ISOTOPE MASSES AND BINDING ENERGY OF NUCLEI IN THE REGION FROM STRONTIUM TO RUTHENIUM

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A double focusing mass spectrograph was employed to measure the masses of stable isotopes in the region $84 \leq A \leq 104$. The average resolving power of the instrument was 50 000 — 60 000. Twenty-five isotope masses were measured. The measurements were performed by checking the "internal consistency" of the results. For this purpose, 60 mass doublets were measured. Most of the isotope masses were determined from measurements of 3 — 4 doublets of different organic composition. Besides the 25 stable isotopes, the masses of 59 radioactive isotopes were determined. The binding energy per nucleon, binding energy of the last neutron (B_n) and proton (B_p), and the neutron (P_n) and proton (P_p) pairing energies were calculated on the basis of these data. The sharp break in the nucleon binding energy curve and the discontinuities in the energies B_n and P_n in nuclei containing 50 neutrons definitely confirm the existence of the "magic" number $N = 50$. Furthermore, anomalously large values of B_n and P_n have been found in nuclei with $N = 48$. On the other hand, no nonmonotonic variation of the above-mentioned parameters has been observed for $Z = 40$, which signifies that no subshell exists for $Z = 40$.

THE measurement of the masses of nuclei in the region from strontium to ruthenium is of interest from the viewpoint of elucidating the variation of the nuclear binding energy close to the magic number $N = 50$. To measure the masses of the isotopes of strontium, yttrium, zirconium, niobium, molybdenum, and ruthenium, we used a double focusing mass-spectrograph. The average resolving power of the intrument was 50 000 — 60 000.

We measured the doublets formed by an organic compound and the isotope under study. We calculated the instrument dispersion in each separate case by using fragments of the organic compounds with a mass difference of one mass of hydrogen. The use of this method made it possible to improve the accuracy of the measurements in this mass region by a factor of about 10 — 15 times, i.e., to obtain $\Delta M/M \approx (2-5) \times 10^{-7}$. A considerable part of the isotope masses in this region has not been measured on a mass spectroscope (Zr^{91}, Zr^{92}, Mo^{95}, Ru^{98}, and others); the values of the masses of the isotopes Ru^{100} and Ru^{101} were measured for the first time.

It is of interest to study the energy variation in the region $Z = 40$, which, in the opinion of several authors, is a "semi-magic" number.[1-4] Conclusions based on the values of six isobaric pairs and one isobaric triplet can be compared with those based on the region of heavier masses ($A \approx 200$).

MASS MEASUREMENTS

1. Strontium. We measured the masses of the strontium isotopes by comparing the masses of strontium evaporated in a tungsten vessel with the organic compounds toluene (C_7H_8, $M = 92$) and isoamyl alcohol ($C_5H_{12}O$, $M = 88$) and their fragments. In the first case, the vapor pressure of the toluene was sufficient to maintain the arc of the ion source. In the case of isoamyl alcohol, the ion-source arc was maintained on helium, and the standard organic compound $C_5H_{12}O$ was introduced additionally into the ion source. The values of the doublets and the isotope masses of strontium were calculated with the "weights" for the measurements shown in Table I.

2. Yttrium. The yttrium ions were obtained by evaporation of a metallic powder of yttrium in a tungsten vessel. The discharge in the ion source was maintained on toluene vapors (C_7H_8, $M = 92$). A toluene fragment (C_7H_5, $M = 89$) was used to measure the mass of the yttrium isotope. The values of the doublet and mass of yttrium are shown in Table I.

3. Zirconium. To obtain the zirconium ions, we evaporated zirconium chloride in a crucible and introduced the vapors into the discharge arc. We measured the zirconium masses by comparison with organic compounds of aniline (C_6H_7N,

