A DRESSED PARTICLE ANALYSIS OF THE $\pi + d \rightleftarrows 2N$ *REACTION*

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The $\pi + d \rightleftarrows 2N$ reaction is treated by a nonrelativistic meson theory of the πN interaction in the P state. Dressed particle techniques are used to express the amplitude for this process in terms of the coupling constant and the P-wave phase shifts for πN scattering. The energy and angle dependence of the calculated cross sections are in qualitative agreement with experiment. The maximum difference between the theoretical and observed cross sections for angles far from 90° and energies that are not too high is 30%. For angles close to 90° and energies much higher than resonant the calculated cross section becomes as much as two times as large as the experimentally observed one. Improvement of the results requires including the S state in the πN interaction, as well as high nucleon velocities.

1. INTRODUCTION

THERE is much experimental material on the $+ d \rightleftarrows 2N$ reaction.[1] Progress in the theoretical explanation of this reaction, on the other hand, is quite modest in scope. The best results are obtained with the phenomenological theory of Mandelstam.[2] He guesses the form of the resonance factor and uses three adjustable parameters, and in this way is able to give a relatively good explanation of the experimental data for incident pion energies from 0 to roughly 300 Mev. Investigations based on meson theory consider the reaction cross section close to threshold.[3,4] The agreement with experiment obtained in this way is satisfactory.[4] From the theoretical point of view, however, these investigations are not very systematic, since the two-nucleon problem is treated in them phenomenologically, rather than in terms of the meson field. Furthermore, in the most interesting region, that of resonance, the cross section is yet to be studied.

In the present paper the $\pi + d \rightleftarrows 2N$ interaction is treated by the dressed-particle method, by means of which the two-nucleon problem is reduced to a calculation involving only single-nucleon matrix elements.[5,6]

As with other authors, our description of the nucleon is not relativistic, and the Hamiltonian we choose is

$$H = H_{N_1} + H_{N_2} + H_\pi + \sum_k \{c_k (G_k^{(1)} + G_k^{(2)}) + \text{Herm. adj.}\},$$
$$G_k^{(s)} = \frac{i\sqrt{4\pi}\, f_0\, v(k)}{2\sqrt{2k_0}} \tau_k^{(s)} \left[\left(\sigma^{(s)}, \mathbf{k} + i\frac{k_0}{M}\nabla_s\right), e^{i\mathbf{k}\mathbf{r}_s}\right]_+, \quad s = 1, 2. \qquad (1)$$

Here H_{N_1}, H_{N_2}, and H_π are the kinetic energies, respectively, of the first and second nucleon and the meson field. The index k includes the meson three-momentum K and its isotopic spin coordinate. We use the system of coordinates in which $\hbar = c = \mu = 1$ (where μ is the pion mass). The rest of the notation is quite common and need not be explained. We remark that the Hamiltonian of (1) describes correctly the resonance interaction of π mesons with nonrelativistic nucleons in the P state. As is known, there is no successful theory of the πN interaction in the S state, and we therefore must neglect this interaction.

The following approximations are made in the calculation.

1. Terms describing two-meson and higher exchange between the nucleons are dropped. Effects related to such terms increase rapidly with increasing distance r between the nucleons (as fast as and faster than e^{-2r}), and therefore for a system as weakly bound as the deuteron one may reliably treat them as small.*

2. In calculating the single-nucleon matrix elements, we drop terms that depend on the square and higher powers of the nucleon velocities and terms linear in the velocities of the nucleon in the deuteron. This latter approximation is probably justified because the nucleons move relatively slowly inside the deuteron. As for neglecting the square of the nucleon velocities in the final state (in the $\pi + d \rightarrow 2N$ reaction), it cannot be justified by simply pointing to low velocities, for they are of the order of 0.3 to 0.5. Essentially one

*Clearly this is not equivalent to perturbation theory.

must assume that the matrix elements depend only weakly on these velocities.

3. We neglect all πN scattering phase shifts other than that for resonance (namely δ_{33}), since there is no reliable way to deal with small phase shifts in the presently existing theory.

4. From similar considerations, the outgoing nucleons are described by plane waves. The error involved is probably not large, because the nucleon energies are so high.

2. TRANSITION AMPLITUDE

In this section we shall briefly describe the derivation of the formulas for the transition amplitude. The technical details are available in other works[5-7] discussing the calculation.

