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THE DYNAMIC EFFECT OF THE NUCLEAR VOLUME IN CONVERSION M1 TRANSITIONS IN EVEN-EVEN NUCLEI FOR THE NONAXIAL ROTATOR MODEL AND FOR THE VIBRATIONAL MODEL OF THE NUCLEUS

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The corrections to the internal conversion coefficient for a M1 nuclear transition arising when the potential produced by the intranuclear electron transition current is taken into account are estimated for the transitions of K, LI, and LII electrons to the $s_{1/2}$ and $p_{1/2}$ states of the continuous spectrum. The calculations are carried out according to the harmonic vibrational model and the nonaxial rotator model (Davydov-Filippov model) of the nucleus.

1. INTRODUCTION

In the internal conversion of orbital electrons, the vector potential due to the electron current $\mathbf{j}_e$ of the transition

$$\mathbf{A}(\mathbf{r}) = \int \mathbf{j}_e(\mathbf{r}') \frac{\exp\{i\omega|\mathbf{r}-\mathbf{r}'|\}}{|\mathbf{r}-\mathbf{r}'|}(d\mathbf{r}') \qquad (1)$$

is composed of the potential $\mathbf{A}_{in}(\mathbf{r})$, which is due only to the electron current $\mathbf{j}_e(\mathbf{r}')$ (where $r' \leq R_0$) distributed over the volume of the nucleus, and of $\mathbf{A}_{ex}(\mathbf{r})$ due mainly to the extranuclear electron current $\mathbf{j}_e(\mathbf{r}')$ (where $r' \geq R_0$):

$$\mathbf{A}(\mathbf{r}) = \mathbf{A}_{in}(\mathbf{r}) + \mathbf{A}_{ex}(\mathbf{r}). \qquad (2)$$

Since the nuclear volume is small as compared with the volume of the atomic shell, we have $|\mathbf{A}_{in}| \ll |\mathbf{A}_{ex}|$. Since the quantity $\mathbf{A}_{in}$ is determined by the probability of penetration of the conversion electron into the nuclear volume, the maximum potential $\mathbf{A}_{in}$ is produced in M1 transitions of an electron of the type $s_{1/2} \rightarrow s_{1/2}$ and $p_{1/2} \rightarrow p_{1/2}$ (e.g., $K \rightarrow s_{1/2}$, $LI \rightarrow s_{1/2}$, and $LII \rightarrow p_{1/2}$), while in these transitions the internal potential has the order of magnitude

$$\mathbf{A}_{in} \sim \frac{Z}{137}\left[R_0 Z \frac{m_e e^2}{\hbar^2}\right]^{2\gamma-1} \mathbf{A}_{ex}, \qquad \gamma = \left[1 - \left(\frac{Z}{137}\right)^2\right]^{1/2}.$$

In other M1 transitions of orbital electrons, $\mathbf{A}_{in}$ is considerably less than the estimate given above, and its contribution to the conversion probability may be neglected.

Usually, in the theory of internal conversion, $\mathbf{A}_{in}$ is not considered, and the conversion transition probability is calculated taking only the potential $\mathbf{A}_{ex}$ into account. In that case, the probability of the γ transition in a nucleus, as well as the probability of the conversion transition, is determined by the same nuclear matrix element, which drops out from the internal conversion coefficient, the latter consequently becoming independent of the nuclear structure.

However, the potentials $\mathbf{A}_{in}(\mathbf{r})$ and $\mathbf{A}_{ex}(\mathbf{r})$ have a qualitatively different radial dependence, and the corresponding nuclear matrix elements of the operators of the interaction of the nuclear current $\mathbf{j}_N$ with this potential ($\mathbf{j}_N\mathbf{A}_{in}$ and $\mathbf{j}_N\mathbf{A}_{ex}$) are, in general, subject to different selection rules determined by the given structure of the excited states of the nucleus. Therefore, in separate transitions, the contribution of $\mathbf{A}_{in}$ may be comparable to that of $\mathbf{A}_{ex}$ irrespective of the fact that $\mathbf{A}_{in} \ll \mathbf{A}_{ex}$. In such cases, we are dealing with forbidden transitions inside the nucleus; the conversion probability is determined by several matrix elements, and the internal conversion coefficient depends on the ratio of the contributions of the operators $\mathbf{j}_N\mathbf{A}_{in}$ and $\mathbf{j}_N\mathbf{A}_{ex}$, i.e., on the nuclear structure.

