This formula gives the field of equivalent photons (amplitude and polarization).

It is not hard to get from it also the spectrum of equivalent photons. Let us determine the number of equivalent photons in the invariant interval d^4q:

$$I(\varkappa,\ q^2)d^4q = -\varkappa^{-1}\ m^{-2}\int T_{ik}\,p_i\,p_k\,dV d^4q, \tag{4}$$

where T_{ik} is the energy-momentum tensor of the field (3), and $-\kappa = m^{-1}(pq)$ is a variable that plays the part of the energy of the quantum. The integration is over a spacelike hypersurface orthogonal to p. It is not hard to see that in the system $\mathbf{p} = 0$ this expression goes over into the ratio of the energy T_{00} of the field to the energy q_0 of one quantum, and consequently Eq. (4) gives an invariant definition of the number of equivalent photons. It is important to note that only with the gauge (2) is the number of photons proportional to the square of the amplitude of the field (as in the usual gauge for free photons). The calculations give

$$I(\varkappa,\ q^2) = \frac{Z^2\alpha}{2\pi^2}\frac{(q_1p)}{(pP)}f^2\left[1+\frac{m^2}{2(q_1p)^2}q_1^2\right]\frac{\delta(qP)}{q^4}. \tag{5}$$

Here we have introduced q_1, which is the component of q orthogonal to $\tilde{e}$ and plays the part of the wave vector of the quantum:

$$q_1\tilde{e} = 0, \qquad q_1 = q - \tilde{e}\,(\tilde{e}\,q), \qquad qp = q_1p.$$

If the condition $q_1^2 \ll (q_1p)^2m^{-2}$ is satisfied, we can neglect the second term in the square brackets in Eq. (5). In the system $\mathbf{p} = 0$ this condition requires that the four-length of the "wave vector" be small in comparison with its energy. When this condition holds the spectrum can be written in the form

$$I(\varkappa,\ q^2) = \frac{Z^2\alpha}{2\pi^2}\frac{(pP)}{(q_1p)}\left(q^2 + P^2\frac{(qp)^2}{(pP)^2}\right)\frac{\delta(qP)}{q^4}. \tag{6}$$

The spectrum can also be written in an invariant three-dimensional form.

To get rid of the δ function, we introduce

$$P = P_{\mathrm{I}} + P_{\mathrm{II}},\ P_{\mathrm{I}} = -m^{-2}(pP)p,\ P_{\mathrm{II}}p = 0.$$

Then

$$\delta(pP) = \delta[-m^{-2}(pP)(qp) + qP_{\mathrm{II}}].$$

We can integrate over the component of q parallel to p [integral over d(qp)]. After this there remains the integration over the three-dimensional space orthogonal to p, under the supplementary condition

$$m^{-1}(qp) = m(qP_{\mathrm{II}})(pP)^{-1} = -(qv),$$

where the vector $\mathbf{v} = -m\,(pP)^{-1}P_{\mathrm{II}}$ has three independent components (because of the condition $vp = 0$) and goes over into the ordinary velocity vector in the reference system in which $\mathbf{p} = 0$. The spectrum is finally written in the form

$$I(\varkappa,\ q^2)d^3q = \frac{Z^2\alpha}{2\pi^2}\frac{m}{(qv)}[q-(qv)v-m^{-1}(qv)p]^2\frac{d^3q}{q^4}$$

$$\text{with } qp=0. \tag{7}$$

It is not hard to see that for $\mathbf{p} = 0$ this expression agrees with the usual one (for $|\mathbf{v}| \sim 1$). The square-bracket expression in this case goes over into $\mathbf{q}_\perp^2$, where $\mathbf{q}_\perp$ is the three-dimensional vector orthogonal to $\mathbf{v}$.

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ON THE PARTICIPATION OF π^0 MESONS IN ELECTROMAGNETIC PROCESSES

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THE direct interaction of π^0 mesons with the electromagnetic field can be written in the form[1]

$$H_{int} = \frac{1}{\mu}\sqrt{\frac{8\pi}{\mu\tau}}\,\psi(x)\varepsilon_{\alpha\beta\gamma\delta}\frac{\partial A^{\alpha}(x)}{\partial x_{\beta}}\frac{\partial A^{\gamma}(x)}{\partial x_{\delta}} \tag{1}$$

where τ is the mean life of the π^0 meson and μ its mass. From this follows that π^0 mesons can be produced in electromagnetic processes:

1) $e + e \to e + e + \pi^0$[2]; 2) $\gamma + e \to \pi^0 + \gamma + e$;

3) $e^+ + e^- \to \pi^0 + \gamma$; 4) $e^+ + e^- \to \pi^0$;

5) $\gamma +$ nucleus $\to \pi^0 +$ nucleus;

6) $e +$ nucleus $\to \pi^0 + e +$ nucleus.

