SOME ISOTOPIC RELATIONS FOR REACTIONS OF THE TYPE $\pi N \to \pi\pi N$

V. N. STREL'TSOV

Joint Institute for Nuclear Research

Submitted to JETP editor October 29, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 1140-1142 (April, 1961)

Isotopic relations are utilized in an analysis of experimental data for reactions of the type $\pi N \to \pi\pi N$ from the point of view of resonant $\pi\pi$ interactions.

THE model of resonant $\pi\pi$ interactions in states with definite isotopic spin ($T_{\pi\pi}$) values for the $\pi\pi$ system[1-7] has recently attracted attention as a means of explaining various experimental results. In particular, these considerations have been used to explain the mechanism of reactions of the type $\pi N \to \pi\pi N$.[4-7] The presence in such reactions of resonant $\pi\pi$ interactions with definite $T_{\pi\pi}$ can be examined with the aid of the isotopic relations which apply in this case (see, for example, references 1 and 2).

We shall analyze the experimental data using the somewhat more general relations, which are satisfied when the $\pi\pi$ interaction in the state with some value of the isotopic spin ($T'_{\pi\pi}$) is much smaller than the interaction in another state (the two other states), i.e.,

$$|a(T'_{\pi\pi})| \ll a|(T''_{\pi\pi})| \qquad (|a(T'_{\pi\pi})| \ll |a(T''_{\pi\pi})|,\ |a(T'''_{\pi\pi})|).$$

Then we have the following relations among the total and differential cross sections:*

I. $|a(0)| \ll a|(2)|$

$$\sigma(\pi^-n \to \pi^-\pi^-p) = 9\sigma(\pi^-p \to \pi^0\pi^0n);$$

II. $|a(1)| \ll a|(2)|$

$$\sigma(\pi^-n \to \pi^-\pi^0n) = \tfrac{1}{4}\sigma(\pi^-n \to \pi^-\pi^-p) = \sigma(\pi^-p \to \pi^-\pi^0p),$$

$$d\sigma(\pi^-p \to \pi^-\pi^0p) = d\sigma(\pi^-p \to \pi^0\pi^-p),$$

$$d\sigma(\pi^-n \to \pi^-\pi^0n) = d\sigma(\pi^-n \to \pi^0\pi^-n);$$

IIa. $|a(1)| \ll |a(0)|,\ |(a(2)| \qquad (|a(1)| \gg |a(0)|,\ |a(2)|)$

$$d\sigma(\pi^-p \to \pi^-\pi^+n) = d\sigma(\pi^-p \to \pi^+\pi^-n);$$

III. $|a(2)| \ll |a(1)|$

$$\sigma(\pi^-n \to \pi^-\pi^-p) = 0,\quad d\sigma(\pi^-p \to \pi^-\pi^0p) = d\sigma(\pi^-p \to \pi^0\pi^-p)$$

$$d\sigma(\pi^-n \to \pi^-\pi^0n) = d\sigma(\pi^-n \to \pi^0\pi^-n).$$

At present, a more or less definite examination of relations I—III for the total cross section is possible only at an energy of about 1.5 Bev. The cross section ratios

$$\beta = \frac{\sigma(\pi^-n \to \pi^-\pi^-p)}{\sigma(\pi^-p \to \pi^0\pi^0n)}, \qquad \gamma = \frac{\sigma(\pi^-n \to \pi^-\pi^-p)}{\sigma(\pi^-p \to \pi^-\pi^0p)},$$

$$\delta = \frac{\sigma(\pi^-n \to \pi^-\pi^0n)}{\sigma(\pi^-p \to \pi^-\pi^0p)}$$

computed from the experimental data[8-10] for cases I and II are given in the table. From the table it is clear that $\beta \ll 9$, i.e., the condition $|a(0)| \ll |a(2)|$ is not fulfilled at 1.5 Bev. The values obtained for γ and δ also differ from those possible in case II.

