# *EQUATIONS FOR THE SPECTRAL FUNCTIONS OF CHARGED PIONS*

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The amplitudes for the scattering of charged pions by charged pions are expressed by means of the Mandelstam representation in terms of the spectral functions. A set of equations is derived for the one-dimensional and two-dimensional spectral functions. One of the methods for obtaining an approximate solution is discussed.

## 1. INTRODUCTION

ONE of the authors (Ter-Martirosyan) has recently proposed a method for constructing a symmetric system of equations for the spectral functions for the two-particle transition amplitude. The method was based on Mandelstam's equations[1] for the two-dimensional spectral functions. The equations were given both in general form,[2] and for a number of simple cases[3] involving the interactions of neutral particles. The resultant system of equations involves only experimentally observable quantities (i.e., in the language of old theory, renormalized quantities) and the parameters that appear are the particle masses and the usual coupling constants (for example the constant g for the meson-nucleon interaction, the constant $\lambda$ for the pion-pion interaction). The theory requires no other parameters and, furthermore, no divergences appear in this formulation.

When the two-particle transition amplitude (in all three channels) is expanded in Legendre polynomials the system of equations for the spectral functions is transformed into an infinite system of coupled equations[4] for partial amplitudes, of the same type as those considered by Chew and Mandelstam[5] and Cini and Fubini.[6] However, when this is done divergences appear (as noted by Efremov et al.[7]), whose degree increases with increasing number of partial waves taken into account in the equations. These divergences are a simple consequence of an illegitimate use of the Legendre expansion in a region (substantial in the equations) in which it fails to converge. In the original equations for the two-dimensional spectral functions these divergences do not appear. Therefore in the solution of our equations one should make use of some method other than the expansion in Legendre polynomials, for example the method of partial iteration of the equations for the spectral functions $A_{ij}(s)$ described previously,[2,3] provided of course that convergent results are obtained.

In this note we obtain equations for the one- and two-dimensional spectral functions for the interaction of charged pions with each other. We use throughout the same notation as was introduced previously (see Mandelstam[1] and Ter-Martirosyan[2]).

## 2. DERIVATION OF THE EQUATIONS

The amplitude describing the interaction of charged pions may be expressed as follows

$$A_{\alpha\beta\gamma\delta}(s_1, s_2, s_3) = \lambda(s_1, s_2; s_3)\delta_{\alpha\gamma}\delta_{\beta\delta} + \lambda(s_3, s_2; s_1)\delta_{\alpha\beta}\delta_{\gamma\delta} + \lambda(s_1, s_3; s_2)\delta_{\alpha\delta}\delta_{\beta\gamma}, \qquad (1)$$

where $\lambda(s_1, s_2; s_3) = \lambda(s_2, s_1; s_3)$ is a function of the invariants

$$s_1 = (p_1 + p_2)^2, \quad s_2 = (p_1 + p_4)^2, \quad s_3 = (p_1 + p_3)^2, \quad s_1 + s_2 + s_3 = 4\mu^2,$$

and $(p_1\alpha)$, $(p_2\beta)$, $(p_3\gamma)$ and $(p_4\delta)$ are the momenta and isospin indices of the four pions. (Fig. 1).

The Mandelstam representation for the function $\lambda$, when written with one subtraction, has the form [see Eq. (4) of Ter-Martirosyan[3]]

$$\lambda(s_1, s_2; s_3) = \lambda_0 + \frac{1}{\pi}\int_0^\infty \{\alpha(\sigma)[\varphi(\sigma, s_1) + \varphi(\sigma, s_2)] + \beta(\sigma)\varphi(\sigma, s_3)\}d\sigma + \frac{1}{\pi^2}\iint_0^\infty \{\rho_c(\sigma, \sigma')\varphi(\sigma, s_1)\varphi(\sigma', s_2) + \rho(\sigma, \sigma')[\varphi(\sigma', s_1) + \varphi(\sigma', s_2)]\varphi(\sigma, s_3)\}d\sigma\, d\sigma', \qquad (2)$$

where

$$\varphi(\sigma, s) = \frac{1}{\sigma - s} - \frac{1}{\sigma - s_0}, \quad s_0 = \frac{4\mu^2}{3}, \quad \lambda_0 = \lambda(s_0, s_0, s_0),$$