This effect of the contribution of $\mathbf{A}_{in}$ has been called the dynamic effect of the nuclear volume, and the corresponding changes in the internal conversion coefficient are called "anomalies."

It should be mentioned that an anomaly may also arise in the case of a strongly forbidden nuclear transition due to the potential $\mathbf{A}_{ex}$ only, i.e., where the matrix element of the operator $\mathbf{j}_N\mathbf{A}_{ex}$ is small as compared with its usual value for allowed transitions, while the matrix element of the operator $\mathbf{j}_N\mathbf{A}_{in}$ has a usual order of magnitude. However, another type of forbidden M1 transitions is possible if the matrix elements of the operators $\mathbf{j}_N\mathbf{A}_{ex}$ and $\mathbf{j}_N\mathbf{A}_{in}$ are small as compared with their values in allowed transitions. In forbidden M1 transitions of such a type, the internal conversion coefficient has its normal listed value, and is practically independent of the nuclear structure.

Thus, the fact that an M1 transition is forbidden is a necessary but not sufficient condition for an anomaly in the internal conversion coefficient.

The dynamic effect of the nuclear volume has first been studied for the independent-particle model by the author.[1] Church and Weneser[2] have, in a more general form, considered the corrections to the internal conversion coefficient in M1 transitions and, in particular, have drawn attention to the forbidden collective M1 transitions in even-even nuclei where one can expect an anomaly in the internal conversion coefficient.

The vibrational nuclear model[3] and the rotator model[3,4] have recently been developed to describe the structure of collective excited states of even-even nuclei. The purpose of the present paper is to estimate the contribution of $\mathbf{A}_{in}(\mathbf{r})$ to the internal conversion coefficient within the framework of these collective nuclear models, and to compare these estimates for nuclei where both the rotator model[4] and the vibrational model are applicable.

Henceforth, all quantities are expressed in relativistic units $\hbar = m_e = c = 1$, $e^2 = 1/137$; the nuclear transition energy ω and the energy of electronic states ϵ_i are expressed in units of $m_e c^2$; and the nuclear radius $R_0 = 0.43\, e^2 A^{1/3}$ corresponds to $R_0 = 1.2 \times 10^{-13} A^{1/3}$ cm in the usual units.

In calculating the electron potentials, the formalism of spherical vector functions[5] is used. In the wave functions of an orbital electron and of an electron in the continuous energy spectrum, the effect of finite dimensions of the nucleus is taken into account assuming, moreover, that the nucleon represents a uniformly charged sphere with radius R_0. The functions of the continuous-spectrum electrons are normalized to the energy interval.

2. POTENTIALS OF THE M1 TRANSITIONS OF K, LI, AND LII ELECTRONS

In the transition of an electron from an orbital state with energy ϵ_1, total momentum j_1, and total orbital momentum $l_1 = j_1 + \lambda_1$ to a continuous-spectrum state (ϵ_2, j_2, λ_2), the potentials of the M1 transitions are found to be

$$\mathbf{A}_{ex}(\mathbf{r}) = ie\left(\sum_M \sqrt{4\pi} C^{j_1\mu_1}_{j_2\mu_2 1M} \mathbf{Y}^0_{1M}(\mathbf{r})\right) f_1(\omega r)\, \mathfrak{M}, \qquad (3)$$

where

$$\mathfrak{M} = i\omega\beta\left\{\int_0^{R_0} dr\, r^2 h_1^{(1)}(\omega r)\,[G_{j_2\lambda_2}F_{j_1\lambda_1} + G_{j_1\lambda_1}F_{j_2\lambda_2}] + \int_{R_0}^{\infty} dr\, r^2 h_1^{(1)}(\omega r)\,[g_{j_2\lambda_2}f_{j_1\lambda_1} + g_{j_1\lambda_1}f_{j_2\lambda_2}]\right\}. \qquad (4)$$