The last two processes are meant to take place in the Coulomb field of the nucleus. The cross sections of these processes are generally small because the mean life of the π^0 meson is large compared with the characteristic time of the electromagnetic interaction. So the cross section of process 1) (see reference 2) at an electron energy of 300 Mev equals roughly 10^{-6} of the elastic scattering cross section.

However, if a π^0 propagator is contained in the amplitude for another process, poles will appear, of which some may lie in the physical region. Then the diagrams containing these singularities can dominate in the expression for the cross sections of that particular process. All such diagrams characteristically contain a part that corresponds to photon-photon scattering via a π^0 meson. Such a part will be contained in the diagrams of the following processes:

a) $e^+ + e^- \to 3\gamma$; b) $e + \gamma \to e + 2\gamma$; c) $e^+ + e^- \to 2\gamma$;

d) $e + e \to e + e$; e) $\gamma + e \to \gamma + e$; f) $e + e \to e + e + 2\gamma$.

The photon-photon scattering was investigated by Oraevskii,[3] who showed that there exists a sharp maximum in the cross section for this process at a photon energy $\omega = \mu/2$. Analogous maxima will exist also in the cross sections of processes c) and d) (in the latter case — in the annihilation diagram of electron-positron scattering). However, due to the small probability of a decay of the π^0 meson into two electrons the contribution of the π^0 intermediate state to these processes is smaller than the purely electrodynamic part even at its peak. We assume here and later that the "widths" of the π^0 "level" are determined by its experimental mean life.

The situation is completely different for the processes a) and b). Here the cross section of the process going via the π^0 meson contains just the additional factor $e^2 = 1/137$ as compared to the case of photon-photon scattering. As a result of this the contribution of the diagrams going via a π^0 meson will dominate the cross sections in the vicinity of the pole of the π^0 propagator. For example, the cross section for the three photon annihilation of a pair at an energy of the positron or electron in the center-of-mass system (c.m.s.) of 140 Mev has the order of magnitude of 10^{-29} — 10^{-30} cm^2/sr. This is 5 — 6 orders of magnitude larger than the corresponding electrodynamic cross section and has the same order of magnitude as the two-photon annihilation cross section. As collision processes which have three particles in the final state are characterized by five parameters the maximum of the scattering amplitude for process a) will lie on the line $\omega = \epsilon - \mu^2/4\epsilon$ (ω — energy of one of the final photons, ϵ — energy of the electron or positron in the c.m.s.) and for process b) on the line $\epsilon_2 = \omega - \mu^2/4\omega$ (ω — energy of the incoming photon in the c.m.s., ϵ — energy of the final electron).

In processes which do not have an annihilation character, and also in the process e), the pole of the π^0 propagator lies in the unphysical region. For example, for process e) it lies at c.m.s. scattering angle with $\cos\vartheta = 1 + \mu^2/2\omega^2$.

Amongst the possible π^0 production processes the process 3) is of particular interest. It is not masked by photonuclear production processes like the processes 5) and 6); it has a larger cross section than processes 1) and 4), and it is easier to observe than process 2). Its cross section in the c.m.s. is given by

$$\sigma(\vartheta) = (e^2/4\pi\mu^3)(q/\varepsilon)^3(1 + \cos^2\vartheta)f^2(\varepsilon^2), \tag{2}$$

where q — momentum of the π^0 meson, ϑ — angle between the momenta of the initial and the final particles, $f(\epsilon^2)$ — form factor of the π^0 meson. The latter makes its appearance because the interaction (1) is not a "fundamental" interaction and, generally speaking, has to be chosen in a nonlocal form. At an energy of 130 Mev the process has a cross section of the order of 10^{-36} cm^2.

Recently the possibility of the existence of a neutral vector meson of mass $M \sim 3\mu$ has been widely discussed. It has been proposed by Nambu[4] to explain the isoscalar part of Hofstadters charge distributions. In this connection we would like to remark that the existence of such a meson could be checked by means of the reaction $e^+ + e^- \to \pi^0 + \gamma$. In this case in the reaction cross section there would appear a resonance denominator of the form $[(2\epsilon - M)^2 + \Gamma^2/4]$ (Γ — "width of the level"). If the mean life of that meson with respect to the decay into $\pi^0 + \gamma$ would be 10^{-20} — 10^{-21} sec then the reaction cross section averaged over an interval of 5 Mev at the energy containing the resonance would be of order 10^{-27} to 10^{-28} cm^2 which is considerably larger than all electrodynamic cross sections at such an energy.

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