Table I

Doublet	ΔM, 10^{-3} amu	Isotope mass, amu	Mean value of mass, amu	Isotope mass from mass-spectroscopic data, amu	Isotope mass from Wapstra's data,[5] amu
C_5H_4O — Sr^{84}	144.148±0.017	83.940088±21			
C_6H_{12} — Sr^{84}	180.583±0.023	83.940041±26	83.940070±17	83.939924±150* [6]	83.939860±250
$C_5H_{10}O$ — Sr^{86}	164.039±0.024	85.936481±26		85.936658±110* [6]	
C_7H_2 — Sr^{86}	106.482±0.042	85.936542±44	85.936497±23	85.935330±430 [7] 85.936670± 50 [8]	85.936620±200
$C_5H_{11}O$ — Sr^{87}	172.251±0.021	86.936411±24			
C_7H_3 — Sr^{87}	114.628±0.021	86.936538±25	86.936472±42	86.936612± 80* [6]	86.936576±150
C_7H_4 — Sr^{88}	125.862±0.038	87.933446±40		87.933956±110* [6]	
C_7H_5 — $Sr^{88}H$	125.822±0.080	87.933486±81	87.933454±16	87.933740±530 [7] 87.933634± 18 [8]	87.933750±200
C_7H_5 — Y^{89}	133.303±0.007	88.934147±15	88.934147±15	88.933998±110* [6]	88.934080±200
C_7H_6 — Zr^{90}	142.323±0.021	89.933269±25		89.932932±250* [6]	
$C_6C^{13}H_6$ — Zr^{90}	137.832±0.036	89.933289±40	89.933271±20	89.931780±630 [9]	89.932840±200
C_6H_4N — Zr^{90}	129.768±0.050	89.933247±52		89.933500±200 [8]	
C_7H_7 — Zr^{91}	149.184±0.028	90.934550±30			
C_6H_5N — Zr^{91}	136.626±0.037	90.934531±40	90.934543±20	—	90.934140±200
C_7H_8 — Zr^{92}	157.539±0.019	91.934337±25			
$C_6C^{13}H_7$ — Zr^{92}	153.078±0.025	91.934327±30	91.934322±22	—	91.933820±210
C_6H_6N — Zr^{92}	145.006±0.029	91.934292±33			
C_7H_{10} — Zr^{94}	172.074±0.063	93.936086±65			
$C_6C^{13}H_9$ — Zr^{94}	167.608±0.036	93.936081±40			
C_6H_6O — Zr^{94}	135.626±0.022	93.936146±25	93.936126±25	—	93.935800±350
$C_5C^{13}H_7N$ — Zr^{94}	154.877±0.028	93.936235±31			
C_7H_{12} — Zr^{96}	185.576±0.044	95.938868±46	95.938868±46		95.938530±500
C_6H_7N — Nb^{93}	151.579±0.020	92.935862±25	92.935862±25	92.935208± 90* [6]	93.935260±240
C_7H_8 — Mo^{92}	155.734±0.024	91.936142±30			
$C_6C^{13}H_7$ — Mo^{92}	151.329±0.029	91.936076±35	91.936098±42	—	91.935210±290
$C_4H_7Cl^{37}$ — Mo^{92}	113.977±0.054	91.935954±60			
C_7H_{10} — Mo^{94}	173.205±0.046	93.934955±50			
C_6H_6O — Mo^{94}	136.572±0.070	93.935200±75	93.935024±52	93.934300±800 [8]	93.934330±270
$C_6C^{13}H_9$ — Mo^{94}	168.686±0.088	93.935003±90			
C_7H_{11} — Mo^{95}	180.271±0.030	94.936031±35			
C_6H_7O — Mo^{95}	143.926±0.040	94.935988±45	94.936015±15	—	94.935700±400
C_7H_{12} — Mo^{96}	189.332±0.022	95.935112±30			
C_6H_8O — Mo^{96}	152.995±0.070	95.935061±75	95.935173±44	95.935970±390 [9]	95.934900±430
$C_6C^{13}H_{11}$ — Mo^{96}	184.721±0.023	95.935252±30			
C_7H_{13} — Mo^{97}	195.804±0.040	96.936782±45			
C_6H_9O — Mo^{97}	159.502±0.060	96.936696±65	96.936755±30	—	96.936470±500
C_7H_{14} — Mo^{98}	204.220±0.041	97.936508±45			
$C_6H_{10}O$ — Mo^{98}	167.759±0.070	97.936581±75	97.936543±16	97.936100±400 [7]	97.936560±500
$C_6C^{13}H_{13}$ — Mo^{98}	199.685±0.046	97.936572±50			
C_7H_{16} — Mo^{100}	217.721±0.031	99.939291±35	99.939291±35	99.938600±400 [9]	99.938280±500
C_7H_{12} — Ru^{96}	186.574±0.062	95.937870±64			
C_6H_4O — Ru^{96}	150.217±0.124	95.937839±125	95.937865±15	—	95.937920±500
$C_6H_{10}N$ — Ru^{96}	173.998±0.094	95.937869±95			
C_7H_{14} — Ru^{98}	204.751±0.301	97.935977±301			
$C_6H_{10}O$ — Ru^{98}	168.659±0.422	97.935681±422			
$C_6H_{12}N$ — Ru^{98}	192.071±0.402	97.936080±402	97.935871±60	—	97.937130±650
C_8H_2 — Ru^{98}	111.041±0.220	97.935803±220			
C_7H_{15} — Ru^{99}	211.593±0.165	98.937277±165			
$C_6H_{11}O$ — Ru^{99}	175.413±0.068	98.937069±70	98.937100±50	—	98.938200±900
$^1/_2$ Hg^{198} — Ru^{99}	77.765±0.302	98.937091±302			
C_7H_{16} — Ru^{100}	220.964±0.094	99.936048±95			
$C_6H_{14}N$ — Ru^{100}	208.264±0.234	99.936171±234			
C_8H_4 — Ru^{100}	127.275±0.258	99.935853±258	99.935932±74	—	—
$^1/_2$ Hg^{200} — Ru^{100}	80.298±0.129	99.935658±130			
C_8H_5 — Ru^{101}	134.076±0.104	100.937194±105			
$C_6H_{13}O$ — Ru^{101}	191.356±0.150	100.937410±150	100.937271±44	—	—
$^1/_2$ Hg^{202} — Ru^{101}	80.182±0.088	100.937294±90			
C_8H_6 — Ru^{102}	142.973±0.070	101.936439±72	101.936439±72	101.935900±600 [10]	101.936410±500
C_8H_8 — Ru^{104}	157.576±0.082	103.938120±84			
C_7H_6N — Ru^{104}	144.939±0.167	103.938180±167	103.938132±23	103.938000±500 [10]	103.937800±700

*The mass values were recalculated for the auxiliary standard values H = 1.008142 ± 1, C = 12.003820 ± 5. The value of the error remains unchanged.