In the above, the contribution of the intranuclear electron current is taken into account in $\mathbf{A}_{ex}(\mathbf{r})$. For a field $\mathbf{A}$, this is identical to the contribution of the extranuclear transition electron current. $G_{ji\lambda i}$ and $F_{ji\lambda i}$ are the large and small radial components of the electron wavefunction in the range $r' \le R_0$, $g_{ji\lambda i}$ and $f_{ji\lambda i}$ are those for the range $r' \gg R_0$; $f_1(\omega r)$ and $h_1^{(1)}(\omega r)$ are spherical Bessel and Hankel functions; $C^{a\alpha}_{b\beta c\gamma}$ is the Clebsch-Gordan coefficient. The coefficient β for transitions $s_{1/2} \to s_{1/2}$ equals $\beta = -\sqrt{2}$, while, for transitions $p_{1/2} \to p_{1/2}$ of an electron, $\beta = +\sqrt{2}$.

The internal potential of an M1 electron transition may be represented by the series

$$\mathbf{A}_{in}(\mathbf{r}) = ie\left(\sum_M \sqrt{4\pi} C^{j_1\mu_1}_{j_2\mu_2 1M} \mathbf{Y}^0_{1M}(\mathbf{r})\right) \sum_{\varkappa=1} B_{2\varkappa+1}\left(\frac{r}{R_0}\right)^{2\varkappa+1}, \qquad (5)$$

where all coefficients $B_{2\kappa+1}$ are real.

The series in κ converges rapidly, so that, for the estimate of the potentials, it is sufficient to limit ourselves to one term with $\kappa = 1$. For the transitions $K \to s_{1/2}$, $LI \to s_{1/2}$, and $LII \to p_{1/2}$, the required coefficients B_3 are given by the equations

$$B_3(K \to s_{1/2}) = -\frac{\sqrt{2}}{30}[3Ze^2 + (\varepsilon_1 + \varepsilon_2 - 2)R_0]\, A(K)\, A(s_{1/2})\, R_0^2,$$
$$B_3(LI \to s_{1/2}) = -\frac{\sqrt{2}}{30}[3Ze^2 + (\varepsilon_1 + \varepsilon_2 - 2)R_0]\, A(LI)\, A(s_{1/2})\, R_0^2,$$
$$B_3(LII \to p_{1/2}) = -\frac{\sqrt{2}}{30}[3Ze^2 + (\varepsilon_1 + \varepsilon_2 + 2)R_0]\, A(LII)\, A(p_{1/2})\, R_0^2. \qquad (6)$$

Neglecting the screening effect, we find the amplitudes of electrons in the K, LI, LII, $s_{1/2}$, and $p_{1/2}$ states to be

$$A(K) = -\left[\frac{(1+\varepsilon_1)}{2\Gamma(2\gamma+1)}\right]^{1/2}(2Ze^2)^{\gamma+1/2}R_0^{\gamma-1}\exp(-Ze^2R_0),$$
$$A(LI) = -\left[\frac{(1+\varepsilon_1)(2\gamma+1)}{N(N+1)\Gamma(2\gamma+1)}\right]^{1/2}\left(\frac{2Ze^2}{N}\right)^{\gamma+1/2}R_0^{\gamma-1}\exp\left(-\frac{Ze^2R_0}{N}\right),$$
$$A(LII) = -\left[\frac{(1-\varepsilon_1)(2\gamma+1)}{N(N-1)\Gamma(2\gamma+1)}\right]^{1/2}\left(\frac{2Ze^2}{N}\right)^{\gamma+1/2}R_0^{\gamma-1}\exp\left(-\frac{Ze^2R_0}{N}\right),$$
$$A(s_{1/2}) = -\left[\frac{\varepsilon_2+1}{\pi p}\right]^{1/2}\frac{(2p)^\gamma\,|\Gamma(\gamma + iZe^2\varepsilon_2/p)|}{\Gamma(2\gamma+1)}R_0^{\gamma-1}\exp\left(\frac{\pi Ze^2\varepsilon_2}{2p}\right),$$
$$A(p_{1/2}) = -\left[\frac{\varepsilon_2-1}{\pi p}\right]^{1/2}\frac{(2p)^\gamma\,|\Gamma(\gamma + iZe^2\varepsilon_2/p)|}{\Gamma(2\gamma+1)}R_0^{\gamma-1}\exp\left(\frac{\pi Ze^2\varepsilon_2}{2p}\right). \qquad (7)$$

