ANGULAR CORRELATIONS IN INELASTIC SCATTERING OF HIGH-ENERGY NUCLEONS

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Angular correlations for the inelastic scattering of high-energy nucleons by nuclei with zero spin and zero isotopic spin are examined. The calculation is carried out in the impulse approximation at small angles. It is shown that the correlation function and its dependence on the nucleon scattering angle are mainly determined by the parity and isotopic spin of the excited level.

A great amount of data has accumulated in recent years on the polarization of high-energy protons in the inelastic scattering by nuclei with excitation of the low-lying levels.[1] Some of the features of these experimental results have been successfully explained by Kerman, McManus, and Thaler[2] with the help of the impulse approximation. The experiments show that in a number of cases the angular distribution of the polarization of the inelastic group of protons is the same or almost the same as for the elastic group. This fact has been regarded as a confirmation of the assumption of the rotational nature of the excited levels, since the Born approximation calculations with a nonspherical optical model potential yield the same polarization for the elastic and inelastic scattering.[3] However, it was shown in reference 2 that the observed regularities have a more general character. In the impulse approximation, which is valid for high energies, the amplitude for the inelastic scattering of the nucleons by nuclei is expressed in terms of the amplitude for nucleon-nucleon scattering and the reduced nuclear matrix elements, which play the role of phenomenological parameters.

In the impulse approximation, the observed similarity between the polarization of the groups of elastically and inelastically scattered nucleons for even-even nuclei connected with the excitation of a level with $\pi = (-)^J$ (J is the spin of the level) is explained in a natural way by the smallness of the ratio of the reduced matrix element with spin flip over the matrix element without spin flip, independently of the nature of the excited level. One can also explain the small polarization connected with the excitation of levels of even-even nuclei with the parity $\pi = (-)^{J+1}$ and a number of other features. It is, therefore, of interest to consider the predictions of the theory of inelastic scattering in the impulse approximation with regard to the p-γ correlations. The investigation of the p-γ correlations in the inelastic scattering of high-energy nucleons is of interest not only as a test of the theory of inelastic scattering, but also from the point of view of using this process for the determination of the spectroscopic properties of the levels. This refers, in particular, to the isotopic spin, which does not play an important role in low energy scattering experiments.

In the first nonvanishing approximation, the inelastic scattering amplitude T is related to the nucleon-nucleon scattering amplitude M in the following way (we consider small angle scattering):

$$T = -\frac{\hbar^2}{2\pi^2 m} N\overline{M}, \qquad \overline{M} = \langle f | Me^{-i\mathbf{QR}} | i\rangle, \tag{1}$$

$$M = A + B(\boldsymbol{\sigma}_0\mathbf{n})(\boldsymbol{\sigma}_1\mathbf{n}) + C(\boldsymbol{\sigma}_0\mathbf{n} + \boldsymbol{\sigma}_1\mathbf{n}) + E(\boldsymbol{\sigma}_0\mathbf{q})(\boldsymbol{\sigma}_1\mathbf{q}) + F(\boldsymbol{\sigma}_0\mathbf{p})(\boldsymbol{\sigma}_1\mathbf{p}). \tag{2}*$$

Here N is the number of particles in the nucleus, f and i are the labels of the excited and ground states of the nucleus,*

$$\mathbf{q} = \mathbf{Q}/Q = (\mathbf{k}_0' - \mathbf{k}_0)/|\mathbf{k}_0' - \mathbf{k}_0|,$$

$$\mathbf{n} = [\mathbf{k}_0\mathbf{k}_0']/|[\mathbf{k}_0\mathbf{k}_0']|, \quad \mathbf{p} = [\mathbf{qn}],$$

and $\mathbf{k}_0$ and $\mathbf{k}_0'$ are the wave vectors of the incident and scattered nucleons. We have omitted the index for the total isotopic spin t = 0, 1 in the system of the two nucleons in the quantities A, B, C, etc. in (2). In place of the quantities A_0 and A_1, B_0 and B_1, etc. it is convenient to introduce the following linear combinations: $A(0) \equiv \frac{1}{4}(3A_1 + A_0)$, $A(1) \equiv \frac{1}{4}(A_1 - A_0)$ and analogously for B, C, etc. The quantities A, B, etc., also depend on Q^2. The lower indices 0 and 1 of the spin operators $\boldsymbol{\sigma}$ denote the nucleons.