$$\rho_c(\sigma, \sigma') = \rho_c(\sigma', \sigma).$$

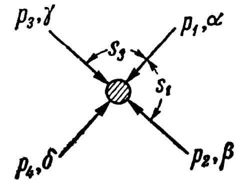


FIG. 1

The equations for the spectral functions $\alpha(\sigma)$, $\beta(\sigma)$ and $\rho_c$, $\rho$ follow (see Ter-Martirosyan[2]) directly from Eq. (2) and the unitarity conditions, when the latter are written out for the invariant function $\lambda$. This can be done by, for example, making use of the relation determining the $\pi\pi$ scattering amplitude $\alpha^{(I)}$ in states with prescribed isospin I.

For transitions in the first channel we have

$$\alpha^{(0)} = 3\lambda(s_3, s_2; s_1) + \lambda(s_1, s_2; s_3) + \lambda(s_1, s_3; s_2),$$
$$\alpha^{(1)} = \lambda(s_1, s_2; s_3) - \lambda(s_1, s_3; s_2),$$
$$\alpha^{(2)} = \lambda(s_1, s_2; s_3) + \lambda(s_1, s_3; s_2). \qquad (3)$$

From the three unitarity conditions for the amplitudes $\alpha^{(I)}$ it is easy to obtain two relations for the absorptive parts $\lambda_1$ and $\lambda_3$ of the amplitude $\lambda$ in the first and third channel:

$$\lambda_1(s_1, s_2; s_3) = \sqrt{\frac{s_1 - 4\mu^2}{s_1}} \int \{\lambda^*(s_1, s_2'; s_3') \lambda(s_1, s_2''; s_3'') + \lambda^*(s_1, s_3'; s_2') \lambda(s_1, s_3''; s_2'')\} \frac{dn}{4\pi} + \Delta_1(s_1, s_2; s_3),$$

$$\lambda_3(s_3, s_2; s_1) = \sqrt{\frac{s_1 - 4\mu^2}{s_1}} \int \{3\lambda^*(s_3', s_2'; s_1) \lambda(s_3'', s_2''; s_1) + \lambda^*(s_3', s_2'; s_1) [\lambda(s_1, s_2''; s_3'') + \lambda(s_1, s_3''; s_2'')] + \lambda(s_3'', s_2''; s_1) [\lambda^*(s_1, s_2'; s_3') + \lambda^*(s_1, s_3'; s_2')]\} + \Delta_3(s_3, s_2; s_1). \qquad (4)$$

Here $\Delta_1$ and $\Delta_3$ refer to the contributions from states containing more than two mesons (i.e. from four-, six-, etc., meson states). The $\Delta_i$ are symmetric functions of their first two arguments, i.e. $\Delta_i(s_1, s_2; s_3) = \Delta_i(s_2, s_1; s_3)$. We use the normalization of Chew and Mandelstam,[5] so that $d\sigma/d\Omega = |4s_1^{-1/2}\lambda|^2$.

As far as its symmetry properties are concerned the amplitude $\lambda$ is of precisely the type for which equations for the spectral functions were formulated by Ter-Martirosyan,[3] Sec. 2. We may therefore make direct use of those results.[3] At the same time we give, for convenience of the reader, a different derivation of these equations in Appendix 1, making use of the unitarity conditions and spectral representations directly for the functions $\alpha^{(I)}(s_1, s_2; s_3)$.

In this way one obtains from Eq. (2) of this work and Eqs. (10) and (11) of Ter-Martirosyan[3] the result

$$\rho(s', s) = Q_1(s', s) + Q_3(s, s'),$$
$$\rho_c(s, s') = Q_2(s, s') + Q_2(s', s), \qquad (5)$$

where

$$Q_1(s', s) = \frac{2}{\pi^2} \iint \Gamma(\sigma', \sigma''; s', s) \{\lambda_2^*(\sigma') \lambda_2(\sigma'') + \lambda_3^*(\sigma') \lambda_3(\sigma'')\} d\sigma' d\sigma'',$$