M = 93), cumene (C_9H_{12}, M = 120), and their fragments. In a number of cases, it was possible to use ions containing the carbon isotope C^{13} for the mass measurements.

The measured doublets, the zirconium isotope masses obtained from each individual doublet, and the mean values of the masses calculated with a "weight" for the measurements are listed in Table I.

4. Niobium. The niobium ions were obtained by evaporation of metallic niobium in a tungsten vessel and subsequent introduction of the vapor into the discharge arc of the ion source. We used aniline (C_6H_7N, M = 93) to measure the mass of Nb^{93}. The instrument dispersion was determined from the aniline fragments. The values of the doublets and the niobium mass are given in Table I.

5. Molybdenum. We used molybdenum chloride to obtain molybdenum ions. This compound is unstable and decomposes in air. We had to use it, however, since it was the only volatile molybdenum compound at our disposal. We determined the molybdenum isotope masses by checking the "internal consistency" of the measurements, i.e., the masses of the six molybenum isotopes were measured by comparison with organic compounds of different composition. We employed heptane (C_7H_{16}, M = 100), cyclohexane ($H_6H_{10}O$, M = 98), and their fragments.

The data on measurements of the isotope masses of molybdenum are shown in Table I.

6. Ruthenium. The ruthenium ions were obtained by evaporation in a tungsten vessel (M = 101, 102, and 104) and from ruthenium chloride. Owing to the fact that the melting point of ruthenium is high (~ 2000° C), only the most abundant isotopes (M = 101, 102, and 104) could be obtained from the evaporation in a tungsten crucible. The masses of these isotopes were measured with the aid of the organic compound sytrene (C_8H_8, M = 104). To obtain the less abundant ruthenium isotopes, we used ruthenium chloride and organic compound heptane (C_7H_{16}, M = 100). With heptane it is possible to obtain doublets for the masses 100, 99, 98, and 96; also, one may use the compound $C_6C^{13}H_{16}$ to obtain a doublet for the mass 101.

Admixtures of organic fragments containing C^{13}, and, consequently, producing the doublets $CH-C^{13}$ require a resolution of 45 000 for a mass of 100. Owing to the fact that the resolving power of the mass spectrograph was equal to 50 000 — 60 000, it was not necessary to correct for the C^{13}, since the doublet, for example, $C_7H_8-C_6C^{13}H_7$, is fully resolved. Hence there is no error associated with an unresolved admixture of C^{13} in a standard base.

Most of the measured isotopes were obtained from 2 — 4 different doublets (organic compounds of different composition). The masses of three ruthenium isotopes, Ru^{99}, Ru^{100}, and Ru^{101} were also measured by means of the doublets formed by doubly ionized ions of mercury, $\frac{1}{2}Hg^{198}$, $\frac{1}{2}Hg^{200}$, and $\frac{1}{2}Hg^{202}$, respectively. In this case, as well as in the case in which organic compounds were used to obtain the doublet, the base was computed from fragments of organic compounds with a difference of one unit mass (H^1).

The relatively good agreement of the results makes it possible to state that the measurements are "internally" consistent, i.e., there are no systematic errors within the limits of the experimental accuracy. The errors in the values of the doublets were calculated in the standard way as the weighted mean square error. The mean value of the error over the entire range of measured masses is $\approx 30\ \mu m$, which corresponds to a relative accuracy of $\Delta M/M \approx 3 \times 10^{-7}$.

For the determination of 25 isotope masses we measured 60 doublets. Moreover, we made a qualitative check of the measurements by measuring the doublets $CH-C^{13}$, CH_2-N, CH_4-O, and OH_8-C_2 for masses of 92 to 104. The results of the measurements of these doublets are shown in Table II. Comparison of the mean values of these mass differences with similar differences measured for masses 13, 14, and 16[11,12] show that the data are in fully satisfactory agreement. This con-

Table II

Mass		Value of doublet, mmu				
		$CH-C^{13}$	CH_2-N	NH_2-O	CH_4-O	C_2-OH_8
$_{38}Sr$	84					57.557±0.050
	86					57.504±0.029
	87				36.427±0.017	
$_{40}Zr$	90	4.459±0.035	12.565±0.054			
	91		12.558±0.042			
	92	4.466±0.012	12.533±0.035			
	94	4.466±0.073			36.434±0.061	
$_{42}Mo$	92	4.517±0.032				
	94	4.519±0.100			36.428±0.140	
	95				36.355±0.060	
	96	4.528±0.013			36.337±0.075	
	97				36.348±0.047	
	98	4.463±0.031			36.461±0.080	
$_{44}Ru$	96		12.613±0.046	23.876±0.168	36.354±0.111	
	98		12.598±0.031	23.723±0.043	36.430±0.315	
	99				36.320±0.092	
	100		12.578±0.070			
	104		12.551±0.068			
Mean with "weight"		4.488±0.008	12.572±0.007	23.762±0.068	36.414±0.022	57.517±0.019
Data from ref. 11, 12		4.471±0.003	12.578±0.004	23.808±0.008	36.388±0.004	57.496±0.006*

*Calculated.

