ROTATIONAL STATES OF ODD NUCLEI WITH SMALL NONAXIALITY

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We compute the dependence of the energy spectrum of the excited states of odd nuclei with spin $^5/_2$ in the ground state on the ratio of the rotational energy to the coupling energy between the external nucleon and nonspherical part of the core potential.

INTRODUCTION

THE presence of an "energy gap" in the spectrum of single-nucleon states of even-even nuclei facilitates the separation of collective excitations that correspond to the rotation of the nucleus and to the change in the form of its surface. In odd nuclei the energy of single-nucleon excitations differs little from the energy of collective excitations, and they can therefore be separated only in some particular cases. The interaction between rotation of the nucleus and the motion of an external nucleon changes the structure of the rotational spectrum corresponding to the adiabatic approximation.

To investigate low excited states of odd atomic nuclei we consider a nucleon model consisting of a core and one outer nucleon. We assume that the form of the nucleus differs little from an ellipsoid of revolution (low nonaxiality).

Nuclei with ground-state spins $^1/_2$ or $^3/_2$ call for a special analysis (see, for example, references 1 and 2). We shall therefore investigate in the present article the case of nuclei whose ground-state spin is $^5/_2$. The same method can be extended to include nuclei with larger spins. It will be shown that the sequence of spins of the first excited states and the ratio of their energies are determined by a single parameter, corresponding to the ratio of the rotational energy to the energy of interaction between the external nucleon and the nonspherical part of the potential of the nuclear core.

1. ENERGY STATES

The Hamiltonian operator of a system consisting of a nuclear core and one nucleon can be written in the form

$$H = H_p + H_r + H_{int}. \tag{1.1}$$

Here H_p is the Hamiltonian operator of the internal state of the nuclear core and the external nucleon in a centrally-symmetrical field;

$$H_r = \sum_i a_i (\hat{I}_i - \hat{j}_i)^2 \tag{1.2}$$

is the rotational-energy operator, where $\hat{I}_i$ and $\hat{j}_i$ are respectively the projections of the total angular momentum and of the momentum of the outer nucleon on the coordinate axes fixed in the nucleus, and a_i are quantities determined by the moments of inertia of the nucleus;

$$H_{int} = -T\beta \{\cos\gamma\,(3\hat{j}_3^2 - \hat{j}^2) + \sqrt{3}\sin\gamma\,(\hat{j}_1^2 - \hat{j}_2^2)\}. \tag{1.3}$$

The Schrödinger equation with the Hamiltonian operator (1.1) reduces to systems of algebraic equations for each value of the total momentum I, if we seek the solution in the form

$$\Psi_\tau(I) = \sum_{K,\Omega>0} A_{K\Omega}^{I\tau} \sqrt{\frac{2I+1}{16\pi^2}} \{D_{mK}^I \varphi_\Omega^j + (-1)^{I-j} D_{m,-K}^I \varphi_{-\Omega}^j\}, \tag{1.4}$$

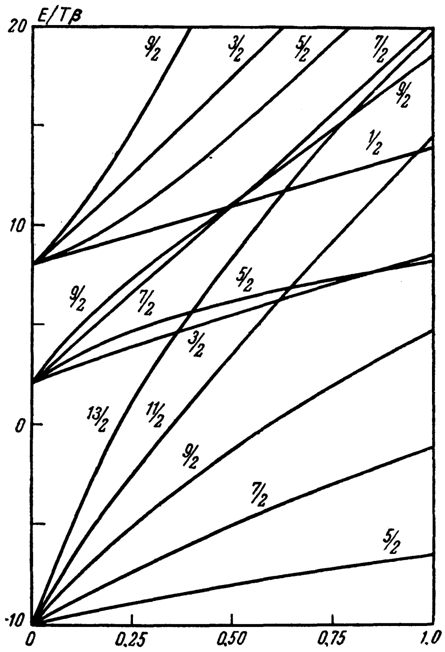
where D_{mK}^I are generalized spherical functions determining the orientation of the nucleus in space; φ_Ω^j are the wave functions of the external nucleon in the coordinate system fixed in the nucleus.

In the case of small nonaxiality $\gamma \approx 0$ and we can retain in the operator (1.3) only the first term, putting $\cos\gamma \approx 1$. We can then put in (1.2) $a_1 = a_2$, and finally it is necessary to put in (1.4) $K = \Omega$, so that the double summation reduces to summation over K only. If we use the matrix elements of operators (1.2) and (1.3) with wave functions (1.4) as given by O. Bohr,[1] the problem of determining the energy of the excited states of the nucleus reduces to a solution of secular equations of the type

$$|\langle l | H_r + H_{int} | K\rangle - E(I)\,\delta_{lK}| = 0 \tag{1.5}$$

for each value of the total angular momentum I.