$$Q_2(s', s) = \frac{2}{\pi^2} \iint \Gamma(\sigma', \sigma''; s', s) \{\lambda_2^*(\sigma') \lambda_3(\sigma'') + \lambda_3^*(\sigma') \lambda_2(\sigma'')\} d\sigma' d\sigma'',$$

$$Q_3(s', s) = \frac{2}{\pi^2} \iint \Gamma(\sigma', \sigma''; s', s) \{3\lambda_1^*(\sigma') \lambda_1(\sigma'') + 2\,\mathrm{Re}\,\lambda_1^*(\sigma') [\lambda_2(\sigma'') + \lambda_3(\sigma'')]\} d\sigma' d\sigma''. \qquad (6)$$

The arguments of the absorptive parts have the following values

$$\lambda_1(\sigma) = \lambda_1(\sigma, 4\mu^2 - s - \sigma; s),$$
$$\lambda_2(\sigma) = \lambda_2(s, \sigma; 4\mu^2 - s - \sigma),$$
$$\lambda_3(\sigma) = \lambda_3(s, 4\mu^2 - s - \sigma; \sigma),$$

and the function $\Gamma$ stands for the spectral function of the box diagram shown in Fig. 2 [see the equation following Eq. (18) of Ter-Martirosyan[3]]:

$$\Gamma(\sigma', \sigma''; s', s) = 2\pi s^{-1/2} \{(s - 4\mu^2) s'^2 - 2s' [(s - 4\mu^2)(\sigma' + \sigma'') + 2\sigma'\sigma''] + (s - 4\mu^2)^2 (\sigma' - \sigma'')^2\}^{-1/2} \theta(s' - s_c),$$

$$s_c = \sigma' + \sigma'' + \frac{2\sigma'\sigma''}{s - 4\mu^2} + \left\{\left(\sigma' + \sigma'' + \frac{2\sigma'\sigma''}{s - 4\mu^2}\right)^2 - (\sigma' - \sigma'')^2\right\}^{1/2},$$

$$\theta(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0 \end{cases}. \qquad (7)$$

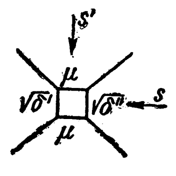


FIG. 2

With the help of Eq. (2) the functions $\lambda_i(\sigma)$ may be expressed in terms of $\alpha$, $\beta$, $\rho$, $\rho_c$:

$$\lambda_1(\sigma) = \alpha(\sigma) + \frac{1}{\pi} \int_0^\infty [\rho(\tau, \sigma) \varphi(\tau, s) + \rho_c(\sigma, \tau) \varphi(\tau, 4\mu^2 - s - \sigma)] d\tau,$$

$$\lambda_2(\sigma) = \alpha(\sigma) + \frac{1}{\pi} \int_0^\infty [\rho(\tau, \sigma) \varphi(\tau, 4\mu^2 - s - \sigma) + \rho_c(\tau, \sigma) \varphi(\tau, s)] d\tau,$$

$$\lambda_3(\sigma) = \beta(\sigma) + \frac{1}{\pi} \int_0^\infty \rho(\sigma, \tau) [\varphi(\tau, s) + \varphi(\tau, 4\mu^2 - s - \sigma)] d\tau. \qquad (8)$$

Together, Eqs. (5), (6) and (8) relate $\rho$ and $\rho_c$ to $\alpha$ and $\beta$.

The remaining two equations [determining the functions $\alpha(\sigma)$ and $\beta(\sigma)$] are obtained by averaging Eq. (4) over the scattering angles. If we denote by $\langle\lambda(s)\rangle_1$ and $\langle\lambda(s)\rangle_3$ the amplitudes averaged over the scattering angles of the first and third channel respectively, then we obtain

$$\langle\lambda_1(s)\rangle_1 = 2\sqrt{(s-4\mu^2)/s}\,|\langle\lambda(s)\rangle_1|^2, \quad (9)$$

$$\langle\lambda_3(s)\rangle_3 = \sqrt{(s-4\mu^2)/s}\{3|\langle\lambda(s)\rangle_3|^2 + 4\mathrm{Re}\,\langle\lambda^*(s)\rangle_3\langle\lambda(s)\rangle_1\}. \quad (10)$$

