QUANTIZED CYCLOTRON RESONANCE IN METALS

I. M. LIFSHITZ

Khar'kov State University

Submitted to JETP editor March 4, 1961

J. Exptl. Theoret. Phys. (U.S.S.R.) **40**, 1235-1236 (April, 1961)

IN metals with a complicated dispersion law for the conduction electrons there is a continuous spectrum of values for the effective masses and, consequently, a whole range of natural frequencies of rotation of the electrons in a magnetic field $H_z = H$ ($\omega_H = \mu H$; $\mu = e/mc$; $2\pi m = \partial S/\partial\epsilon$, where $S = S(\epsilon, p_z)$ is the cross-section area of the isoenergetic surface by the plane p_z = const).[1]

Under the conditions for cyclotron resonance (i.e., in a magnetic field parallel to the surface of the metal and in a high-frequency electric field of frequency ω),[2] electrons of mass m contribute to all the high frequency characteristics of the metal a term with a "resonance" denominator $\omega - \mu H + i/\tau$ (τ is the relaxation time). This means that the corresponding characteristics (such as the conductivity of surface impedance) are of the form, for $\omega\tau \gg 1$

$$M = \int \frac{\chi(\mu)\, d\mu}{\omega - \mu H + i/\tau} \approx -i\frac{\pi}{H}\chi\left(\frac{\omega}{H}\right) + \int \frac{\chi(\mu)\, d\mu}{\omega - \mu H}, \quad (1)$$

where $\chi(\mu)$ is a smooth function of μ over the whole range of variation of effective masses, except for points where the density of states has a singularity or a discontinuity; in particular near the extremal values $\mu = \mu_g$: $\chi(\mu) \sim (\mu - \mu_g)^{-1/2}$. As can be seen from (1) it is just these singularities $\omega_g = \mu H$ which are also resonances for M, and for all other natural frequencies M is a smooth function of ω.

At low temperatures ($kT \ll \mu\hbar H \ll \epsilon_0$) when the integration in (1) is only carried out for electrons at the Fermi surface $\epsilon = \epsilon_0$, quantization of p_z in a magnetic field leads to discrete values of the effective mass and, as a result, to the replacement of (1) by a sum of the form

$$M' = \sum_n \frac{\chi(\mu_n)\, \delta\mu_n}{\omega - \mu_n H + i/\tau}. \quad (2)$$

By considering the cross section area of the Fermi surface as a function of μ ($S = S(\mu)$), we can express the condition for quasi-classical quantization as

$$S(\mu_n) = ehnH/c.$$

From this $\delta\mu = ehH/c\,(\partial S/\partial\mu)$, and near the point $\mu H = \omega$

$$\delta\mu/\mu = h\omega/(\partial S/\partial m). \quad (3)$$

If the relative distance between the levels $\delta\mu/\mu$ is of the same order as or larger than the attenuation constant $1/\omega\tau$, i.e.,

$$h\omega/(\partial S/\partial m) \lesssim 1/\omega\tau, \quad (4)$$

then the separate frequencies $\mu_n H$ in the sum (3) are split and M′ is no longer a smooth function of ω/H. A simple calculation gives*

$$M' = \frac{\pi}{H}\chi(\mu)\,\mathrm{ctg}\,\pi\left\{\frac{cS(\mu)}{ehH} + i\frac{\mu}{\omega\tau\delta\mu}\right\}\Big|_{\mu=\omega/H} + \int \frac{\chi(\mu)\, d\mu}{\omega - \mu H}. \quad (5)^*$$

For $\mu/\omega\tau\delta\mu \gg 1$, $\cot \pi\{\ldots\} \to -i$ and (5) goes over into (1). On the other hand, for $\mu/\omega\tau\delta\mu \lesssim 1$, M′ has oscillations of large amplitude $\Delta M'/M' \sim (\delta\mu/\mu)\,\omega\tau$ which correspond to resonance at each of the discrete frequencies $\omega_n = \mu_n H$. The periods of these oscillations, in the inverse of the magnetic field Δ are

$$\Delta = \frac{eh}{c} \Big/ \left(S + \mu\frac{\partial S}{\partial\mu}\right)\Big|_{\mu=\omega/H}. \quad (6)$$

The period Δ is thus a function of the magnetic field.