*$(\boldsymbol{\sigma}_0\mathbf{p}) = \boldsymbol{\sigma}\cdot\mathbf{p}$; $[\mathbf{qn}] = \mathbf{q}\times\mathbf{n}$.

We shall use a coordinate system with the z axis along **q** and the x and y axes along the vectors **n** and **p**, respectively. It is convenient to separate out the components of M which transform according to an irreducible representation of the rotation group:

$$M = M^{(0)} + \sum_{\mu} (-)^{\mu} M^{(1)}_{-\mu}\sigma^{(1)}_{1\mu}, \qquad \sigma^{(1)}_{10} = \sigma_{1z},$$
$$\sigma^{(1)}_{1,\pm 1} = \mp (\sigma_{1x} \pm i\sigma_{1y}) \, / \, \sqrt{2}\,.$$

In our system of coordinates, we have

$$M^{(0)} = A + C\boldsymbol{\sigma}_0\mathbf{n}, \qquad M^{(1)}_1 = -(C + B\boldsymbol{\sigma}_0\mathbf{n} + iF\boldsymbol{\sigma}_0\mathbf{p})/\sqrt{2},$$
$$M^{(1)}_0 = E\boldsymbol{\sigma}_0\mathbf{q}, \qquad M^{(1)}_{-1} = (C + B\boldsymbol{\sigma}_0\mathbf{n} - iF\boldsymbol{\sigma}_0\mathbf{p})/\sqrt{2}.$$

We then obtain the following expression for M in terms of the reduced matrix elements:

$$M = \sum_{\alpha=0,1} M^{(0)}_{(\alpha)} \sum_{l} \frac{(J_0M_0l0\,|\,JM)}{(2J+1)^{1/2}} N_{l\alpha}$$
$$+ \sum_{\mu} (-)^{\mu} M^{(1)}_{-\mu}(\alpha) \sum_{l,k} \frac{(1\mu l0\,|\,k\mu)(J_0M_0k\mu\,|\,JM)}{(2J+1)^{1/2}} Q_{kl\alpha}\,. \tag{3}$$

Here the quantities $M(\alpha)$ are expressed in terms of $A(\alpha)$, $B(\alpha)$, etc., and

$$N_{l0} = \langle JT \,\|\, \rho_l Y \,\|\, J_0T_0\rangle, \quad N_{l1} = f(TT_0)\,\langle JT \,\|\, \rho_l Y_l \tau^{(1)} \|\, J_0T_0\rangle, \tag{4}$$
$$Q_{kl0} = \langle JT \,\|\, \rho_l T_k(l) \,\|\, J_0T_0\rangle,$$
$$Q_{kl1} = f(TT_0)\,\langle JT \,\|\, \rho_l T_k(l)\; \tau^{(1)} \|\, J_0T_0\rangle;$$
$$e^{-i\mathbf{QR}} = \sum_{l} \rho_l Y_{l0}, \quad e^{-i\mathbf{QR}}\sigma^{(1)}_{1\mu} = \sum_{lk} (1\mu l0\,|\,k\mu)\, \rho_l T_{k\mu}(l), \tag{5}$$
$$f(TT_0) = \pm (T_0T_310\,|\,TT_3)/(2T+1)^{-1/2}$$

(the plus sign corresponds to protons, the minus sign, to neutrons); J_0, M_0, and T_0 are the spin, its projection, and the isotopic spin of the initial state of the nucleus; J, M, and T are the corresponding quantities for the final state of the nucleus. Processes involving charge exchange are excluded in the expression for $f(TT_0)$.