Here the expressions appearing on the left sides of Eqs. (9) and (10) can be written as follows [see Eq. (9′) of Mandelstam[1]]:

$$\langle\lambda_1(s)\rangle_1 = \alpha(s) + \frac{1}{\pi}\int_0^\infty [Q_1(\sigma, s) + Q_2(\sigma, s)]\, l(\sigma, s)\, d\sigma, \quad (11)$$

$$\langle\lambda_3(s)\rangle_3 = \beta(s) + \frac{2}{\pi}\int_0^\infty Q_3(\sigma, s)\, l(\sigma, s)\, d\sigma. \quad (12)$$

The function $l(\sigma, s)$ has the form[2]

$$l(\sigma, s) = \frac{1}{s-4\mu^2}\ln\left(1+\frac{s-4\mu^2}{\sigma}\right) - \frac{1}{\sigma - s_0}.$$

In order to find $\langle\lambda(s)\rangle_{1,3}$ we make use of dispersion relations, following directly from Eq. (2):

$$\lambda(s_1, s_2; s_3) = \Phi(s_1) + \frac{1}{\pi}\int_0^\infty [\lambda_2(s_1, \sigma)\,\varphi(\sigma, s_2) + \lambda_3(s_1, \sigma)\,\varphi(\sigma, s_3)]\, d\sigma. \quad (13)$$

The functions $\lambda_2(\sigma)$ and $\lambda_3(\sigma)$ are given by Eq. (8) and stand for the imaginary parts of the amplitude in the second and third channel respectively:

$$\Phi(s_1) = \lambda_0 + \frac{1}{\pi}\int_0^\infty \varphi(\sigma, s_1)\,\alpha(\sigma)\, d\sigma + \frac{1}{\pi^2}\iint_0^\infty \frac{\rho(\sigma, \sigma')(s_0 - s_1)\, d\sigma\, d\sigma'}{(\sigma+\sigma'+s_1-4\mu^2)(\sigma - s_0)(\sigma' - s_0)}. \quad (14)$$

Therefore

$$\langle\lambda(s)\rangle_1 = \Phi(s) + \frac{1}{\pi}\int_0^\infty l(\sigma, s)\,[\lambda_2(s, \sigma) + \lambda_3(s, \sigma)]\, d\sigma. \quad (15)$$

Similarly

$$\lambda(s_1, s_2; s_3) = F(s_3) + \frac{1}{\pi}\int_0^\infty \lambda_1(s_3, \sigma)\,[\varphi(\sigma, s_1) + \varphi(\sigma, s_2)]\, d\sigma; \quad (16)$$

here $\lambda_1$ is determined by Eq. (8) and

$$F(s) = \lambda_0 + \frac{1}{\pi}\int_0^\infty \beta(\sigma)\,\varphi(\sigma, s)\, d\sigma + \frac{1}{\pi^2}\iint_0^\infty \frac{\rho_c(\sigma, \sigma')(s_0 - s)\, d\sigma\, d\sigma'}{(\sigma+\sigma'+s_1-4\mu^2)(\sigma - s_0)(\sigma' - s_0)}. \quad (17)$$

Hence

$$\langle\lambda(s)\rangle_3 = F(s) + \frac{2}{\pi}\int_0^\infty \lambda_1(s, \sigma)\, l(\sigma, s)\, d\sigma. \quad (18)$$

## 3. DISCUSSION

Together, the four equations (5), (6), (9) and (10) determine the four functions $\alpha$, $\beta$, $\rho$ and $\rho_c$. It is natural to try to solve the system of equations approximately, ignoring in the first approximation the contributions due to the functions $\rho$ and $\rho_c$ in comparison with the contributions due to $\alpha$ and $\beta$. Instead of $\alpha$ and $\beta$ one may introduce the S wave scattering amplitudes $\alpha_0^{(0)}$ and $\alpha_0^{(2)}$ in the isospin I = 0 and 2 states respectively.

