ON NEUTRON TRANSFER IN NUCLEAR COLLISIONS

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Neutron transfer in nuclear collisions is investigated in the case when the effect of the Coulomb field is significant. At energies above the Coulomb barrier the possibility of formation of a compound nucleus is taken into account. The differential cross section attains a maximum which shifts towards smaller angles as the energy is increased. The angular dependence of the cross section depends weakly on the angular momentum of the state into which the neutron is captured. The calculation is restricted to the case when the Q of the reaction is small compared to the energy of the colliding nuclei.

NEUTRON transfer occurring in the bombardment of atomic nuclei by nitrogen ions has been studied in recent years in a number of experimental papers (cf. references 1—4). A study was made of the energy dependence of the neutron transfer cross section[5-7] and of the angular distribution in the reaction $N^{14}(N^{14}N^{13})N^{15}$ at energies of 23.3, 21.1, 19.2, and 16.3 Mev.[1] At an energy of 23.3 Mev, Reynolds and Zucker[1] have observed a maximum at 50° in the center of mass system (c.m.s.). The angle corresponding to the maximum number of scattered particles was found to increase as the energy was decreased.

At energies below the Coulomb barrier the transfer of a nucleon can be due to the penetration through the Coulomb barrier. At high energies when the distance of closest approach of the colliding nuclei becomes equal to or less than the sum of the nuclear radii the formation of a compound nucleus becomes possible. In this case the angular distribution of the inelastic scattering of nuclei accompanied by nucleon transfer is characterized by a sharp falling off at large angles corresponding to small nuclear impact parameters.

Good agreement with experimental data on the elastic scattering of charged particles was obtained by means of the semiclassical theory based on the assumption according to which the l-th partial wave is completely absorbed if the impact parameter associated with it is less than or equal to the sum of the radii of the colliding nuclei, while the wave whose impact parameter is greater than the sum of the nuclear radii remains unperturbed and has the phase corresponding to Coulomb scattering.[8,9] Such a semiclassical theory was first proposed by Akhiezer and Pomeranchuk[10] in the study of diffraction scattering of fast charged particles.

We can assume that the hypothesis just quoted holds also in the case of inelastic scattering. We shall investigate in the c.m.s. collisions of nuclei of mass numbers A_1 and A_2 as a result of which a neutron is transferred from the nucleus A_1 to the nucleus A_2 ($A_1 \gg 1$, $A_2 \gg 1$). The problem is considerably simplified in the limiting case $\eta_i = Z_1Z_2e^2/\hbar v_i > 1$, $\eta_f = Z_1Z_2e^2/\hbar v_f > 1$, when the quasiclassical approximation is applicable.

The amplitude for neutron transfer, similarly to the amplitude for stripping,[11] can be written in the form

$$f = -\frac{\mu}{4\pi\hbar^2}\int \Phi_{l'}^*(\rho')\,\psi_{k_f}^{(-)*}(\mathbf{r}')\,V(\rho)\,\psi_{k_i}^{(+)}(\mathbf{r})\,\varphi_l(\rho)\,d\mathbf{r}\,d\rho. \quad (1)$$

Here $\boldsymbol{\rho}$ is the position vector of the neutron with respect to the core $A_1 - 1$ in the nucleus A_1, $\boldsymbol{\rho}'$ is the position vector of the neutron with respect to the core A_2 in the nucleus $A_2 + 1$, $\mathbf{r}$ and $\mathbf{r}'$ are respectively the position vectors of the center of mass of the nucleus A_1 and of the core $A_1 - 1$ with respect to the center of mass of the nucleus A_2, μ is the reduced mass of the colliding nuclei, $V(\boldsymbol{\rho})$ is the potential for the nuclear interaction between the neutron and the core $A_1 - 1$, $\varphi_l(\boldsymbol{\rho})$ is the wave function for the neutron of angular momentum l in the nucleus A_1, $\Phi_{l'}(\boldsymbol{\rho}')$ is the wave function for the neutron of angular momentum l' in the nucleus $A_2 + 1$. $\psi_{\mathbf{k}_i}^{(+)}(\mathbf{r})$ denotes the Coulomb wave function for the relative motion of the nuclei A_1 and A_2 which at infinity consists of a plane wave with the propagation vector $\mathbf{k}_i$ and a spherical diverging wave; $\psi_{\mathbf{k}_f}^{(-)}(\mathbf{r}')$ is the Coulomb wave function for the relative motion of the nuclei $A_1 - 1$ and $A_2 + 1$, which at infinity contains a wave with propagation vector $\mathbf{k}_f$ and a converging spherical wave