As is known, the angular distribution of the γ quanta emitted by a system of oriented nuclei is connected with the spin tensors defining the polarization of the nuclei in the following way:

$$w(\vartheta, \varphi) = \sum_{LL'Fq} (2F+1)^{-1/2} G_F(LL'J_fJ)\, C_L C_{L'} \rho_{Fq} Y_{Fq}(\vartheta, \varphi),$$
$$G_F(LL'J_fJ) = (-)^{J_f - J - 1}(2J+1)^{1/2}(2L+1)^{1/2}(2L'+1)^{1/2}$$
$$\times (L1L' - 1\,|\,F0)\, W(JJLL';\; FJ_F), \tag{6}$$

where the W are the Racah coefficients. The multipole transitions $L(L')$ with amplitude $C_L(C_{L'})$ go from a level with spin J to a level with spin J_f. Here ρ_{Fq} are spin tensors which define the polarization of the nuclei.

The problem therefore reduces to the calculation of the spin tensors ρ_{Fq}, for which we have the expression

$$\rho_{Fq} I_0 = \sum_{M_0MM'} (-)^{J-M'}(JMJ - M'\,|\,Fq)\ \mathrm{Sp}\ \overline{M}_{M_0M}\overline{M}^{+}_{M_0M'}\,,$$

where

$$I_0 = \sum_{M_0M} \mathrm{Sp}\ \overline{M}_{M_0M}\overline{M}^{+}_{M_0M}$$

agrees with the cross section for the process except for a factor.

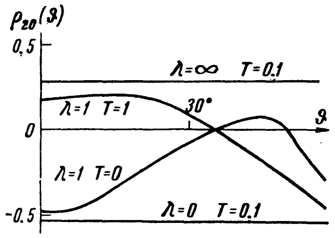
Let us consider the angular correlations for a nucleus with vanishing spin and isotopic spin, $J_0 = T_0 = 0$. In this case the angular correlations are essentially determined by the parity and isotopic spin of the excited level. The selection rules permit the values $T = 0, 1$ for the isotopic spin of the level. For $T = 0$ only the terms with $\kappa = 0$ are different from zero, and for $T = 1$ the only nonvanishing terms are those with $\alpha = 1$. Furthermore, if the parity of the level is "normal," $\pi = (-)^J$, we have the selection rule $l = J$ for the reduced matrix elements without spin flip and $l = k = J$ for the matrix elements with spin flip. On the other hand, if the parity of the level is "anomalous," $\pi = (-)^{J+1}$, the matrix elements without spin flip vanish, and the matrix elements with spin flip are subject to the selection rule $k = J$, $l = J-1, J+1$. As a result we obtain the following expressions for the spin tensors defining the polarization of nuclei with an excited level with parity $\pi = (-)^J$:

$$\rho_{F0} = (-)^J (J0J0\,|\,F0)\,[|A|^2 + |C|^2 - \tfrac{1}{2}(|B|^2 + |C|^2$$
$$+ |F|^2)\,\lambda\ (J1J-1\,|\,F0)/(J0J0\,|\,F0)]\,K^{-1},$$
$$\lambda = |Q_{JJ\alpha}|^2/|N_{J\alpha}|^2,$$
$$K = |A|^2 + |C|^2 + \tfrac{1}{2}(|B|^2 + |C|^2 + |F|^2)\,\lambda; \tag{7}$$

$$\rho_{F1} = i(-)^J\sqrt{2}\,(11J0\,|\,J1)(J0J1\,|\,F1)\,|AC^*$$
$$+ CB^*|\sqrt{\lambda}\,K^{-1}\sin(\Phi - \Phi_0), \tag{8}$$
$$\Phi = \arg(AC^* + CB^*), \qquad \Phi_0 = \arg N_{J\alpha} - \arg Q_{JJ\alpha};$$

$$\rho_{F2} = \frac{(-)^{J+1}}{2}(J1J1\,|\,F2)(11J0\,|\,J1)^2(|B|^2$$
$$+ |C|^2 - |F|^2)\,\lambda K^{-1}. \tag{9}$$