Energy E_{lab}, Bev	I	II	
	β	γ	δ
1.37 [8]		0.7±0.6	3.1±1.1
1.5 [9,10]	$0.41\pm0.12 \leqslant \beta^* \leqslant 2.0\pm0.8$	0.36±0.09	2.1±0.3

*The value of β is obtained from only the data of Walker et al.[10] The limits given for β correspond to two different assumptions on the magnitude of the cross section $\sigma_1(\pi^-p \to \pi^0n)$ [namely, $\sigma_1 = 0$ and $\sigma_1 = \sigma(\pi^-p \to \pi^-p)$], since there are experimental results only for the sum of the cross sections σ_1 and $\sigma(\pi^-p \to \pi^0\pi^0n)$.

Condition II, moreover, is also in disagreement with the different shapes of the angular distributions of π^0 and π^- mesons in the $\pi^-p \to \pi^-\pi^0p$ reaction. It follows, therefore, that $|a(1)|$ cannot be small. The condition that the $\pi^-n \to \pi^-\pi^-p$ reaction cross section be small (III) is apparently not fulfilled by any of the data on the $\pi^-n\,(\pi^+p)$ interaction at 0.5, 1.37, and 1.5 Bev.[8-11]* Thus, the data under consideration give no indication that any of the quantities $|a(T_{\pi\pi})|$ is small.

By using only the conditions on the differential cross sections, we can analyze the data on the π^-p interaction separately in a similar manner. The results obtained by Alles-Borelli et al.[13] at 960 Mev show that the energy distributions of π^+ and

*The equality of differential cross sections, for example $d\sigma(\pi^-p \to \pi^-\pi^0p) = d\sigma(\pi^-p \to \pi^0\pi^-p)$, indicates clearly that the angular and energy distributions of the π^0 and π^- mesons are identical in the $\pi^-p \to \pi^-\pi^0p$ reaction.

*It should be noted that, at 0.75 Bev,[12] only one case of $\pi^+p \to \pi^+\pi^+n$ has been observed, compared with eight $\pi^-p \to \pi^-\pi^+n$ cases. However, the statistics are clearly not good enough to allow a definite conclusion.

π^- mesons in the $\pi^- p \rightarrow \pi^-\pi^+ n$ interaction cannot be resonant in the states with $T_{\pi\pi} = 0$ or 2. To see whether the interaction in the state with $T_{\pi\pi} = 1$ is strong, it would be necessary to measure the cross section for the $\pi^- n \rightarrow \pi^-\pi^- p$ reaction, which would be zero if such were the case. However, it turns out that along with this, the equality

$$d\sigma(\pi^- p \rightarrow \pi^-\pi^+ n) = d\sigma(\pi^- p \rightarrow \pi^+\pi^- n)$$

again must hold, and, since it does not hold, the $\pi\pi$ interaction in the $T_{\pi\pi} = 1$ state can likewise not be resonant. Similar conclusions follow also from other experiments performed at energies around 1 Bev[14] and 1.85 Bev.[15]

Thus, the analysis of experimental data in reactions of the type $\pi N \rightarrow \pi\pi N$ at energies 0.96 — 1.85 Bev gives no indication whatsoever of the presence of resonant $\pi\pi$ interaction in a state with definite $T_{\pi\pi}$.* Moreover, $|a(2)|$ is not small even at 500 Mev.†

This approach allows us to obtain some information on the nature of the interaction of the system of two π mesons in $\pi N \rightarrow \pi\pi N$ reactions near the meson production threshold. Near threshold the π mesons are in a relative S state and therefore the isotopic spin $T_{\pi\pi}$ cannot be 1. It is interesting to see where P-waves begin to be important, i.e., at what energy $|a(1)|$ is no longer small. The difference in the angular and energy distributions of π^+ and π^- mesons in the $\pi^- p \rightarrow \pi^-\pi^+ n$ reaction at 290 Mev[16] apparently indicates the presence of P-waves in the $\pi\pi$ system. If the $\pi\pi$ system at threshold is in a mixture of only $T = 0$ and $T = 2$ states, then an admixture of P states will show itself as a difference in the angular and energy distributions of π^+ and π^- mesons in this reaction.