For the process we are dealing with, the transition amplitude is of the form

$$T = \langle \Psi_\alpha^{(-)} | (H - E) c_q^{+} | \Psi_{d,P} \rangle,$$

where $\Psi_\alpha^{(-)}$ is the incoming two-nucleon state with spins and momenta indicated by α, $\Psi_{d,P}$ is the deuteron state with momentum $\mathbf{P}$, q is the π-meson momentum and spin, and E is the energy of the system. By using dressed particle techniques one can reduce the calculation of T to one involving single-nucleon matrix elements.[6] The final result, which is all we shall present here, is

$$T = \sum_{\beta,\gamma} f_\alpha^{(-)*}(\beta)\, K_{\beta\gamma}\, f_{d,P}(\gamma), \qquad K_{\beta\gamma} = \sum_{s=0}^{4} K_{\beta\gamma}^{(s)},$$

$$K_{\beta\gamma}^{(0)} = \langle \beta | G_q^{(1)} + G_q^{(2)} | \gamma \rangle, \qquad K_{\beta\gamma}^{(1)} = \langle \beta | N (G_q^{(1)} + G_q^{(2)}) | \gamma \rangle,$$

$$K_{\beta\gamma}^{(2)} = -\tfrac{1}{2} \sum_{\delta} \{ \langle \beta | G_q^{(1)} + G_q^{(2)} | \delta \rangle N_{\delta\gamma} + N_{\beta\delta} \langle \delta | G_q^{(1)} + G_q^{(2)} | \gamma \rangle \},$$

$$K_{\beta\gamma}^{(3)} = -\left\langle \beta \left| (G_q^{(1)} + G_q^{(2)}) \frac{1 - P_{00}}{H_1 + H_2 - E_\gamma + i0} U_{12} \right| \gamma \right\rangle,$$

$$K_{\beta\gamma}^{(4)} = -\left\langle \beta \left| U_{12}^{+} \frac{1 - P_{00}}{H_1 + H_2 - E_\beta - i0} (G_q^{(1)} + G_q^{(2)}) \right| \gamma \right\rangle. \quad (2)$$

Here $f_\alpha^{(-)}(\beta)$ is the wave function of the incoming unbound nucleons, and $f_{d,P}(\gamma)$ is the deuteron wave function. The variables β and γ include the nucleon momenta ($\mathbf{p}_1'\mathbf{p}_2'$ and $\mathbf{p}_1$, $\mathbf{p}_2$, respectively) and spins. The remaining notation is the same as is usually used in the dressed-particle method.[5,6] The first term in (2) gives the impulse approximation, and the remaining terms arise from single-meson exchange.

The calculation of the single-nucleon matrix elements in (2) is not particularly difficult, (see, for instance, Novozhilov and Terent'ev[7]). Some peculiarities arise in calculating $K_{\beta\gamma}^{(4)}$. This is because the denominator $H_1 + H_2 - E_\beta$ may vanish, and therefore standard methods would lead to difficult singular integrals over the πN scattering cross section. A typical term in $K_{\beta\gamma}^{(4)}$ may be of the form

$$\sum_k \left\langle \beta \left| G_k^{(1)+} c_k^{(2)} \frac{1 - P_{00}}{H_1 + H_2 - E_\beta - i0} G_q^{(1)} \right| \gamma \right\rangle$$

$$= \sum_k \left\langle \beta_1 \left| G_k^{(1)+} \frac{1 - P_0}{H_1 + E_{\gamma_2} - E_\beta - i0} G_q^{(1)} \right| \gamma_1 \right\rangle \langle \beta_2 | c_k^{(2)+} | \gamma_2 \rangle$$

