the above mentioned authors[4,5] that the position of the nearest singularity for $G(k^2)$ is determined as

$$\bar{k}^2 = \begin{cases} (m_1+m_3)^2, & \text{if } \theta_{12}+\theta_{23} \leqslant \pi \\ m_1^2+m_3^2-2m_1 m_3\cos(\theta_{12}+\theta_{23}), & \text{if } \theta_{12}+\theta_{23} > \pi. \end{cases}$$

*The majorization of these diagrams is based on the circumstance that $m+\kappa > \lambda + 2\mu$, where m, κ λ, μ are the nucleon, kaon, hyperon and pion masses respectively.

Let us find the position of the nearest singularity for the vertex part $\Lambda\Lambda\pi$:

$$\theta_{12} = \theta_{23} = 86.5^\circ, \qquad \theta_{12}+\theta_{23} < \pi, \qquad \bar{k}^2 = 4\mu^2;$$

for the vertex part $\Lambda\Sigma\pi$:

$$\theta_{12} = 86.5^\circ, \qquad \theta_{23} = 120^\circ, \ \theta_{12}+\theta_{23} > \pi, \qquad \bar{k}^2 = 3.78\,\mu^2;$$

and for the vertex part $\Sigma\Sigma\pi$:

$$\theta_{12} = \theta_{23} = 120^\circ, \qquad \theta_{12}+\theta_{23} > \pi, \qquad \bar{k}^2 = 3\mu^2.$$

The author is grateful to N. N. Bogolyubov and A. A. Logunov for valuable advice and remarks.

[1] Y. Nambu, Nuovo cimento **9**, 610 (1958).

[2] K. Symanzik, Progr. Theoret. Phys. **20**, 690 (1958).

[3] N. N. Bogolyubov and D. V. Shirkov, Introduction to Quantum Field Theory, Interscience, 1959.

[4] Karplus, Sommerfield, and Wichmann, Phys. Rev. **114**, 376 (1959).

[5] J. Tarski, J. Math. Phys. **1**, 149 (1960).

[6] P. Matthews, Relativistic Quantum Theory of the Interactions of Elementary Particles (Russ. Transl.), IIL, 1959.

Translated by A. M. Bincer
189