$$\psi_{\mathbf{k}_i}^{(+)}(\mathbf{r}) = \exp\left(-\tfrac{1}{2}\pi\eta_i + i\mathbf{k}_i\mathbf{r}\right)\Gamma(1 + i\eta_i)\,F(-i\eta_i,\ 1;\ i(k_ir - \mathbf{k}_i\mathbf{r})),$$

$$\psi_{\mathbf{k}_f}^{(-)}(\mathbf{r}') = \exp\left(-\tfrac{1}{2}\pi\eta_f + i\mathbf{k}_f\mathbf{r}'\right)\Gamma(1 - i\eta_f)\,F(i\eta_f,\ 1;\ -i(k_fr' + \mathbf{k}_f\mathbf{r}')). \quad (2)$$

If we take the neutron mass M to be much smaller than the mass of the core $A_1 - 1$, we can set $\mathbf{r}' \approx \mathbf{r}$ and $\boldsymbol{\rho}' \approx \mathbf{r} + \boldsymbol{\rho}$. Further, on noting that if the energy of the relative motion of the colliding nuclei is not great (lower than the Coulomb barrier), then in the evaluation of (1) the region inside the nucleus $A_2 + 1$ is not important, we see that, therefore, the function $\Phi_{l'}(\mathbf{r} + \boldsymbol{\rho})$ in (1) can be replaced by its value in the exterior region

$$\Phi_{l'}(\mathbf{r} + \boldsymbol{\rho}) = N_{l'}Y_{l'm'}(\theta', \Phi')\,k_{l'}(\alpha\,|\mathbf{r} + \boldsymbol{\rho}|), \quad |\mathbf{r} + \boldsymbol{\rho}| > R', \quad (3)$$

where $k_{l'}(x) = \sqrt{\pi/2x}\,K_{l'+\frac{1}{2}}(x)$ is Macdonald's spherical harmonic, $\alpha = \sqrt{2M\epsilon'}/\hbar$ (ϵ' is the binding energy of the captured neutron in the nucleus $A_2 + 1$), $Y_{l'm'}(\theta', \Phi')$ is the spherical harmonic corresponding to the state of angular momentum l'; θ', Φ' are the angles specifying the orientation of the vector $\mathbf{r} + \boldsymbol{\rho}$, $N_{l'}$ is a normalization constant, R' is the radius of the nucleus $A_2 + 1$.

If $\alpha R'$ exceeds $l'(l' + 1)/2$, then the function in (3) can be replaced by its asymptotic expression

$$k_{l'}(x) = \pi e^{-x}/2x. \quad (4)$$

For example, in the case of neutron transfer from a N^{14} nucleus to another N^{14} nucleus $\alpha R' = 2.8$, $l' = 1$, and, therefore, formula (4) is a good approximation.

By choosing for the potential $V(\boldsymbol{\rho})$ the rectangular well model

$$V(\rho) = \begin{cases} -V_0, & \rho < R \\ 0, & \rho > R \end{cases}$$

(R is the radius of the nucleus A_1), the function $\varphi_l(\boldsymbol{\rho})$ can be written in the form

$$\varphi_l(\boldsymbol{\rho}) = N_l j_l(\chi\rho)\,Y_{lm}(\vartheta_\rho, \varphi_\rho), \qquad \rho \leqslant R,$$
$$\chi = \sqrt{2M(V_0 - \varepsilon)/\hbar^2}, \quad (5)$$

where $j_l(\chi\rho)$ is the spherical Bessel function of order l, (ϵ is the neutron binding energy in the nucleus A_1) and N_l is a normalization constant.