(where β_1 is the spin and momentum of the first nucleon in the β state, and E_{β_1} is its energy, etc.) The second term is calculated in the usual way. The first one is subjected to the unitary transformation $|\gamma_1\rangle = \exp[i(\mathbf{s} - \mathbf{u})\mathbf{r}_1] | \gamma_1)$ (where $\mathbf{s}$ is the total momentum and $\mathbf{u}$ is the momentum of the meson field), which transforms into

$$\delta(\mathbf{p}_1' + \mathbf{k} - \mathbf{q} - \mathbf{p}_1) \left(\beta_1 \left| \widetilde{G}_k^{(1)+} \frac{1 - \widetilde{P}_0}{\widetilde{H}_1 + E_{\gamma_2} - E_\beta - i0} \widetilde{G}_q^{(1)} \right| \gamma_1 \right),$$

where we have used the notation

$$\widetilde{A} \equiv \exp[-i(\mathbf{s} - \mathbf{u})\mathbf{r}_1]\, A \exp[i(\mathbf{s} - \mathbf{u})\mathbf{r}_1].$$

The matrix element here depends only quadratically on the nucleon velocities in the β_1 and γ_1 states. Then according to our approximations, we may replace the β_1 state by some other state $\widetilde{\beta}_1$ with the same spin, but with another momentum which we shall choose later. We note now that in our approximation we may write

$$E_{\gamma_2} - E_\beta \approx E_{\gamma_2} - E \approx E_{\gamma_2} - (q_0 + E_\gamma) = -q_0 - E_{\gamma_1}$$

and that

$$\frac{1}{\widetilde{H}_1 - q_0 - E_{\gamma_1} - i0} \widetilde{G}_q^{(1)} | \gamma_1) = c_q^{(1)+} | \gamma_1) - | q\gamma_1)^{(+)},$$

where $|q\gamma_1)^{(+)}$ is an outgoing state containing a single nucleon (spin and momentum γ_1) and a single meson (spin and momentum q). Of the two matrix elements obtained, the first, which contains no singular denominators, is calculated in the usual way, while the second gives the T matrix for πN scattering, but with neither energy nor momentum conservation. We now use the arbitrariness in the choice of $\widetilde{\beta}_1$ and the explicit form of the T matrix for the P-wave πN interaction to achieve conservation, and thus the matrix element can be written in terms of the πN scattering phase shifts.

The final expression for $K_{\beta\gamma}$ is then

$$K_{\beta\gamma} = i\delta(\mathbf{p}_1' + \mathbf{p}_2' - \mathbf{q} - \mathbf{P}) \left(1 - \frac{q_0}{2M}\right) \Big\{ \delta(\mathbf{p}_1' - \mathbf{p}_1) \times \frac{f v(q)}{2\pi \sqrt{q_0}} \left(\sigma^{(2)}, \mathbf{q} + \frac{q_0}{M} \mathbf{p}_1'\right) + F(q_0) \int d^3k \frac{v^2(k)}{k_0^2} \delta(\mathbf{k} - \mathbf{p}_2' + \mathbf{p}_2) \times \boldsymbol{\sigma}^{(2)}\mathbf{k}\, [3\mathbf{k}\mathbf{q} - (\boldsymbol{\sigma}^{(1)}\mathbf{k})(\boldsymbol{\sigma}^{(1)}\mathbf{q})] - (1 \rightleftarrows 2) \Big\}. \quad (3)$$

The resonance factor $F(q_0)$ is given by

$$F(q_0) = \frac{8}{3} \frac{fv(q)}{(2\pi)^3 \sqrt{q_0}} \left[\frac{e^{i\delta_{33}(\bar{q}_0)} \sin \delta_{33}(\bar{q}_0)}{\bar{q}^2 q v^2(q)} - \frac{3}{2} \frac{f^2}{q_0} \right]. \quad (4)$$

Here $\bar{q}_0$ and $\bar{q}$ are the energy and the magnitude of the momentum in the π-nucleon center-of-mass system.

3. NUMERICAL RESULTS AND DISCUSSION

We now take the absolute square of the amplitude T of Eq. (2) (dropping the δ function, which gives total momentum conservation), sum over final spins, and average over initial. If we denote by Z the expression obtained in the laboratory system ($\mathbf{P} = 0$), the differential cross section obtained in the center-of-mass system for the $\pi + d \rightleftarrows 2N$ reaction is

$$\frac{d\sigma(\pi + d \to 2N)}{d\Omega} = \frac{8\pi^4 q_0 l}{q} \sqrt{M^2 + l^2 Z}$$

$$\frac{d\sigma(2N \to \pi + d)}{d\Omega} = \frac{3}{2} \frac{\eta^2}{l^2} \frac{d\sigma(\pi + d \to 2N)}{d\Omega}. \quad (5)$$

Here l^2 and η^2 are the squares of the nucleon and π momenta in the center-of-mass system for this reaction.

