tion in the contribution of fast holes, and the second with that of electrons.

The form of the $V_x(H)$ dependence also changes with changing temperature. On increasing the temperature, the field corresponding to the first change decreases, while the field corresponding to the second increases. There is thus no first passing of the Hall constant through zero for $t > 60°$ C (curve 5). A decrease in the temperature leads to a reduction in the number of free electrons and to an increase in the mobility of the current carriers. As a result, the role of electrons in the Hall effect decreases, and the field range in which the influence of fast holes becomes appreciable narrows. Our results agree with the deductions from classical theory of galvanomagnetic phenomena not only qualitatively but quantitatively. In fact, if we take the concentration of light holes to be 5% of that of heavy holes, and the ratio of their mobilities to be 8.0, then the experimental field dependence of the Hall constant agrees well with theory. Numerical comparison was made for specimen No. 1 at 55, 62.5, and 76° C. The experiments show that in the temperature range studied the ratio of mobilities of "light" and "heavy" holes does not change, while the ratio of their concentrations is, to a first approximation, proportional to the absolute temperature.

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THE WEIZSÄCKER-WILLIAMS RELATION FOR MATRIX ELEMENTS

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FERMI[1] was evidently the first to use the analogy between the electromagnetic field of a moving charge and a radiation field. This method was developed by Weizsäcker[2] and Williams,[3] who obtained the spectrum of photons equivalent to the field of a charge. Recently there have been a number of papers in which authors have used this method (cf. Low,[4] and especially Pomeranchuk and Shmushkevich[5]).

In these papers, however, the method is applied only to cross sections averaged over polarizations. We shall show how a virtual quantum can be replaced by a real one directly in the matrix element.

Let us consider a diagram describing some process that occurs in the field of another particle (and with the diagram connected with the source of the field by one photon line). The momentum of the virtual photon is $q = P_1 - P_2$ (the difference of the four-momenta of the heavy particle before and after the collision). The Fourier components of such a field are given by

$$A(q) = -2\pi Zeq^{-2}P\delta(qP), \qquad (1)$$

where $P = P_1 + P_2$, and the δ function expresses the obvious relation $(P_1 - P_2)(P_1 + P_2) = 0$; we use throughout $ab = \mathbf{a}\cdot\mathbf{b} - a_0b_0$. In the system in which $\mathbf{P} = 0$, Eq. (1) goes over into the Fourier component of index 4 for the static Coulomb field.

We call one of the external lines of the diagram the incident particle [momentum $p(p_0, \mathbf{p})$, mass m]. In order for the expression (1) to go over into a free field, we must choose a new gauge. In the calculation of formulas with a free photon one usually chooses the gauge so that the photon will have no scalar component in some chosen coordinate system (usually in the system $\mathbf{p} = 0$). This condition can be written in the form $ep = 0$. Therefore let us choose a new polarization vector $\tilde{e}$ by the formula

$$\tilde{e} = [P - q(qp)^{-1}(pP)]f^{-1}, \quad f^2 = P^2 + q^2(qp)^{-2}(pP)^2, \qquad (2)$$

so that $\tilde{e}p = 0$. Then the field (1) is replaced by

$$A(q) = -2\pi Zeq^{-2}\delta(qP)f\tilde{e}. \qquad (3)$$

This formula gives the field of equivalent photons (amplitude and polarization).

It is not hard to get from it also the spectrum of equivalent photons. Let us determine the number of equivalent photons in the invariant interval d^4q:

$$I(\varkappa,\ q^2)d^4q = -\varkappa^{-1}\ m^{-2}\int T_{ik}\,p_i\,p_k\,dV d^4q, \qquad (4)$$

where T_{ik} is the energy-momentum tensor of the field (3), and $-\kappa = m^{-1}(pq)$ is a variable that plays the part of the energy of the quantum. The integration is over a spacelike hypersurface orthogonal to p. It is not hard to see that in the system $\mathbf{p} = 0$ this expression goes over into the ratio of the energy T_{00} of the field to the energy q_0 of one quantum, and consequently Eq. (4) gives an invariant definition of the number of equivalent photons. It is important to note that only with the gauge (2) is the number of photons proportional to the square of the amplitude of the field (as in the usual gauge for free photons). The calculations give

$$I(\varkappa,\ q^2) = \frac{Z^2\alpha}{2\pi^2}\frac{(q_1p)}{(pP)}f^2\left[1+\frac{m^2}{2(q_1p)^2}q_1^2\right]\frac{\delta(qP)}{q^4}. \qquad (5)$$