A first approximation for $\rho$ and $\rho_c$ then follows from Eqs. (9) and (10) (with only $\alpha$ and $\beta$ on their right hand sides). The expressions for $\rho$ and $\rho_c$ obtained in this manner are given in Appendix 2. Given $\rho$ and $\rho_c$ we can obtain corrections to the S wave amplitudes, as well as P, D, etc., wave amplitudes. The S wave dominant solution found by Chew, Mandelstam and Noyes[8] is precisely characterized by the property that the P and higher waves are very small in comparison with the S wave. It is therefore clear that the above described procedure will lead in essence to the Chew, Mandelstam and Noyes solution and will differ from it only by corrections from higher order approximations. Should it turn out that it is the P wave (or any other wave with $l > 0$) that is large then the functions $\rho$ and $\rho_c$ cannot be ignored in our equations and it is necessary to solve the four equations with $\rho$ and $\rho_c$ taken into account.

## APPENDIX 1

We present here a derivation of the equations for the spectral functions that is different from that given by Ter-Martirosyan.[3] Let us write

$$\alpha_1^{(l)}(s_1, s_2, s_3) = \sqrt{\frac{s_1-4\mu^2}{s_1}}\int \alpha^{(l)*}(s_1, s_2', s_3')\,\alpha^{(l)}(s_1, s_2'', s_3'')\,\frac{dn_1}{4\pi} + \Delta^{(l)}(s_1, s_2, s_3). \quad (A.1)$$

The first term on the right hand side of Eq. (A.1) may be expressed in the form

$$\frac{1}{\pi}\int_0^\infty Q_l(\sigma, s_1)\,\varphi(\sigma, s_3)\, d\sigma + \frac{1}{\pi}\int_0^\infty P_l(\sigma, s_1)\,\varphi(\sigma, s_2)\, d\sigma, \quad (A.2)$$

where

$$Q_l(\sigma, s_1) = \frac{1}{\pi^2}\iint_0^\infty \Gamma(\tau, \tau'; \sigma, s_1)\,[\alpha_2^{(l)*}(\tau)\,\alpha_2^{(l)}(\tau') + \alpha_3^{(l)*}(\tau)\,\alpha_3^{(l)}(\tau')]\, d\tau\, d\tau',$$

$$P_l(\sigma, s_1) = \frac{1}{\pi^2}\iint_0^\infty \Gamma(\tau, \tau'; \sigma, s_1)\,[\alpha_2^{(l)*}(\tau)\,\alpha_3^{(l)}(\tau') + \alpha_3^{(l)*}(\tau)\,\alpha_2^{(l)}(\tau')]\, d\tau\, d\tau' \quad (A.3)$$

and $\alpha_2^{(I)}$ denotes the jump in $s_2$ of the function $\alpha^{(I)}(s_1, s_2, s_3)$. The arguments of the functions $\alpha_2^{(I)}$ and $\alpha_3^{(I)}$ are as follows:

$$\alpha_2^{(I)} = \alpha_2^{(I)}(s_1, \tau, 4\mu^2 - s_1 - \tau),$$

$$\alpha_3^{(I)} = \alpha_3^{(I)}(s_1, 4\mu^2 - s_1 - \tau, \tau).$$

Evaluating on both sides of Eq. (A.1) the jump in $s_2$ we obtain

$$\alpha_{12}^{(I)}(s_1, s_2, s_3) = P_I(s_2, s_1) + v_I(s_2, s_1); \qquad \text{(A.4)}$$

$v_I(s_2, s_1) = 0$ for $s_1 \le 16\mu^2$. With the help of Eqs. (2) and (3) the functions $\alpha_{12}^{(I)}$, $\alpha_2^{(I)}$ and $\alpha_3^{(I)}$ are easily expressed in terms of $\rho$ and $\rho_c$. Then Eq. (A.4) for I = 0, 1, 2 takes on the form

$$3\rho(s_1, s_2) + \rho(s_2, s_1) + \rho_c(s_1, s_2) = L_0, \qquad \text{(A.5)}$$