As was mentioned in the introduction, the $f_\alpha(\beta)$ we use are free-particle functions. For the deuteron function we take the expression given by Hulthén and Sugawara,[8] which gives the probability $p_D = 4\%$ for the D wave, and a hard-core radius $r_c = 0.306$. The cutoff factor is chosen as $v(k) = a^2/(a^2 + k^2)$ with $a = 7$. The cross section depends weakly on a.

We have tabulated our theoretical results alongside the experimental[1] ones for the $2p \to \pi^+ + d$ reaction for several values of η and scattering angle θ in the center-of-mass system. It should be emphasized that it is not possible for us to calculate in any reliable way the contribution to the cross section from the triplet state of the nucleons. This is because the most important part of this contribution comes from the S-state part of the πN interaction, and we do not include this. For this reason we tabulate the results both with and without the triplet-state contribution.

As is seen from the table, the energy and angle dependence of the calculated cross section agree qualitatively with experiment. The total cross section exhibits a resonant behavior with a maximum for η in the region of 1.5 to 1.8, which is supported by the experimental data.

As for a quantitative comparison with experiment, we must emphasize that in view of the third of the approximations we make (see Introduction) the calculated cross section can be considered reliable (assuming the other approximations valid) only where the resonant terms with the δ_{33} phase shift are large compared to the small-phase shift terms we have not included. This is true only for angles far from 90° and energies below the resonance in the πN scattering. Therefore it is only in these regions that our theory may attempt to reproduce the experimental results.

Then excluding angles close to 90° and the point $\eta = 2.05$, we find that the theory deviates from experiment by at most about 30%. In our

Differential and total cross sections for the $2p \to \pi^+ + d$ reaction

$\eta/\mu c$	θ, deg	Differential cross sections (10^{-28} cm^2/sr): theory, including $d\sigma_t$	theory, not including $d\sigma_t$	experiment	Total cross sections (10^{-27} cm^2): theory, including σ_t	theory, not including σ_t	experiment
0.54	0	0.256	0.239	0.312±0.031	0.151	0,116	0,156±0.016
	45	0.153	0.123	0.145±0.018			
	90	0.054	0.027	0.028±0.011			
1.15	0	1.72	1.70	2,85±0,32	1.32	0.92	1,53±0.23
	45	1.17	0.87	1.70±0.22			
	90	0.77	0.40	0.55±0,12			
1.55	0	5.11	5,07	5.55±0,52	3.80	2.71	3.15±0,25
	45	3.40	2.63	3,26±0.32			
	90	2.17	1.04	0,99±0.13			
1.74	0	6.68	6.55	5,05±0,42	4,63	3.44	3,06±0,31
	30	5.28	4,76	4,05±0,35			
	45	4.15	3,35	3.09±0.30			
	60	3.22	2,22	2.14±0,24			
	90	2.54	1,25	1.18±0.18			
2,05	0	5.41	5,03	2.73±0,22	3,14	2,44	1.89±0.17
	45	3,03	2.57	1,81±0.17			
	90	1.30	0.59	0,86±0.11			

opinion this is quite satisfactory when one recalls the approximations made.

If all angles and energies are included, the theoretical differential cross section differs from experiment by at most 100%. For the total cross section this figure is about 50%. Better results are obtained if one neglects the triplet state of the nucleons. Then, as the table shows, the total cross section differs from experiment by no more than 30% for all energies, and 10% at resonance.

We note also that around 90° the cross section depends strongly on the choice of p_D and r_c. We have chosen the values generally used, but they are almost definitely not the best ones from the point of view of obtaining agreement with experiment for our calculation. A final choice, however, can be made only after one takes account of other factors influencing the cross section, especially the S-state πN interaction and the low phase shifts in the πN scattering. For this reason we do not vary p_D and r_c in our calculation.

In conclusion we wish to emphasize that our calculation is necessarily a first approximation, presenting only the roughest outlines of the mechanism by which the $\pi + d \rightleftarrows 2N$ reaction takes place. A real improvement of the calculations will become possible only when we learn to include the high nucleon velocities characteristic of this reaction, i.e., when there exists a relativistic theory of strong interactions.

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