Table III

Isotope	Z	N	Isotope mass, amu	Binding energy, Mev	B_n, Mev	B_p, Mev	P_n, Mev	P_p, Mev
Rb^{84}		47	83.941090±25	728.583	—	—	—	—
Rb^{85}		48	84.940175±165	737.801	9.218	—	—	—
Rd^{86}		49	85.938387±25	747.832	10.031	—	—	—
Rb^{87}		50	86.936522±170	757.935	10.103	—	0.072	—
Rb^{88}	37	51	87.939034±80	763.962	6.253	—	—	—
Rb^{89}		52	88.939887±115	771.535	7.573	—	1.320	—
Rb^{90}		53	89.942510±340	777.458±0.316	5.923	—	—	—
Rb^{91}		54	90.943120	785.257	7.799	—	1.876	—
Sr^{84}*		46	83.940070±17	728.748	—	—	—	—
Sr^{85}		47	84.940428±400	736.780±0.372	8.033	8.197	—	—
Sr^{86}*		48	85.936497±23	748.807	12.027	11.006	3.994	—
Sr^{87}*		49	86.936472±42	757.197	8.390	9.365	—	—
Sr^{88}*	38	50	87.933454±16	768.373	11.176	10.603	2.786	—
Sr^{89}		51	88.935717±20	774.632	6.259	10.670	—	—
Sr^{90}		52	89.936410±100	782.354	7.721	10.819	1.462	—
Sr^{91}		53	90.939053±20	788.259	5.905	10.801	—	—
Sr^{92}		54	91.940232±80	795.527	7.268	10.270	1.363	—
Y^{86}		47	85.942947±65	742.016	—	5.236	—	—
Y^{87}		48	86.938490±220	754.533±0.205	12.516	5.726	—	—
Y^{88}		49	87.937400±300	763.914±0.280	9.381	6.717	—	—
Y^{89}*		50	88.934147±15	775.310	11.395	6.936	2.014	—
Y^{90}	39	51	89.935840±100	782.099	6.790	7.467	—	—
Y^{91}		52	90.936193±12	790.137	8.038	7.783	1.248	—
Y^{92}		53	91.938172±65	796.661	6.524	8.402	—	—
Y^{93}		54	92.939586±280	803.710±0.260	7.050	8.183	0.526	—
Y^{94}		55	93.941896±320	809.926±0.298	6.215	—	—	—
Zr^{87}		47	86.942250±260	750.247±0.242	—	8.230	—	2.994
Zr^{88}		48	87.937835±300	762.724±0.280	12.477	8.191	—	2.465
Zr^{89}		49	88.937136±150	771.741	9.017	7.827	—	1.110
Zr^{90}*		50	89.933271±20	783.706	11.965	8.397	2.948	1.461
Zr^{91}*		51	90.934543±10	790.888	7.182	8.789	—	1.322
Zr^{92}*		52	91.934322±22	799.460	8.572	9.324	1.390	1.540
Zr^{93}	40	53	92.936266±120	806.017	6.556	9.356	—	0.954
Zr^{94}*		54	93.936126±25	814.514	8.497	10.803	1.941	2.620
Zr^{95}		55	94.939208±25	820.010	5.496	10.084	—	—
Zr^{96}*		56	95.938868±46	828.690	8.680	—	3.184	—
Zr^{97}		57	96.940635±70	835.414	6.724	—	—	—
Nb^{89}		48	88.941286±200	767.092	—	4.368	—	—
Nb^{90}		49	89.939891	776.757	9.665	5.016	—	—
Nb^{91}		50	90.936063±50	788.688	11.931	4.982	2.266	—
Nb^{92}		51	91.936046±50	797.070	8.382	6.182	—	—
Nb^{93}*	41	52	92.935862±25	805.608	8.538	6.147	0.156	—
Nb^{94}		53	93.937182±80	812.745	7.137	6.728	—	—
Nb^{95}		54	94.937008±20	821.274	8.528	6.760	1.391	—
Nb^{96}		55	95.938523±50	828.229	6.956	8.219	—	—
Nb^{97}		56	96.938795±60	836.342	8.113	7.652	1.157	—
Mo^{90}		48	89.942611	773.440	—	6.348	—	1.980
Mo^{91}		49	90.940813±200	783.480	10.041	6.723	—	1.707
Mo^{92}*		50	91.936098±42	796.237	12.757	7.549	2.716	2.567
Mo^{93}		51	92.936520±100	804.210	7.973	7.140	—	0.958
Mo^{94}*		52	93.935024±52	813.970	9.759	8.362	1.786	2.215
Mo^{95}*		53	94.936015±15	821.413	7.443	8.668	—	1.940
Mo^{96}*	42	54	95.935173±44	830.564	9.150	9.290	1.707	2.530
Mo^{97}*		55	96.936755±30	837.457	6.893	9.228	—	1.009
Mo^{98}*		56	97.936543±16	846.021	8.564	9.678	1.671	2.026
Mo^{99}		57	98.938884±55	852.207	6.186	—	—	—
Mo^{100}*		58	99.939291±35	860.194	7.987	—	1.801	—
Mo^{101}		59	100.942031±80	866.010	5.815	—	—	—
Tc^{94}		51	93.939601±130	808.923	—	4.713	—	—
Tc^{95}		52	94.938000±150	818.780	9.857	4.810	0.097	—
Tc^{96}		53	95.938333±325	826.836±0.303	8.056	5.423	—	—
Tc^{97}		54	96.936800	836.630	9.794	6.066	1.738	—
Tc^{98}		55	97.937691±85	844.167	7.537	6.710	—	—
Tc^{99}	43	56	98.937414±50	852.791	8.624	6.770	1.087	—
Tc^{100}		57	99.938812±250	859.856±0.233	7.065	7.648	—	—
Tc^{101}		58	100.939021±60	868.027	8.172	7.833	1.107	—
Tc^{102}		59	101.940719±350	874.812±0.325	6.785	8.803	—	—
Ru^{95}		51	94.940570±300	815.602±0.280	—	6.679	—	1.966
Ru^{96}*		52	95.937865±15	826.487	10.885	7.707	—	2.897
Ru^{97}		53	96.937238	835.437	8.950	8.601	—	3.178
Ru^{98}*		54	97.935871±60	845.076	9.639	8.446	0.689	2.380
Ru^{99}*		55	98.937100±50	852.298	7.222	8.132	—	1.422
Ru^{100}*	44	56	99.935932±74	861.752	9.454	8.961	2.232	2.191
Ru^{101}*		57	100.937271±44	868.872	7.120	9.016	—	1.368
Ru^{102}*		58	101.936439±72	878.013	9.141	9.986	2.021	2.153
Ru^{103}		59	102.937921	884.999	6.986	10.187	—	1.384
Ru^{104}*		60	103.938132±23	893.169	8.170	—	1.184	—
Rh^{98}		53	97.940531±180	839.952	—	4.515	—	—
Rh^{99}		54	98.939330±70	849.437	9.485	4.361	—	—
Rh^{100}	45	55	99.939700±250	857.459±0.233	8.022	5.160	—	—
Rh^{101}		56	100.939188±350	866.302±0.325	8.845	4.550	0.821	—
Rh^{102}		57	101.938560±300	875.253±0.280	8.951	6.381	—	—
Rh^{103}		58	102.937121	884.959	9.706	6.946	0.755	—
Rh^{104}		59	103.938320	892.209	7.250	7.210	—	—

