QUANTUM COUNTERS

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Noise and characteristic transient times in quantum counters are considered.

THE sensitivity of detectors of long-wave radiation can be increased by converting the photons to high-frequency photons in the visible or ultraviolet.[1] It is of interest to analyze the operation of devices of this kind in some detail.

Consider the system shown in the figure. If this system is at a very low temperature, only the lower level (1) is populated. When radiation at a frequency $\nu_{21} = (E_2 - E_1)/h$ is applied to the system level 2 is populated. Suppose there is auxiliary radiation at a frequency $\nu_{32} = (E_3 - E_2)/h$. This radiation causes the transfer of population from level 2 to level 3; because of spontaneous emission, there is then a transfer to the lower level 1, with the emission of photons of frequency ν_{31}. In this way, photons characterized by $h\nu_{21}$ are converted to photons characterized by $h\nu_{31}$.

Let n_i be the population in the i-th level while n is the total population in the system; w_{ik} and w_{ki} are the probabilities for transitions from the i-th level to the k-th level and vice versa, $W_{ik} = W_{ki}$ is the probability for a transition from the i-th level to the k-th level under the effect of the external radiation. We assume that the probabilities w_{13} and w_{23} are so small that they can be set equal to zero, while w_{31} is simply the probability for spontaneous emission; it is assumed that this last process predominates in the present case.

The population equations for levels 1 and 3 are written in the form

$$dn_1/dt = a_{11}n_1 + a_{12}n_3 - (a_{11} + w_{12} + W_{12})\, n,$$
$$dn_3/dt = a_{21}n_1 + a_{22}n_3 - a_{21}n, \qquad (1)$$

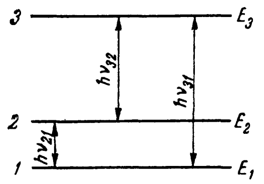
where

$$a_{11} = -2W_{12} - w_{12} - w_{21}, \qquad a_{12} = w_{31} - w_{21} - W_{12},$$
$$a_{22} = -2W_{32} - w_{31} - w_{32}, \qquad a_{21} = -W_{32}.$$

The solution of the system in (1) is

$$n_1 = Ae^{\alpha_1 t} + Be^{\alpha_2 t} + n_1^0,$$
$$n_3 = A\frac{\alpha_1 - a_{11}}{a_{12}}e^{\alpha_1 t} + B\frac{\alpha_2 - a_{11}}{a_{12}}e^{\alpha_2 t} + n_3^0, \qquad (2)$$

where A and B are constants which are deter-



mined from the initial conditions while α_1, α_2, n_1^0 and n_3^0 have the following meaning:

$$\alpha_{1,2} = \tfrac{1}{2}(a_{11} + a_{22}) \pm \left[\tfrac{1}{4}(a_{11} - a_{22})^2 + a_{12}a_{21}\right]^{1/2},$$
$$n_1^0 = [1 + a_{22}(w_{12} + W_{12})/(a_{11}a_{22} - a_{12}a_{21})]\, n,$$
$$n_3^0 = -a_{21}(w_{12} + W_{12})\, n\, /\, (a_{11}a_{22} - a_{12}a_{21}). \qquad (3)$$

We consider stationary states of the system. Let w_{12}, w_{21}, $W_{12} \ll w_{31} \ll W_{32}$; $w_{32} \ll W_{32}$ so that the population in level 3 is

$$n_3^0 = (w_{12} + W_{12})\, n/w_{31}.$$

The number of photons emitted spontaneously from level 3 to level 1 is

$$N = n_3^0 w_{31} = (w_{12} + W_{12})\, n. \qquad (4)$$

It follows from Eq. (4) that when $W_{12} = 0$ there is a "dark" radiation background; in this case the number of emitted photons is

$$N_d = nw_{12}. \qquad (5)$$

If the minimum number of photons of frequency ν_{31} recorded by the detection device is N_{min}, the sensitivity of the quantum counter is a maximum when the condition $N_{max} \ll N_{min}$ is satisfied; in accordance with Eq. (5), this means when

$$nw_{12} \ll N_{min}. \qquad (6)$$

The population required in the system n is determined from the condition that every photon $h\nu_{21}$ be absorbed. If the absorption coefficient is α, this condition can be written in the form

$$\alpha l = 1, \qquad (7)$$

where l is the path length traversed by the photon.