By utilizing the well known expansion

$$\frac{\exp(-\alpha|\mathbf{r}+\boldsymbol{\rho}|)}{|\mathbf{r}+\boldsymbol{\rho}|} = \frac{4\pi}{\sqrt{r\rho}}\sum_{\lambda,\nu}(-1)^\lambda I_{\lambda+1/2}(\alpha\rho)\,K_{\lambda+1/2}(\alpha r)\,Y^*_{\lambda\nu}(\vartheta, \varphi) \times Y_{\lambda\nu}(\vartheta_\rho, \varphi_\rho), \qquad \rho < r, \quad (6)$$

where $I_{\lambda+\frac{1}{2}}(x)$ is a Bessel function of imaginary argument, ϑ, φ are the angles specifying the orientation of the vector $\mathbf{r}$ and ϑ_ρ, φ_ρ are the angles specifying the orientation of the vector $\boldsymbol{\rho}$, we rewrite the amplitude f in the following form:

$$f = N_l N^* \frac{\pi\mu V_0}{2\hbar^2\sqrt{\alpha\chi}}\sum_{\lambda,\nu}(-1)^\lambda\int_0^R \frac{1}{\rho}I_{\lambda+1/2}(\alpha\rho)\,J_{l+1/2}(\chi\rho) \times Y_{lm}(\vartheta_\rho,\varphi_\rho)\,Y^*_{\lambda\nu}(\vartheta_\rho,\varphi_\rho)\,d\rho\int\psi_{\mathbf{k}_f}^{(-)*}(\mathbf{r})\,k_\lambda(\alpha r)\,\psi_{\mathbf{k}_i}^{(+)}(\mathbf{r}) \times Y^*_{l'm'}(\theta', \Phi')\,Y_{\lambda\nu}(\vartheta, \varphi)\,d\mathbf{r}. \quad (7)$$

It should be noted that the angles θ', Φ' are functions of ϑ, φ, ϑ_ρ, φ_ρ, ρ, r. However, approximately we can set $\theta' \approx \vartheta$ and $\Phi' \approx \varphi$. Such a replacement is valid if the effective value of $\boldsymbol{\rho}$ is small in comparison with the effective value of $\mathbf{r}$. We can make an estimate of the error made as the result of such replacement in the case of the example $l = l' = 0$. In this case the exactly calculated amplitude can be compared with the amplitude obtained if we neglect in the function $\Phi_0^*(\mathbf{r}+\boldsymbol{\rho})$ of formula (1) the quantity $\boldsymbol{\rho}$ compared to $\mathbf{r}$. It can be easily seen that the replacement of $\Phi_0^*(\mathbf{r}+\boldsymbol{\rho})$ by $\Phi_0(\mathbf{r})$ does not affect the nature of the angular distribution for $l' = 0$. We should, therefore, expect that the replacement $\theta' \to \vartheta$ and $\Phi' \to \varphi$ in the general case $l' \neq 0$ will also not lead to any significant change in the angular distribution.

Thus, for the amplitude of the reaction we obtain the following expression:

$$f = N_l N^*_{l'}\frac{\pi\mu V_0 B_l}{2\hbar^2\sqrt{\alpha\chi}}(-1)^l\int\psi_{\mathbf{k}_f}^{(-)*}(\mathbf{r})\,k_l(\alpha r)\,\psi_{\mathbf{k}_i}^{(+)}(\mathbf{r})\,Y_{l'm'}(\vartheta, \varphi) \times Y_{lm}(\vartheta, \varphi)\,d\mathbf{r},$$

$$B_l = \frac{R}{\chi^2 + \alpha^2}\left\{J_{l+1/2}(\chi R)\frac{d}{dR}I_{l+1/2}(\alpha R) - I_{l+1/2}(\alpha R)\frac{d}{dR}J_{l+1/2}(\chi R)\right\}. \quad (8)$$

In evaluating the integral remaining in (8) we can again make use of the asymptotic expression (4) for $k_l(\alpha r)$ if the condition $\alpha r_{\text{eff}} > l(l + 1)/2$ is satisfied. If in making the estimate we make use of the value

$$r_{\text{eff}} = Z_1Z_2e^2/E + [\varepsilon'(A - 1) - \varepsilon A]^2/4\varepsilon' A,$$
$$A = A_1A_2/(A_1 + A_2),$$

obtained in evaluating the integral in (8) by the saddle point method, it can be easily seen that at energies of the order of the Coulomb barrier

$$\alpha r_{\text{eff}} \approx \tfrac{2}{3}\left(\sqrt[3]{A_1} + \sqrt[3]{A_2}\right).$$

In the case of the reaction $N^{14}(N^{14}N^{13})N^{15}$ at $E \approx 9$ Mev, $\epsilon' = 10.8$ Mev, $\epsilon = 10.6$ Mev (cf. reference 11) we have $\alpha r_{\text{eff}} \approx 5$.