*Isotopes measured in the present experiment.

clusion also applies to the calculated doublet OH_8-C_2.

As is seen from Table I, half of the isotope masses reported in the present article have not been measured with a mass spectroscope. The masses of the isotopes Ru^{100} and Ru^{101} were measured for the first time.

Comparison of the values of the masses obtained in the present experiment with the mass spectroscopic data and the data of Wapstra[5] (col-

umns 5 and 6 of Table I) indicates that there is no systematic deviation of one group of measurements with respect to the other. In most cases, the differences fall within the combined error of measurement. The exceptions are the isotopes Nb^{93}, Zr^{90}, and Zr^{92}. The best agreement occurs for the results of Isenor et al.[8] (1960). There is satisfactory agreement on the masses of molybdenum and ruthenium with the data of Duckworth et al., but the errors in the measured masses are large (0.4 — 0.8 mmu).[4,7,9,10]

BINDING ENERGY OF NUCLEONS IN THE NUCLEUS

The mass values of the nuclei obtained in the present experiment make it possible to determine with better accuracy the binding energy of nucleons in the nucleus, the binding energy of the last neutron and proton, and the pairing energy of neutrons and protons.

Besides measuring the masses of 25 stable isotopes of Sr, Y, Zr, Nb, Mo, and Ru, we calculated the masses and binding energies of 59 radioactive isotopes. The calculation was made by means of the Q values measured in nuclear reactions and α and β transitions.[13,14] The isotope masses measured in the present experiment served as reference values. The masses were calculated from all the data available in the sources already referred to. Wherever possible, we calulated the masses by several independent methods. The final value was obtained by averaging the results with a "weight" for the measurement. In all, 86 Q values were used. In six cases, the Q values were discarded as incompatible with the results of other reactions and mass-spectroscopic measurements.