From Eq. (7) we find the number of particles

$$n = Shc\Delta\nu / 8\pi^2 \nu_{21} |\mu_{12}|^2, \qquad (8)$$

where $\Delta\nu$ is the line width, μ_{12} is the matrix element for the transition between levels 1 and 2 and S is the area of the sample.

The expression in (8) is obtained under the assumption that the radiation passes through the system only once. If the system is a resonator with plane-parallel walls and a quality factor Q, we may say that the photon traverses an effective path length

$$l^* = Q\lambda / 2\pi. \qquad (9)$$

The quality factor Q is given by the formula[2]

$$Q = 2\pi l / \lambda (1 - k), \qquad (10)$$

where k is the reflection coefficient. From Eqs. (9) and (10) we have

$$l^* = l / (1 - k).$$

Consequently, the population found from Eq. (8) must be multiplied by the factor $(1-k)$. It is obvious that the smaller the value we obtain for n the larger the value we can take for w_{12} to satisfy the condition in (6).

We now consider the quantity w_{12} in greater detail. If $W_{23} = 0$, the relaxation processes between levels 1 and 2 take place without the participation of level 3. In a two-level system of this kind the relaxation time T_1 is given by the expression

$$T_1 = 1 / (w_{12} + w_{21}).$$

Since $w_{12} = w_{21} \exp\{-h\nu_{21}/kT\}$ then $T_1 \approx 1/w_{21}$ and the quantity w_{12} can be written in the form

$$w_{12} = T_1^{-1} \exp\{-h\nu_{21} / kT\}. \qquad (11)$$

If the temperature T is given, in order to reduce w_{12} we must choose a system with a long relaxation time between levels 1 and 2. We may note that it may be possible to achieve the required value of w_{12} by lowering the temperature.

According to Eq. (4), the number of radiated photons is proportional to the signal power if $W_{12} \gg w_{12}$.

We now consider transient processes in the system. As we have shown above, for good sensitivity the quantities w_{12} and w_{21} must be small. If we neglect W_{12}, w_{21}, and w_{12}, in accordance with Eq. (3) the quantities $\alpha_{1,2}$ assume the following values:

$$\alpha_{1,2} = -\tfrac{1}{2}(2W_{32} + w_{32} + w_{31}) \pm \left[W_{32}^2 + w_{32}W_{32} + \tfrac{1}{4}(w_{32} + w_{31})^2\right]^{1/2}.$$

We consider two particular cases:

1) If $W_{32} \ll w_{31}$, w_{32} then

$$\alpha_1 = -w_{31}W_{32}/(w_{31} + w_{32}), \qquad \alpha_2 = -(w_{31} + w_{32}). \qquad (12)$$

2) If $W_{32} \gg w_{31}$, w_{32} then

$$\alpha_1 = -w_{31}/2, \qquad \alpha_2 = -2W_{32}. \qquad (13)$$

Thus, under the most favorable conditions, where $W_{32} \gg w_{31}$, w_{32}, the rate at which the steady state is established is determined by the slowest exponential factor, i.e., the quantity $\alpha_1 = -w_{31}/2$; in this case, the relaxation time for the transient processes is $\tau = 2/w_{31}$. Consequently the relaxation time for the transient processes is always greater than the lifetime of the system in the excited state.

[1] N. Bloembergen, Phys. Rev. Letters **2**, 84 (1959).

[2] A. M. Prokhorov, JETP **34**, 1658 (1958), Soviet Phys. JETP **7**, 1140 (1958).

Translated by H. Lashinsky
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