By measuring the $\Delta(\omega/H)$ dependence we can obtain a variety of information about the Fermi surface, in particular we can find the area of any cross-section as a function of the inverse of the mass μ:

$$\mu S(\mu) = \frac{eh}{c}\int_{\mu_0}^{\mu} \frac{d\mu}{\Delta(\mu)}, \qquad S(\mu_0) = 0.$$

Condition (4) is most easily fulfilled for the "anomalous" electron groups with long period oscillations in the de Haas–van Alphen effect (in this case it is apparently fully realized for frequencies $\omega \sim 10^{11}\ \mathrm{sec}^{-1}$). It is still not quite clear for which cases it can be realized for the normal electron groups.

We should remark that the expression $\partial S/\partial m$ appearing in the denominators of (3) and (4) can, in general, change sign and be zero at certain points ($\partial S/\partial m = v_z^0/(\partial \ln m/\partial p_z)$; $v_z^0(p_z)$ is the mean value of v_z along the contour p_z = const). Near points where $v_z^0 = 0$, $\partial m/\partial p_z \neq 0$, $\partial S/\partial m$

*If the $S(\mu)$ dependence has several branches, then the sum in (5) must be taken over all these branches.

*ctg = cot.

will be $\sim m - m_0$, and the relative distance between levels will be $\delta\mu/\mu \sim \sqrt{\hbar\omega/\epsilon_0}$. Near the critical points p_z^k, corresponding to self-intersecting trajectories (of the figure-of-eight type), $m \sim -\ln|p_z - p_z^k|$ (see reference (1)), and a simple calculation gives $\delta\mu/\mu \sim 1/\ln(\epsilon_0/\hbar\omega)$. In these cases the condition (4) goes over to a considerably weaker condition $\omega\tau \gtrsim \ln(\epsilon_0/\hbar\omega)$, which is almost always realized.

The oscillations connected with the splitting of the resonance frequencies differ considerably from the usual quantum oscillations of the de Haas–van Alphen and Shubnikov–de Haas effect, and also from the high frequency conductivity oscillations considered by Azbel'. All these oscillations have a universal period, independent of magnetic field, $\Delta = eh/cS_{extr}$, determined by the extremal Fermi surface cross-section; the form of these oscillations is also described by a single universal function.[3] In calculating the sum in (2), these oscillations arise from terms lying in the region of singular points in the density of states $\nu(p_z)dp_z = \nu(p_z)(dp_z/d\mu)d\mu$, i.e., singular points of the function $\chi(\mu)$. These points, naturally, are independent of H and ω and correspond to extremal cross-sections S_{extr}. The oscillations determined by them have very small amplitude and can be neglected if condition (4) holds, as has been done in (5). On the other hand, for the opposite limiting case $\delta\mu/\mu \ll 1/\omega\tau$, when there is no splitting of the resonance frequencies, quantum effects enter only into these small oscillations.

[1] I. M. Lifshitz and M. I. Kaganov, Usp. Fiz. Nauk **69**, 419 (1959), Soviet Phys.-Uspekhi **2**, 831 (1960).

[2] M. Ya. Azbel' and É. A. Kaner, JETP **30**, 811 (1956) and **32**, 896 (1957), Soviet Phys. JETP **3**, 772 (1956) and **5**, 730 (1957).

[3] I. M. Lifshitz and A. M. Kosevich, JETP **29**, 730 (1955), Soviet Phys. JETP **2**, 636 (1956); Izv. Akad. Nauk SSSR, Ser. Fiz. **19**, 395 (1955), Columbia Tech. Transl. p. 353; I. M. Lifshitz, JETP **32**, 1507 (1957), Soviet Phys. JETP **5**, 1227 (1957); M. Ya. Azbel', JETP **34**, 969 (1958), Soviet Phys. JETP **7**, 669 (1958).

Translated by R. Berman
210