From these data, we calculated the binding energies of the nuclei, the binding energies per nucleon (E/A), the binding energies of the last neutron B_n and of the last proton B_p, and the pairing energies of neutrons P_n and protons P_p. The results of the calculations are listed in Table III. Column 4 of this table gives the masses of the measured and calculated isotopes with the corresponding errors of measurement. Where the errors are not shown (eight cases), only the upper or lower limit of the Q value for the given reaction was known. The errors in the binding energies of the nuclei (column 5) are given only if they exceed 200 kev. The values of B_n and B_p, in most cases, have errors not exceeding 0.3 Mev. Correspondingly, errors in the values of P_n and P_p do not exceed 0.4 Mev. As auxiliary standards,

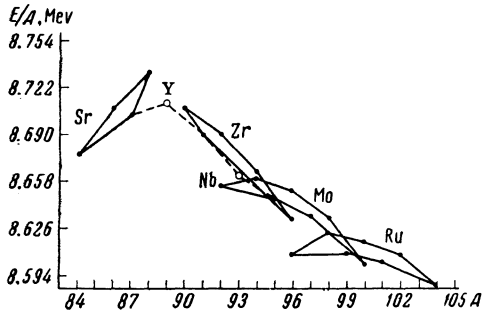

FIG. 1. Binding energy per nucleon in the range $84 \leq A \leq 104$.

we used the following masses:[11,12] $n_0 = 1.008985 \pm 1$; $H^1 = 1.008142 \pm 1$; $C^{12} = 12.003820 \pm 5$; $C^{13} = 13.007491 \pm 3$; $N^{14} = 14.007527 \pm 4$. For the conversion from atomic mass units to energy units, we used the relation: 1 amu = 931.145 ± 0.010 Mev. The symbols H, C, N, and O everywhere denote the isotopes occurring with greatest abundance, i.e., H^1, C^{12}, N^{14}, and O^{16}.

The binding energy per nucleon gives a general characteristic of the binding energy of nucleons in the nucleus. A graphical picture of the average binding energy per nucleon for stable nuclei in the region $84 \leq A \leq 104$ is shown as a function of A in Fig. 1. Binding energies of isotopes with the same Z are joined by solid lines. The binding energies of nuclei with odd Z (Y, Nb) are denoted by circles and are joined by broken lines to elements with odd A and even Z. The maximum errors in the values of E/A are 0.6 — 0.8 kev, i.e., they were too small to show in the figure (on the corresponding scale).

The mass region $84 \leq A \leq 104$ is of interest in connection with the fact that it includes the "magic" number N = 50. There should be a break in the curve of binding energy per nucleon in the region of the "magic" number. As seen in Fig. 1, a sharp break in the curve of binding energy per nucleon is observed at N = 50 for both even and odd Z (nuclei $_{38}Sr^{88}_{50}$, $_{39}Y^{89}_{50}$, and $_{40}Zr^{90}_{50}$). In this connection, it should be stressed that certain patterns of variation in the curve of binding energy per nucleon noted in previous studies[15,16] are observed in the nuclear range under consideration. Thus, in particular, there is a marked difference in the binding energies of the nucleons between nuclei with even and odd mass numbers. The curve of binding energy per nucleon for odd A and Z = const nowhere intersects the curve for even A.

A jump of about 3 — 3.5 Mev in the binding energy of the (N+1)th neutron after the neutron shells are filled (N = 50) is clearly observed. This is 30 — 40% (depending on the nucleus) of

Table IV

Isobar	Value and error, mmu	
	Geiger et. al.[17]	Present experiment
$_{42}Mo^{92}_{50} - {}_{40}Zr^{92}_{50}$	1.340±0.260	1.776±0.048
$_{40}Zr^{94}_{54} - {}_{42}Mo^{94}_{52}$	1.220±0.200	1.102±0.058
$_{40}Zr^{96}_{56} - {}_{42}Mo^{96}_{54}$	3.630±0.280	3.695±0.064
$_{44}Ru^{96}_{52} - {}_{42}Mo^{96}_{54}$	3.020±0.220	2.692±0.050
$_{40}Zr^{96}_{56} - {}_{44}Ru^{96}_{52}$	0.610±0.360	1.003±0.050
$_{42}Mo^{98}_{56} - {}_{44}Ru^{98}_{54}$	—	0.672±0.065
$_{42}Mo^{100}_{58} - {}_{44}Ru^{100}_{56}$	—	3.359±0.080

the value of the binding energy of the nearest even neutron. It is of interest to analyze the numerical data for the values of B_n, B_p, P_n, and P_p in this mass region. Over the entire range of measured and calculated masses, the average binding energy is 8.470 Mev for the last neutron and 7.670 Mev for the last proton. Consequently, the observation[15] that the binding energy of the last neutron is greater than the binding energy of the last proton is valid for this mass region, too. The average binding energy of the last neutron B_n is 8.480 Mev for even Z and 8.440 Mev for odd Z, i.e., equal within the limits of measurement error. The picture for the average binding energy of the last proton is quite different: B_p is equal to 8.960 Mev for even Z and 6.380 Mev for odd Z. As we see, the difference (~ 2.580 Mev) is far beyond the limits of experimental error.

The average pairing energy is P_n = 1.580 Mev for neutrons and P_p = 1.930 Mev for protons. Although the difference lies within the limits of experimental error, it is nevertheless seen that the average proton pairing energy is higher than the average neutron pairing energy.