When this condition is satisfied the integral (8) can be written in the form[11]

$$\int \psi_{\mathbf{k}_f}^{(-)*}(\mathbf{r})\, k_l(\alpha r)\, \psi_{\mathbf{k}_i}^{(+)}(\mathbf{r})\, Y_{l'm'}(\vartheta,\varphi)\, Y_{lm}(\vartheta,\varphi)\, d\mathbf{r}$$
$$= \frac{\pi}{2\alpha} Y_{lm}(\vartheta_0,\varphi_0)\, Y_{l'm'}(\vartheta_0,\varphi_0) \int \frac{e^{-\alpha r}}{r} \psi_{\mathbf{k}_f}^{(-)*}(\mathbf{r})\, \psi_{\mathbf{k}_i}(\mathbf{r})\, d\mathbf{r}. \tag{9}$$

Here $\mathbf{r}_0(r_0, \vartheta_0, \varphi_0)$ is the saddle point, where the function $F(\mathbf{r}) = -\alpha r + \ln(\psi_{\mathbf{k}_f}^{(-)*}(\mathbf{r})\, \psi_{\mathbf{k}_i}^{(+)}(\mathbf{r}))$ has an extremum.

We expand the Coulomb wave functions $\psi_{\mathbf{k}_i}^{(+)}(\mathbf{r})$ and $\psi_{\mathbf{k}_f}^{(-)}(\mathbf{r})$ into partial waves:[13]

$$\psi_{\mathbf{k}}^{(\pm)}(\mathbf{r}) = 4\pi \sum_{lm} i^l \exp\left[\pm i\delta_l(\eta)\right] (kr)^{-1} Y_{lm}^*(\mathbf{k})\, Y_{lm}$$
$$\times (\vartheta,\varphi)\, F_l(kr), \tag{10}$$

where $\delta_l(\eta) = \arg\Gamma(l + 1 + i\eta)$ is the Coulomb phase shift, and $F_l(kr)$ is the regular solution of the radial wave equation for angular momentum l:

$$F_l(kr) = e^{-\pi\eta/2} \frac{|\Gamma(l+1+i\eta)|}{2\Gamma(2l+2)} (2kr)^{l+1} e^{-ikr} F(l+1$$
$$- i\eta,\ 2l+2,\ 2ikr). \tag{11}$$

On substituting (10) into the integral (9) and on carrying out the integration over the angles we obtain

$$\int \frac{e^{-\alpha r}}{r} \psi_{\mathbf{k}_f}^{(-)*}(\mathbf{r})\, \psi_{\mathbf{k}_i}^{(+)}(\mathbf{r})\, d\mathbf{r} = 4\pi \sum_l (2l+1)$$
$$\times \exp\left[i\delta_l(\eta_i) + i\delta_l(\eta_f)\right] P_l(\cos\theta) \frac{1}{k_i k_f}$$
$$\times \int_0^\infty F_l(k_i r) \frac{e^{-\alpha r}}{r} F_l(k_f r)\, dr \tag{12}$$

($P_l(\cos\theta)$ is a Legendre polynomial, θ is the angle between the vectors $\mathbf{k}_i$ and $\mathbf{k}_f$).

In the quasiclassical case ($\eta_i, \eta_f > 1$) we can utilize the following approximate expression for the radial functions:[14]

$$F_l(kr) = \left[\frac{f(r)}{k^2}\right]^{-1/4} \sin\varphi, \qquad \varphi = \frac{\pi}{4} + \int_{\tilde{r}}^{r} [f(r)]^{1/2} dr,$$
$$f(r) = k^2 - \frac{2k\eta}{r} - \frac{l(l+1)}{r^2}, \tag{13}$$

where $\tilde{r}$ is the classical turning point determined by the condition $f(\tilde{r}) = 0$. If we restrict ourselves to the consideration of the case when the Q of the reaction $Q = E_{A_1-1} - E_{A_1} = \epsilon' - \epsilon$ is much smaller than the kinetic energy of the colliding nuclei we have