where

$$\gamma = \sqrt{1 - Ze^2}, \qquad N = \sqrt{2 + 2\gamma}, \qquad p = \sqrt{\varepsilon_2^2 - 1},$$

and p is the electron momentum.

3. GENERAL FORMULA FOR THE INTERNAL CONVERSION COEFFICIENT IN M1 TRANSITIONS OF THE NUCLEUS, TAKING THE DYNAMIC EFFECT OF THE NUCLEAR VOLUME INTO ACCOUNT

Using Eqs. (3) and (5) for the potentials $\mathbf{A}_{ex}$ and $\mathbf{A}_{in}$ and the normalized potential of the M1 quantum, we obtain a general formula for the internal conversion coefficient (ICC), valid for nuclear transitions $I_1M_1 \rightarrow I_2M_2$ with an arbitrary transition energy ω and an arbitrary selection rule

$$\mathrm{ICC} = \frac{\pi e^2}{3\omega} |\mathfrak{M}|^2 \left| 1 + \sum_{\varkappa=1}^{\infty} \frac{B_{2\varkappa+1}}{\mathfrak{M}} \frac{\langle I_2M_2 | (r/R_0)^{2\varkappa+1} (\mathbf{j}_N \mathbf{Y}_{1M}^0) | I_1M_1 \rangle}{\langle I_2M_2 | f_1(\omega r) (\mathbf{j}_N \mathbf{Y}_{1M}^0) | I_1M_1 \rangle} \right|^2. \quad (8)$$

Neglecting the contribution of $\mathbf{A}_{in}(\mathbf{r})$, we find the usual value of the internal conversion coefficient, as listed, for example, by Sliv[6]

$$(\mathrm{ICC})_0 = \pi e^2 |\mathfrak{M}|^2/3\omega, \quad (9)$$

and hence we determine the absolute value of the integral $\mathfrak{M}$ expressed in terms of the listed quantity $(\mathrm{ICC})_0$

$$|\mathfrak{M}| = \sqrt{3\omega\,(\mathrm{ICC})_0/\pi e^2}. \quad (10)$$

It is important to note that the integral $\mathfrak{M}$ is a complex quantity, and Eq. (10) only enables us to find its absolute value, leaving the phase φ undetermined. Only in the case $\omega \ll Ze^2$, where we can limit ourselves to the approximation $h_1'(\omega r) \approx 1/i\,(\omega r)^2$, is the integral $\mathfrak{M}$ real. In the general formula for the internal conversion coefficient obtained by Church and Weneser[2] and afterwards used by Reiner[7] in the analysis of the anomalies of the internal conversion coefficient in M1 transitions of odd deformed nuclei, the phase φ of the integral $\mathfrak{M}$ has been neglected. In transitions $I_1 \rightarrow I_2$ of even-even nuclei, we have as a rule $\omega \approx Ze^2$ or $\omega > Ze^2$, so that, in the analysis of the internal conversion coefficient in these transitions, it is necessary to take the phase φ into account.