The obtained experimental data permit one to calculate the mass difference for seven isobaric pairs. In Table IV, the resulting values of the mass differences are compared with those available in the literature. The binding energies of the isobars in which two protons are replaced by two neutrons are not always large, as was noted earlier.[16] In the region under study, the following two isobars are exceptions:

$$_{40}Zr^{96}_{56} - {}_{42}Mo^{96}_{54} \text{ and } {}_{42}Mo^{100}_{58} - {}_{44}Ru^{100}_{56}.$$

This result can be explained qualitatively by considering the parabolic curves of the binding energies of the isobars constructed for mass numbers A = 92, 94, 96, 98, and 100 (see Fig. 2). A group of parabolas corresponding to isobaric pairs is shown in the figure. The value of the symmetric

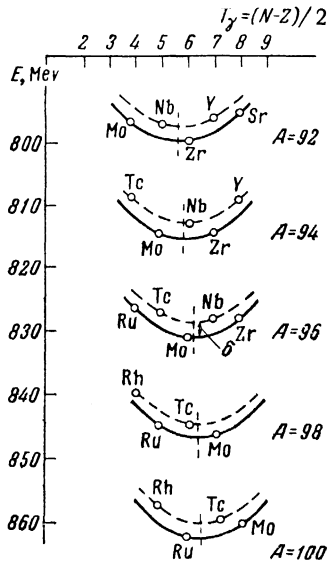

FIG. 2. Binding energy of nucleus as a function of the neutron excess T_γ.

term T_γ is laid off along the abscissa axis and the value of the binding energy of the nucleus in Mev is laid off along the ordinate axis.

In addition to the parabolas for even-even nuclei (solid lines), we constructed parabolas corresponding to radioactive odd-odd nuclei (dotted lines). For the construction of the parabolas, we used the experimental data for the binding energies of nuclei measured in the present work. Moreover, we used an expression for the calculation of the symmetry energy $T_\gamma = (N - Z)/2$ in the semiempirical formula for the binding energy[18] at

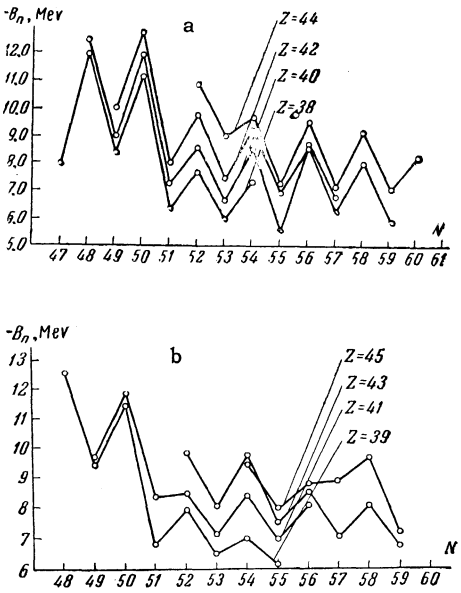

FIG. 3. Binding energy of the last neutron as a function of the number of neutrons N: a – for even Z; b – for odd Z.

Table V

Isotope	$A=92$; $T_{\gamma\,min}=5.71$					$A=94$; $T_{\gamma\,min}=5.88$				
	Sr	Y	Zr	Nb	Mo	Y	Zr	Nb	Mo	Tc
T_γ	8	7	6	5	4	8	7	6	5	4
E, Mev	795,7	796.7	799.5	797.1	796,2	809.9	814.5	812.7	814.0	808.9

Isotope	$A=96$; $T_{\gamma\,min}=6.08$					$A=98$; $T_{\gamma\,min}=6.36$				$A=100$; $T_{\gamma\,min}=6.50$			
	Zr	Nb	Mo	Tc	Ru	Mo	Tc	Ru	Rh	Mo	Tc	Ru	Rh
T_γ	8	7	6	5	4	7	6	5	4	8	7	6	5
E, Mev	828.7	828.2	830.6	826.8	826.5	846.0	844.2	845.1	840.0	860.2	859,9	861.8	875.5

which the binding energy is at a minimum for a given mass number A. Following reference 18, we use the expression

$$T_{\gamma min}=\frac{U_c(A-1)\,A^{-1/3}-1/4\,(M_nc^2-M_pc^2)}{2\,(U_\tau A^{-1}+U_cA^{-1/2})},$$

to obtain the value of T_γ for the isobar. The values of the binding energies T_γ and $T_{\gamma\,min}$ are listed in Table V. The values of the parameters $U_c = 0.177$ Mev and $U_\tau = 23.7$ Mev were taken from reference 19. As is seen from Fig. 2, the position of the isobaric pairs relative to the vertex of the parabola (T_γ = min) changes with A. Thus, for example, molybdenum shifts from the left branch of the parabola (A = 92; excess of protons with respect to neutrons) through the center of the parabola (A = 96) to the right branch (A = 100; excess of neutrons with respect to protons). Similar variations occur for zirconium and ruthenium.

All comparisons of isobaric pairs made in earlier studies[15,16] were based on the left branch of the parabolas, since in the region of heavier masses ($A \approx 200$) there are no stable isotopes corresponding to the right branch of a similar parabola, owing to the instability of the nuclei as regards β decay.