$$\frac{1}{k_i k_f} \int_0^\infty F_l(k_i r) \frac{e^{-\alpha r}}{r} F_l(k_f r)\, dr$$
$$= \frac{1}{2k^2} \exp\left(-\frac{\alpha\eta}{k} + \xi \operatorname{arctg}\frac{\xi k}{\alpha\eta}\right) K_{i\xi}(\varepsilon c),$$
$$\xi = \eta_f - \eta_i, \qquad k = (k_i + k_f)/2, \qquad \eta = (\eta_i + \eta_f)/2,$$
$$\varepsilon = \sin^{-1}(\theta/2), \qquad c = \sqrt{(\alpha\eta/k)^2 + \xi^2} \approx \alpha\eta/k. \tag{14*}$$

*arctg = $\tan^{-1}$.

For $\xi \ll 1$ and $\eta \gg 1$ we have

$$\exp\left[\xi \operatorname{arctg}(\xi k/\alpha\eta)\right] \approx 1, \qquad K_{i\xi}(\varepsilon c) \approx (\pi/2\varepsilon c)^{1/2} e^{-\varepsilon c}$$

(cf. reference 15).

Therefore

$$\frac{1}{k_i k_f} \int_0^\infty F_l(k_i r) \frac{e^{-\alpha r}}{r} F_l(k_f r)\, dr$$
$$= \frac{1}{2k^2} \left(\frac{\pi}{2\alpha a\varepsilon}\right)^{1/2} \exp\left[-\alpha a\left(1 + \sin^{-1}(\theta/2)\right)\right], \tag{15}$$

where $a = \eta/k = Z_1 Z_2 e^2/\mu v^2$. Thus, on substituting (15) into (12) we obtain the following expression for the amplitude for neutron transfer in nuclear collisions:

$$f(\theta) = i\sqrt{\frac{\pi^7}{2\chi a}} \frac{N_l N_{l'}^* V_0 \mu}{k\hbar^2 \alpha^2} B_l (-1)^l Y_{lm}(\vartheta_0,\varphi_0)$$
$$\times Y_{l'm'}^*(\vartheta_0,\varphi_0) \sqrt{\sin\frac{\theta}{2}}\, f_0'(\theta) \exp\left\{-\alpha a\left(1 + \sin^{-1}(\theta/2)\right)\right\}, \tag{16}$$

$$f_0(\theta) = \sum_l \frac{1}{2ik}(2l+1) \exp\left[2i\delta_l(\eta)\right] P_l(\cos\theta). \tag{17}$$

At energies much lower than the Coulomb barrier $f_0(\theta)$ is the ordinary amplitude for Coulomb scattering. As can be seen from (16) the angular dependence of the cross section is given by the expression $\sin^{-3}(\theta/2)\exp(-2\alpha a/\sin(\theta/2))$ (cf. also reference 7). This expression increases with increasing angle θ reaching a maximum,* after which a falling off begins determined by the factor $\sin^{-3}(\theta/2)$. As the energy increases the maximum of the angular distribution shifts towards smaller angles.

At energies greater than the Coulomb barrier it is necessary to take into account the possibility of formation of a compound system in nuclear collisions. In the quasiclassical case the formation of the system is possible if the orbital angular momentum l is less than l_{max} determined by the condition

$$l_{max}(l_{max} + 1)$$
$$= 2\mu\hbar^{-2}(R_{A_1} + R_{A_2})\left(E - Z_1 Z_2 e^2/(R_{A_1} + R_{A_2})\right). \tag{18}$$

In this case in formula (17) which determines the elastic scattering amplitude we must restrict our summation to values of $l \geq l_{max}$.

Owing to the possibility of formation of a compound system, the angular distribution of nuclei in the case of neutron transfer will fall off sharply at angles greater than $\theta = \theta_0$, where θ_0 can be roughly estimated by means of the following formula:

$$R_{A_1} + R_{A_2} = \frac{Z_1 Z_2 e^2}{2E}\left(1 + \sin^{-1}\frac{\theta_0}{2}\right). \tag{19}$$

*The angle θ_{max} is determined by the condition $\sin(\theta_{max}/2) = 2\alpha a/3$.