Here we have introduced q_1, which is the component of q orthogonal to $\tilde{e}$ and plays the part of the wave vector of the quantum:

$$q_1\tilde{e} = 0, \qquad q_1 = q - \tilde{e}\,(\tilde{e}\,q), \qquad qp = q_1p.$$

If the condition $q_1^2 \ll (q_1p)^2m^{-2}$ is satisfied, we can neglect the second term in the square brackets in Eq. (5). In the system $\mathbf{p} = 0$ this condition requires that the four-length of the "wave vector" be small in comparison with its energy. When this condition holds the spectrum can be written in the form

$$I(\varkappa,\ q^2) = \frac{Z^2\alpha}{2\pi^2}\frac{(pP)}{(q_1p)}\left(q^2 + P^2\frac{(qp)^2}{(pP)^2}\right)\frac{\delta(qP)}{q^4}. \qquad (6)$$

The spectrum can also be written in an invariant three-dimensional form.

To get rid of the δ function, we introduce

$$P = P_{\rm I} + P_{\rm II},\ P_{\rm I} = -m^{-2}(pP)p,\ P_{\rm II}p = 0.$$

Then

$$\delta(pP) = \delta[-m^{-2}(pP)(qp) + qP_{\rm II}].$$

We can integrate over the component of q parallel to p [integral over d(qp)]. After this there remains the integration over the three-dimensional space orthogonal to p, under the supplementary condition

$$m^{-1}(qp) = m(qP_{\rm II})(pP)^{-1} = -(qv),$$

where the vector $\mathbf{v} = -m\,(pP)^{-1}P_{\rm II}$ has three independent components (because of the condition $vp = 0$) and goes over into the ordinary velocity vector in the reference system in which $\mathbf{p} = 0$. The spectrum is finally written in the form

$$I(\varkappa,\ q^2)d^3q = \frac{Z^2\alpha}{2\pi^2}\frac{m}{(qv)}[q-(qv)v-m^{-1}(qv)p]^2\frac{d^3q}{q^4}$$

$$\text{with } qp = 0. \qquad (7)$$

It is not hard to see that for $\mathbf{p} = 0$ this expression agrees with the usual one (for $|\mathbf{v}| \sim 1$). The square-bracket expression in this case goes over into $\mathbf{q}_\perp^2$, where $\mathbf{q}_\perp$ is the three-dimensional vector orthogonal to $\mathbf{v}$.

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ON THE PARTICIPATION OF π^0 MESONS IN ELECTROMAGNETIC PROCESSES

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THE direct interaction of π^0 mesons with the electromagnetic field can be written in the form[1]

$$H_{int} = \frac{1}{\mu}\sqrt{\frac{8\pi}{\mu\tau}}\,\psi(x)\varepsilon_{\alpha\beta\gamma\delta}\frac{\partial A^\alpha(x)}{\partial x_\beta}\frac{\partial A^\gamma(x)}{\partial x_\delta} \qquad (1)$$

where τ is the mean life of the π^0 meson and μ its mass. From this follows that π^0 mesons can be produced in electromagnetic processes:

1) $e + e \to e + e + \pi^0$[2]; 2) $\gamma + e \to \pi^0 + \gamma + e$;

3) $e^+ + e^- \to \pi^0 + \gamma$; 4) $e^+ + e^- \to \pi^0$;

5) γ + nucleus $\to \pi^0$ + nucleus;

6) e + nucleus $\to \pi^0 + e$ + nucleus.

The last two processes are meant to take place in