From an analysis of Fig. 2, it is readily seen that there is a difference δ equal to the difference in the binding energy for even-even and odd-odd nuclei. The mean value of $\frac{1}{2}\delta$ for this mass region (A = 92 — 100) is 1.05 Mev. This value is in very good agreement with the value of the term taking into account the pairing effect in the semiempirical formula for the nuclear binding energy. This term, according to reference 19, is expressed in the form: $C_{pair}A^{-3/4}\delta$, where $C_{pair} = 34$ Mev and $\delta = +1$ or -1, depending on the parity of the nucleus. The mean value of this expression for the given mass region is 1.11 Mev. The difference between the experimental and theoretical values is equal to 0.06 Mev, which lies within the limits of experimental error.

Figure 3 shows the values of B_n as functions of N for Z = const for even and odd Z. As seen from Fig. 3, the value of B_n is a maximum at N = 48, and not at N = 50. The difference is rather considerable (~ 1.0 mmu) and exceeds the measurement error. Nuclei with N = 50 have somewhat smaller values of B_n, but these values are significantly larger than any even or odd number of neutrons for N > 50. There is good agreement with the parity rule: the binding energy of

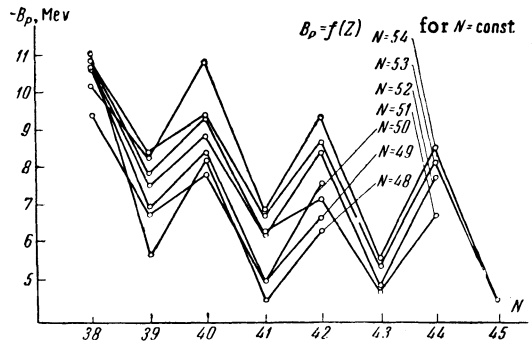

FIG. 4. Binding energy of the last proton as a function of the number of protons Z (N = const).

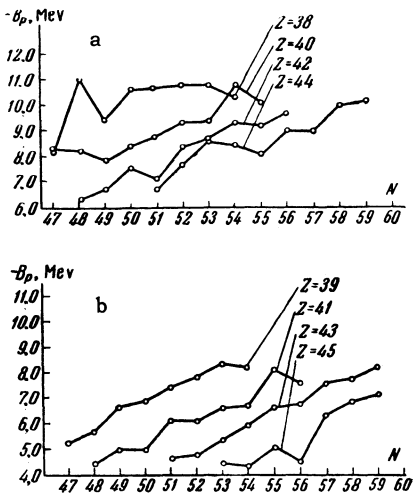

FIG. 5. Binding energy of the last proton as a function of the number of neutrons N: a — for even Z; b — for odd Z.

the last neutron is always greater for nuclei with even N than for neighboring nuclei with odd N. With an increase in the number of neutrons in the nucleus, the values of B_n decrease for both even and odd Z. The values of B_n increase with Z.

Figure 4 gives the dependence of the binding energy of the last proton on the number of protons for N = const. In this case, too, the parity rule is strictly fulfilled. The character of the curves shows no change whatsoever as a function of the number of neutrons. There are no visible changes as one goes from the region with unfilled neutron shells (N = 48, 49) to the region of filled shells (N = 51, 52). Figure 5 shows the dependence of B_p on the number of neutrons for Z = const. There is a visible monotonic decrease in B_p (for both even and odd N) with an increase in Z. The values of $B_p(N)$ for even and odd Z increase with the number of neutrons. This increase, however, is not always monotonic (Z = 42, 44).

The neutron pairing energies for the "magic" number N = 50 have a maximum for Z = 39, 41, and 42. At the same time, the value of P_n for N = 48 attains values higher than the maximum for N = 50.

In some nuclei with N = 50 ($_{40}Zr^{90}_{50}$, $_{41}Nb^{91}_{50}$, $_{42}Mo^{92}_{50}$), the values of B_n and P_n are the maximum values as compared to nuclei which do not have a "magic" number of neutrons. It is of interest to note, however, that nuclei with 48 neutrons have anomalously high values of B_n and P_n ($_{38}Sr^{86}_{48}$, $_{39}Y^{87}_{48}$, $_{40}Zr^{88}_{48}$), higher than for nuclei with the magic number of neutrons N = 50.

The proton pairing energies, in the great majority of cases, are greater in the case of even-even nuclei, but there are two exceptions ($_{40}Zr^{87}_{47}$ and $_{44}Ru^{97}_{53}$). It is possible that these exceptions are connected with the fact that the calculation of the proton pairing energy for even-odd nuclei is not entirely valid, since in this case, owing to the presence of an odd nucleon of a different type in the nucleus, a so-called residual np interaction occurs. It is not appropriate to identify this interaction with pairing energy.[20]

Analysis of the data in Table III and Figs. 3, 4, and 5 for the binding energies of the last neutron and proton and the values of the pairing energies makes it possible to state that there are no appreciable changes of these parameters for Z = 40. Consequently, the suggested existence of a "subshell" or "semi-magic" number at Z = 40 is not confirmed by the experimental data. A similar conclusion was already made in a number of papers of a statistical character.[20,21]

In conclusion, the authors consider it their obligation to express their gratitude to E. E. Baroni and V. M. Soĭfer for the preparation of the chloride compounds of a number of elements used in the present work and to G. A. Dorokhova for aid in the work.

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