In conclusion, we give some isotopic relations which may be useful in future analyses of experimental data. They are for the case that the interaction in the πN subsystem in the final state occurs predominantly in a state with definite isotopic spin, for example, in the $T_{\pi N} = 3/2$ ("isobar") state, and in addition the inequality $|a(T'_{\pi\pi})| \ll |a(T''_{\pi\pi})|$ holds for the $\pi\pi$ system. For $|a(2)| = 0$,* we obtain the following equations for the total cross sections:

$$\sigma(\pi^- n \rightarrow \pi^-\pi^0 n) = \sigma(\pi^- n \rightarrow \pi^-\pi^- p) = 0,$$

$$\sigma(\pi^- p \rightarrow \pi^-\pi^0 p) = \sigma(\pi^- p \rightarrow \pi^0\pi^0 n) = \tfrac{2}{5}\,\sigma(\pi^- p \rightarrow \pi^-\pi^+ n).$$

For $|a(0)| = 0$ these relations are identical with those which hold in the case $T_{\pi N} = T = 3/2$, where T is the isotopic spin of the initial state.

The author thanks L. I. Lapidus, V. I. Ogievetskii, and M. I. Podgoretskii for appraising these results and for valuable comments, and also S. A. Bunyatov and A. V. Efremov for aid in this work.

*It should be noted that if the $\pi N \rightarrow \pi\pi N$ reaction goes only through the nucleon "isobar" ($J = T = 3/2$), then a resonant $\pi\pi$ interaction in a state with definite $T_{\pi\pi}$ is obviously impossible. Therefore, if the "isobar" model is valid in the energy range 1–1.5 Bev (as is apparently indicated by the results in references 14 and 17) such a resonant $\pi\pi$ interaction is impossible.

†Of course, these conclusions do not exclude the possibility that the $\pi\pi$ interaction is resonant for certain values of the relative momentum of the π mesons.

*The case $|a(1)| = 0$ is impossible; that is, if the πN interaction is strong in a state with definite $T_{\pi N}$, then the $\pi\pi$ interaction in the $T = 1$ state cannot be small.

[1] F. J. Dyson, Phys. Rev. **99**, 1037 (1955).

[2] G. Takeda, Phys. Rev. **100**, 440 (1955).

[3] W. R. Fraxer and I. R. Fulco, Phys. Rev. Lett. **2**, 365 (1959); Phys. Rev. **117**, 1609 (1960).

[4] F. Bonsignori and F. Sellery, Nuovo cimento **15**, 465 (1960).

[5] F. Sellery, Nuovo cimento **16**, 1775 (1960).

[6] P. Carruthers and H. A. Bethe, Phys. Rev. Lett. **4**, 536 (1960).

[7] Itabashi, Kato, Nakagawa, and Takeda, preprint.

[8] V. P. Kenney, Phys. Rev. **104**, 784 (1956).

[9] W. D. Walker and J. Crussard, Phys. Rev. **98**, 1416 (1955).

[10] Walker, Crussard, and Koshiba, Phys. Rev. **95**, 852 (1954).

[11] W. J. Willis, Phys. Rev. **116**, 753 (1959); Blevis, Block, and Leitner, Phys. Rev. **112**, 1287 (1958).

[12] M. Blau and A. R. Oliver, Phys. Rev. **102**, 489 (1956).

[13] Alles-Borelli, Bergia, Perez Ferreira, and Waloschek, Nuovo cimento **14**, 211 (1959).

[14] Derado, Lütjens, and Schmitz, Ann. Physik **4**, 103 (1959); I. Derado and N. Schmitz, Phys. Rev. **118**, 309 (1960).

[15] R. C. Whitten and M. M. Block, Phys. Rev. **111**, 1676 (1958).

[16] Batusov, Bogachev, and Sidorov in B. M. Pontecorvo's report to the Ninth High-Energy Physics Conference, Kiev, 1959.

[17] S. I. Lindenbaum and R. M. Sternheimer, Phys. Rev. **106**, 1107 (1957).

Translated by M. Bolsterli
191