$$-\rho_c(s_1, s_2) + \rho(s_2, s_1) = L_1, \qquad \text{(A.6)}$$

$$\rho(s_2, s_1) + \rho_c(s_1, s_2) = L_2, \qquad \text{(A.7)}$$

where we have introduced the notation

$$L_i = \frac{2}{\pi^2}\iint_0^\infty \Gamma(\tau, \tau'; s_2, s_1) A_i(\tau) A_i(\tau')\, d\tau\, d\tau' + v_i(s_2, s_1) \quad (i = 1, 2, 3);$$

$$A_0(\sigma) \equiv \alpha_2^{(0)}(s_1, \sigma, 4\mu^2 - s_1 - \sigma) = 4\alpha(\sigma) + \beta(\sigma) + \frac{1}{\pi}\int_0^\infty [3\rho(\sigma', \sigma) + \rho(\sigma, \sigma') + \rho_c(\sigma, \sigma')]\varphi(\sigma', s_1)\, d\sigma' + \frac{1}{\pi}\int_0^\infty [3\rho_c(\sigma, \sigma') + \rho(\sigma', \sigma) + \rho(\sigma, \sigma')]\varphi(\sigma', 4\mu^2 - s_1 - \sigma)\, d\sigma', \qquad \text{(A.8)}$$

$$A_1(\sigma) \equiv \alpha_2^{(1)}(s_1, \sigma) = \alpha(\sigma) - \beta(\sigma) + \frac{1}{\pi}\int_0^\infty [\rho_c(\sigma', \sigma) - \rho(\sigma, \sigma')]\varphi(\sigma', s_1)\, d\sigma' + \frac{1}{\pi}\int_0^\infty [\rho(\sigma', \sigma) - \rho(\sigma, \sigma')]\varphi(\sigma', 4\mu^2 - s_1 - \sigma)\, d\sigma', \qquad \text{(A.9)}$$

$$A_2(\sigma) \equiv \alpha_2^{(2)}(s_1, \sigma) = \alpha(\sigma) + \beta(\sigma) + \frac{1}{\pi}\int_0^\infty [\rho(\sigma, \sigma') + \rho_c(\sigma, \sigma')]\varphi(\sigma', s_1)\, d\sigma' + \frac{1}{\pi}\int_0^\infty [\rho(\sigma, \sigma') + \rho(\sigma', \sigma)]\varphi(\sigma', 4\mu^2 - s_1 - \sigma)\, d\sigma'. \qquad \text{(A.10)}$$

Subtracting Eq. (A.6) from Eq. (A.7) and making use of the symmetry of $\rho_c(s_1, s_2)$ we find for $\rho_c(s_1, s_2)$ an expression which coincides exactly with Eq. (5). Adding Eqs. (A.6) and (A.7) we get $\rho(s_2, s_1)$ in the region $s_1 \ge 16\mu^2$, $s_2 < 16\mu^2$, whereas by subtracting Eq. (A.7) from Eq. (A.5) we obtain $\rho(s_1, s_2)$ in the same region. From this point on it is easy to get the expression for $\rho(s_1, s_2)$ which coincides with Eq. (5).

## APPENDIX 2

Let us find the first approximation for $\rho$ and $\rho_c$. We make use of Eqs. (5) and (6), with $\lambda_1$, $\lambda_2$, $\lambda_3$ determined with the help of Eq. (8). Leaving in $\lambda_1$, $\lambda_2$ and $\lambda_3$ only $\alpha(\sigma)$ and $\beta(\sigma)$ we get

$$\rho(s', s) = \frac{2}{\pi^2}\iint_0^\infty \Gamma(\tau, \tau'; s', s)\{\alpha(\tau)\alpha(\tau') + \beta(\tau)\beta(\tau')\}\, d\tau\, d\tau' + \frac{2}{\pi^2}\iint_0^\infty \Gamma(\tau, \tau'; s, s') \times \{3\alpha(\tau)\alpha(\tau') + 2\alpha(\tau)[\alpha(\tau') + \beta(\tau')]\}\, d\tau\, d\tau', \qquad \text{(A.11)}$$

$$\rho_c(s, s') = \frac{4}{\pi^2}\iint_0^\infty \{\Gamma(\tau, \tau'; s, s') + \Gamma(\tau, \tau'; s', s)\}\alpha(\tau)\beta(\tau')\, d\tau\, d\tau'. \qquad \text{(A.12)}$$

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