F takes the value 2 for dipole transitions (electric and magnetic) and the values 2, 4 for electric quadrupole transitions. The value $\rho_{00} = (2J+1)^{-1/2}$ follows from the normalization of the density matrix. The quantities ρ_{F0} and ρ_{F2}, which are measured directly in experiment, are real and depend, just like the polarization of the elastically scattered nucleons, on the single parameter λ, the ratio of the reduced matrix element with spin flip over the



ρ_{20} as a function of the scattering angle of the nucleon at the energy 156 Mev. The 2^+ level is excited. The curves refer to the isotopic spin values T = 0 and T = 1 and three values of λ: 0, 1, and ∞. For λ = 1, the curves corresponding to different values of the isotopic spin are very different from each other.

reduced matrix element without spin flip. The quantity ρ_{F1} depends, owing to the interference of the last two processes, also on the additional parameter Φ_0, which, however, like λ, is independent of the scattering angle of the nucleon. The angular dependence is contained in the quantities A, B, etc., which have been derived by Kerman, McManus, and Thaler[2] as functions of the scattering angle in the center of mass system of the two nucleons.

Thus the measurement of the polarization of the inelastically scattered nucleons allows us to determine the spin tensors, i.e., the form of the correlation function.

The investigation of the polarization of the protons in the inelastic scattering from the levels 4.43 Mev of the nucleus C^{12} and 6.14 Mev of the nucleus O^{16} indicates that the parameter λ is small.[2] The spin tensors ρ_{F1} and ρ_{F2} must in this case be small to explain the angular correlations, and ρ_{F0} (F = 2, 4) is close to the value $(-)^J(J0J0 \mid F0)$ and depends weakly on the scattering angle of the proton.

In the case when the reduced matrix elements with spin flip are of the same order as those without spin flip, the spin tensors depend strongly on the isotopic spin of the excited level, more strongly than the polarization of the nucleons. This situation is illustrated by the figure, where we show the dependence of the 2^+ level on the scattering angle of the nucleon for T = 0 and 1 (the nucleus is regarded as infinitely heavy). If λ = 1, the values of ρ_{20} for T = 0 and T = 1 are radically different. In the two opposite limits λ = 0 and λ = ∞, the dependence on T disappears.

For transition with "anomalous" change of parity, the spin tensors have the form

$$\rho_{F0} = (-)^{J+1}(J1J-1 \mid F0)(|B|^2+|C|^2+|F|^2)(1-\mu)K'^{-1}, \qquad (10)$$

$$\rho_{F1} = 0,$$

$$\rho_{F2} = \frac{(-)^J}{2}(J1J1 \mid F2)(|B|^2+|C|^2-|F|^2)(1-\mu)K'^{-1};$$

$$K' = (|B|^2+|C|^2+|F|^2)(1-\mu)+2|E|^2\mu,$$

$$\mu = \left(\sum_l \frac{|Q_{Jl\alpha}|^2}{2l+1}\right)^{-1} \sum_{ll'} \frac{(10l0 \mid J0)(10l'0 \mid J0)}{2J+1} Q_{Jl\alpha} Q^*_{Jl'\alpha}. \qquad (11)$$

In the case of transitions with anomalous change of parity, the spin tensors also depend only on the single parameter μ. ρ_{F1} is in this case identically equal to zero, and the term with sin φ in the correlation function is absent.

The formulas above are, of course, also applicable to those cases where the levels are de-excited by the emission of an α particle or a β particle instead of a γ ray. Only formula (6) will have to be modified.

Our discussion has shown that the measurement of the correlation function in the inelastic scattering of high-energy nucleons yields information on the parity of the excited states. That is, if we observe a term of the form $P_{21}(\cos\vartheta)\sin\varphi$ in the correlation function (this implies that $\rho_{F1} \neq 0$), we know that the parity changes in the normal way $[\pi = (-)^J]$. If instead we observe a term of the form $P_{22}(\cos\vartheta)\cos 2\varphi$, the parity changes in an anomalous manner $[\pi = (-)^{J+1}]$. Moreover, the investigation of the dependence of the coefficients of the correlation function on the scattering angle of the proton permits us in certain appropriate cases (λ sufficiently large) to determine the isotopic spin of the level.

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238