In a similar manner Reynolds and Zucker[9] have succeeded in explaining the angular distribution of the elastically scattered particles in the reaction $N^{14}(N^{14}N^{14})N^{14}$ at an energy in the laboratory system (l.s.) of 21.5 Mev, by setting $l_{max} = 6$ and $R = 4 \times 10^{-13}$ cm.

Thus, at energies above the Coulomb barrier the angular dependence of the neutron transfer cross section will be given by the expression

$$\frac{d\sigma}{d\Omega}(\theta) \sim \Big| \sin^{-3/2}\frac{\theta}{2} \exp\left(-i\eta \ln \sin^2\frac{\theta}{2} + 2i\delta_0 - \alpha a/\sin\frac{\theta}{2}\right) + \frac{1}{i\eta}\sin^{1/2}\frac{\theta}{2}\exp\left(-\alpha a/\sin\frac{\theta}{2}\right)\sum_{l=0}^{l=l_{max}}(2l+1) \times \exp(2i\delta_l)P_l(\cos\theta)\Big|^2. \quad (20)$$

In collisions of identical nuclei it is necessary to take recoil into account. In this case the angular distribution can be roughly obtained in the following manner:

$$d\sigma/d\Omega = |f(\theta)|^2 + |f(\pi-\theta)|^2. \quad (21)$$

Evidently there will be observed two maxima, symmetric with respect to $\theta = \pi/2$, the separation between which increases with increasing energy.

A more exact result can be obtained by treating the identical nuclei quantum mechanically. In the case of the reaction $N^{14}(N^{14}N^{13})N^{15}$ the angular distribution is given by the following expression[7]

$$d\sigma/d\Omega = \tfrac{2}{3}(2d\sigma_a/d\Omega + d\sigma_s/d\Omega), \quad (22)$$

where $d\sigma_a/d\Omega$ and $d\sigma_s/d\Omega$ are the cross sections for the antisymmetric and the symmetric states:

$$d\sigma_{a,s}/d\Omega = |f(\theta) \mp f(\pi-\theta)|^2. \quad (23)$$

On substituting (16) into (23), on noting that $l = l'$ $= 1$, and on taking into account the weak dependence of ϑ_0 on the scattering angle θ we obtain

$$\frac{d\sigma_{s,a}}{d\Omega} = \frac{1}{\sqrt{2}}\frac{63}{2048}\pi^5|N_1|^2|N_1'|^2\left(\frac{Ze}{E}\right)^2 V_0^2\hbar^3 M^{-3/2}\varepsilon'^{-2}(V_0-\varepsilon)^{-1/2} \times B_1^2 e^{-2\alpha a}\Big|\sin^{-3/2}\frac{\theta}{2}\exp\left(-i\eta\ln\sin^2\frac{\theta}{2} + 2i\delta_0 - \alpha a/\sin\frac{\theta}{2}\right) \pm \cos^{-3/2}\frac{\theta}{2}\exp\left(-i\eta\ln\cos^2\frac{\theta}{2} + 2i\delta_0 - \alpha a/\cos\frac{\theta}{2}\right) + \frac{1}{i\eta}\sin^{1/2}\frac{\theta}{2}\exp\left(-\alpha a/\sin\frac{\theta}{2}\right) \times \sum_{l=1}^{l=l_{max}}(2l+1)\exp(2i\delta_l)P_l(\cos\theta) \pm \frac{1}{i\eta}\cos^{1/2}\frac{\theta}{2}\exp\left(-\alpha a/\cos\frac{\theta}{2}\right)\sum_{l=0}^{l=l_{max}}(2l+1) \times \exp(2i\delta_l)P_l(-\cos\theta)\Big|^2. \quad (24)$$

The energy dependence of the neutron transfer cross section can be roughly estimated by integrating $|f(\theta)|^2$ from $\theta = 0$ to $\theta = \theta_0$. We obtain the result

$$\sigma(E) \sim \frac{1}{E}\exp\left[-\frac{\alpha Z_1 Z_2 e^2}{E}\left(1 + \sin^{-1}\frac{\theta_0}{2}\right)\right]. \quad (25)$$

In order to improve the results given by formula (25) we can utilize the experimentally observed dependence of θ_0 on energy.

It should be noted that at energies below the Coulomb barrier formula (25) leads to too rapid a falling off of the cross section with decreasing energy (cf., reference 7).

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