Taking into account that, in the usual nuclear transitions, the condition $\omega R_0 \ll 1$ is always satisfied, and limiting ourselves to the first term of the series in κ, we obtain a more convenient formula for the internal conversion coefficient in M1 nuclear transitions:

$$\mathrm{ICC}_1 = (\mathrm{ICC})_0 \left| 1 + \frac{B_3}{\omega R_0} \sqrt{\frac{3\pi e^2}{\omega\,(\mathrm{ICC})_0}}\, e^{-i\varphi} X \right|^2, \quad (11)$$

where

$$X = \frac{\langle I_2M_2 | (r/R_0)^3 (\mathbf{j}_N \mathbf{Y}_{1M}^0) | I_1M_1 \rangle}{\langle I_2M_2 | (r/R_0) (\mathbf{j}_N \mathbf{Y}_{1M}^0) | I_1M_1 \rangle}. \quad (12)$$

The quantities B_3 are determined by Eqs. (6) and (7).

4. ESTIMATE OF X FOR M1 TRANSITIONS OF EVEN-EVEN NUCLEI IN MODELS WITH COLLECTIVE QUADRUPOLE EXCITATION

Both in the vibrational nuclear model[3] and in the rotator model,[3,8] the transition current in the nucleus $\mathbf{j}_N$ is determined in the hydrodynamical approximation by the flow velocity of the nuclear liquid

$$\mathbf{V} = \frac{1}{2} \sum_m \dot{\alpha}_{2m} \nabla (r^2 Y_{2m}), \quad (13)$$

where α_{2m} are the parameters of nuclear deformations in the laboratory coordinate system:

$$R(\vartheta, \varphi) = R_0 \left[1 + \alpha_0 + \sum_m \alpha_{2m} Y_{2m} \right].$$

In this approximation for the ratio X, we have

$$X = \frac{\langle I_2M_2 | \hat{J}_{1M}^{(3)} | I_1M_1 \rangle}{\langle I_2M_2 | \hat{J}_{1M}^{(1)} | I_1M_1 \rangle}, \quad (14)$$

where

$$J_{1M}^{(\varkappa)} = \int d\Omega \int_0^{R(\vartheta,\varphi)} dr r^2 \left(\frac{r}{R_0}\right)^{\varkappa} \sum_m \dot{\alpha}_{2m} (\mathbf{Y}_{1M}^0 \nabla r^2 Y_{2m}). \quad (15)$$

Integrating Eq. (15) and retaining only the first term in $\alpha_{2\mu}$ which gives a nonvanishing matrix element of the M1 transition, we obtain for the operator $\hat{J}_{1M}^{(\kappa)}$ the equation

$$\hat{J}_{1M}^{(\varkappa)} = (\varkappa + 3) \frac{3\sqrt{5}}{8\pi} R_0^4 C_{20\,20}^{20} \sum_{m\nu\mu_1\mu_2} C_{2m1M}^{2\nu} C_{2\mu_1 2\mu_2}^{2\nu} \dot{\alpha}_{2m} \overset{*}{\alpha}_{2\mu_1} \overset{*}{\alpha}_{2\mu_2}. \quad (16)$$

Equation (16) is valid for any model with a collective quadrupole excitation, and is valid as long as the current $\mathbf{j}_N$ is determined by the velocity (13). In the rotator model, the operators $\dot{\alpha}_{2m}$ and $\alpha_{2\mu}$ are acting on the angles of orientation of the deformed nucleus in space, while, in the vibrational model (harmonic or anharmonic), $\dot{\alpha}_{2m}$ and $\alpha_{2\mu}$ are expressed in terms of phonon creation and destruction operators. However, independent of the actual choice of the operators $\dot{\alpha}_{2m}$ and $\alpha_{2\mu}$, we find the ratio X for the collective models to be $X = 3/2$.

5. CONCLUSIONS

1. For collective M1 transitions in even-even nuclei, the ratio of the matrix elements of M1 transitions is $X = 3/2$, independent of the actual model used.

2. Since $\mathbf{A}_{in} \approx Ze^2 (R_0 Ze^2)^{2\gamma-1} \mathbf{A}_{ex}$ and

$X = {}^3/_2$, the correction to the internal conversion coefficient in collective M1 transitions of even nuclei has an order of magnitude of $Ze^2(R_0Ze^2)^{2\gamma-1}$, i.e., ~ 1%, and consequently no anomaly, as compared with the listed values of $(ICC)_0$, should be observed in the internal conversion coefficient of collective M1 transitions.

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