Equation (1.5) will contain only two parameters: $D = a_1 + a_2$ and $T\beta$. Thus, the excitation energies, measured in $T\beta$ units, depend only on one parameter $\xi = D/\beta T$. The figure shows the results of the calculations for the case of nuclei with $j = {}^5/_2$.



In using the figure, it is necessary to reckon the energy from the first line corresponding to a nuclear ground-state spin $^5/_2$. When $\xi < 0.25$, the spectrum of the excited states breaks up into three bands. The first band with spin sequence $^5/_2$, $^7/_2$, $^9/_2$, $^{11}/_2$, and $^{13}/_2$ can be called the "main rotational band." However, the rules governing the spacing of these levels differ from the rules derived from the adiabatic theory of rotational states for axially-symmetrical nuclei:

$$E_J = A\{I(I+1) - I_0(I_0+1)\}. \quad (1.6)$$

In particular, when $I = ^5/_2$ the interval rule following from (1.6) is

$$E_{^7/_2} : E_{^9/_2} : E_{^{11}/_2} : E_{^{13}/_2} : \ldots = 1 : 2.29 : 3.86 : 5.76 : \ldots .$$

Table I. Spins and energies (kev) of the levels of U^{233}

Theory (ξ=0.25)	Experiment[3]	Theory (ξ=0.25)	Experiment[3]
$^5/_2$ 0	$^5/_2$ 0	$^5/_2$ 342	$^5/_2$ 341
$^7/_2$ 40	$^7/_2$ 40	$^7/_2$ 387	$^1/_2$ 400
$^9/_2$ 92.5	$^9/_2$ 92	$^9/_2$ 405	$^3/_2$ 417
$^3/_2$ 318	$^3/_2$ 313		

As can be seen from the figure, the relative distances between levels of the "main rotational band" depend on the value of ξ. When $\xi = 0.25$, the interval rule for these levels reduces to 1 : 2.3 : 4.0 : 6.3, and when $\xi = 1$, the interval rule has the form 1 : 2.1 : 3.9 : 4.8.

Tables I, II, and III list the theoretical and experimental values of spins and energies of the levels of the odd nuclei U^{233}, Np^{237}, and Th^{229}. The parameter ξ is determined from the first two experimental values of the energies of the levels with the same parity.

2. PROBABILITIES OF ELECTROMAGNETIC TRANSITIONS BETWEEN EXCITED STATES

To each value of the total momentum $I = ^1/_2, ^3/_2, \ldots$ there will correspond, generally speaking several energy levels which can be distinguished by the index $\tau = 1, 2, \ldots$ Consequently, the excited states will be characterized by two numbers, I and τ. Knowing the energies $E_{I\tau}$, we can readily calculate the coefficients $A_K^{I\tau} = A_{KK}^{I\tau}$, which determine with the aid of (1.4) the wave functions of the excited states of the nucleus, and then calculate the probabilities of electromagnetic transitions between these states.

In the case of nuclei with small nonaxiality, the operator of electric quadrupole transitions has the form

$$Q_{2\mu} = e\,Q_0 D^2_{\mu 0}, \quad (2.1)$$

where e is the unit electric charge and Q_0 is the internal quadrupole moment. The reduced probability of the electric quadrupole transitions is expressed in terms of the coefficients $A_K^{I\tau}$ with the aid of the formula

$$B(E2;\ I\tau \to I'\tau') = \frac{5e^2Q_0^2}{16\pi}\left|\sum_K A_K^{I'\tau'} A_K^{I\tau}(2I0K\,|\,I'K)\right|^2. \quad (2.2)$$

The operator of magnetic dipole transition is

$$\mathfrak{M}(1\nu) = (-1)^\nu \sqrt{\frac{3}{4\pi}}\,\mu_0 (g_j - g_R) \sum_{\mu=-1}^{1} D^1_{\nu\mu} \hat{j}_\mu, \quad (2.3)$$

Table II. Spins and energies (kev) of levels of Np^{237}

Levels with positive parity		Levels with negative parity	
Theory (ξ=0.30)	Experiment[4]	Theory (ξ=0.45)	Experiment[4]
$^5/_2$ 0	$^5/_2$ 0	$^5/_2$ 0	$^5/_2$ 0+59.6
$^7/_2$ 33.2	$^7/_2$ 33.2	$^7/_2$ 43.4	$^7/_2$ 43.4+59.6
$^9/_2$ 76.7	$^9/_2$ 76.4	$^9/_2$ 99.5	$^9/_2$ 98.9+59.6
$^1/_2$ 324	$^1/_2$ 332.3	$^{11}/_2$ 173	$^{11}/_2$ 165.4+59.6
$^5/_2$ 352	$^5/_2$ 368.5	$^3/_2$ 213	$^3/_2$ 207.9+59.6
$^3/_2$ 390	$^3/_2$ 371	$^5/_2$ 226	— —
		$^{13}/_2$ 245	$^{13}/_2$ 345.4+59.6

Table III. Spins and energies (kev) of levels of Th^{229}

Levels of positive parity		Levels with negative parity	
Theory (ξ = 0.25)	Experiment[5]	Theory (ξ = 0.50)	Experiment[5]
$^5/_2$ 0	$^5/_2$ 0	$^5/_2$ 0	$^5/_2$ 0+29
$^7/_2$ 42.3	$^7/_2$ 42.3	$^7/_2$ 43	$^7/_2$ 43+29
$^9/_2$ 97.7	$^9/_2$ 97	$^9/_2$ 96.5	$^9/_2$ 97+29
$^{11}/_2$ 169	$^{11}/_2$ 163	$^{11}/_2$ 166	$^{11}/_2$ 166+29
$^{13}/_2$ 267	$^{13}/_2$ 237		
$^3/_2$ 335	$^3/_2$ 316		
$^5/_2$ 359	$^5/_2$ 364		

Table IV. Reduced probabilities of E2 and M1 transitions for nuclei with $\xi = 0.25$ between the states $I\tau \rightarrow I'\tau'$

Transition $I\tau \rightarrow I'\tau'$	$\frac{B(E2)}{e^2Q_0^2}$	$\frac{B(M1)}{\mu_0^2(g_j - g_R)^2}$	Transition $I\tau \rightarrow I'\tau'$	$\frac{B(E2)}{e^2Q_0^2}$	$\frac{B(M1)}{\mu_0^2(g_j - g_R)^2}$
$^9/_2 1 \rightarrow {}^5/_2 1$	1.0	—	$^5/_2 2 \rightarrow {}^3/_2 1$	3.0	0.22
$^9/_2 1 \rightarrow {}^7/_2 1$	3.0	0.37	$^5/_2 2 \rightarrow {}^7/_2 1$	0.002	0.27
$^7/_2 1 \rightarrow {}^5/_2 1$	3.5	0.27	$^5/_2 2 \rightarrow {}^5/_2 1$	0.005	0.12
$^3/_2 1 \rightarrow {}^5/_2 1$	0.041	0.41	$^5/_2 2 \rightarrow {}^9_{12} 1$	0.03	
$^3/_2 1 \rightarrow {}^7/_2 1$	0.067	—			

where μ_0 is the nuclear magneton, g_j and g_R are respectively the gyromagnetic ratios for the single-nucleon and collective motions, and $\hat{j}_\mu$ are the spherical projections of the operator of angular momentum of the nucleon and the coordinate system fixed in the nuclear core. The action of the operator $\hat{j}_\mu$ on the wave function of the nucleon is determined by the equation

$$\hat{j}_\mu \varphi^j_K = (-1)^\mu \sqrt{j(j+1)}\,(j1K+\mu, -\mu\,|\,jK)\,\varphi_{\mu+K}.$$

With the aid of the functions (1.4) and the operator (2.3) we determine the reduced probability of magnetic dipole transition

$$B(M1;\ I\tau \rightarrow I'\tau') = (3/4\pi)\mu_0^2(g_j - g_R)^2 \times \Big|\sum_{\mu, K} (-1)^\mu A^{I'\tau'}_{K+\mu} A^{I\tau}_K (j1K+\mu, -\mu\,|\,jK)(1I\mu K\,|\,I', K+\mu) - (-1)^{I'-j} A^{I'\tau'}_{1/2} A^{I\tau}_{1/2} (j1 - {}^1/_2 1\,|\,j\,{}^1/_2)(1I - 1\,{}^1/_2\,|\,I', -{}^1/_2)\Big|^2. \quad (2.4)$$

When summing over μ and K in (2.4), it is necessary to sum over all positive values of K when $\mu = 0$ and 1, and to sum only over the values $K \geq {}^3/_2$ when $\mu = -1$.

Table IV lists the values of the reduced probabilities of quadrupole electric and dipole magnetic transitions between the first collective excited states of nuclei with ground-spin state $^5/_2$ and $\xi = 0.25$, as calculated from formulas (2.2) and (2.4).

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