THERMAL CONDUCTIVITY AND DIFFUSION IN WEAK He³—He⁴ SOLUTIONS IN THE TEMPERATURE RANGE FROM THE λ POINT TO 0.6°K

T. P. PTUKHA

Institute for Physics Problems, Academy of Sciences, U.S.S.R.

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Using a thermal flow method, measurements have been made of the effective thermal conductivity coefficient κ_{eff} for He³-He⁴ solutions over a range in He³ molar concentration from 10^{-4} to 10^{-2}. The thermal conductivity coefficients κ and diffusion coefficients D have been derived from these values of κ_{eff}. The thermal conductivity coefficient increases as the temperature is decreased; for any given temperature, κ becomes smaller as the solution concentration increases. Various quantities characterizing the interaction of the impurity and thermal excitations have been computed on the basis of the data obtained. The results are found to be in satisfactory agreement with the Khalatnikov-Zharkov theory.

Landau and Pomeranchuk have shown[1] that impurities (He³ atoms) dissolved in liquid helium below the λ point do not take part in superfluid motion, but enter into the normal component of the solution. For weak solutions, in which the interaction of the impurity particles with one another is negligibly small, the solution can be treated as a mixture of three gases of elementary excitations: a roton gas, a phonon gas, and a gas of impurity excitations. The thermal (rotons and phonons) and impurity excitations, which transfer energy and momentum upon colliding with one another, are responsible for the kinetic processes in the solution.

In the present work a thermal flow method was used to investigate the phenomena of diffusion and thermal conductivity in weak He³-He⁴ isotopic mixtures. From the results of experiments to study these processes one can compute certain quantities characterizing the interaction and scattering laws for the impurity and thermal excitations.

The processes of heat transfer and diffusion of He³ in He⁴ below the λ point may be represented in the following form: at the initial moment, when the heat flux is zero, the impurities have a uniform distribution throughout the thermal excitation gas; the partial pressures of the impurity and thermal excitation gases are everywhere the same. When power is applied to the heater, the thermal and impurity excitations will move to the cold end of the reservoir. The heat carried by them is

$$q = \rho \sigma T v_n, \tag{1}$$

where ρ is the density and σ the entropy per unit mass of the mixture, and v_n is the velocity of the normal component of the liquid. As a result of this process there is developed a concentration gradient ∇c. In the steady-state condition it is necessary that the osmotic pressure arising as a result of the presence of a concentration gradient be balanced by the thermomechanical pressure (thermo-osmosis):

$$(kT/m_3) \nabla c = - \sigma_0 \nabla T, \tag{2}$$

where m_3 is the mass of the He³ atom, σ_0 is the entropy per unit mass of pure He⁴, k is the Boltzmann constant, and $c = N_3 m_3/(N_3 m_3 + N_4 m_4)$ is the concentration (m_4 is the mass of the He⁴ atom; N_3 and N_4 are the numbers of He³ and He⁴ atoms per unit volume); for weak mixtures, $c = N_3 m_3/N_4 m_4 = \epsilon m_3/m_4$, where ϵ is the molar concentration.

The phenomenon of thermo-osmosis has been observed experimentally. The most complete quantitative measurements have been made by Wansink and Taconis.[2]

Thus, in the presence of a thermal current there appear in the reservoir that contains the mixture a concentration gradient ∇c directed from the cold to the warm end, and a temperature gradient ∇T in the opposite direction, which give rise to inverse processes — diffusion of the impurities and of the thermal excitations. The additional thermal flow arising from the transfer of heat by the diffu-
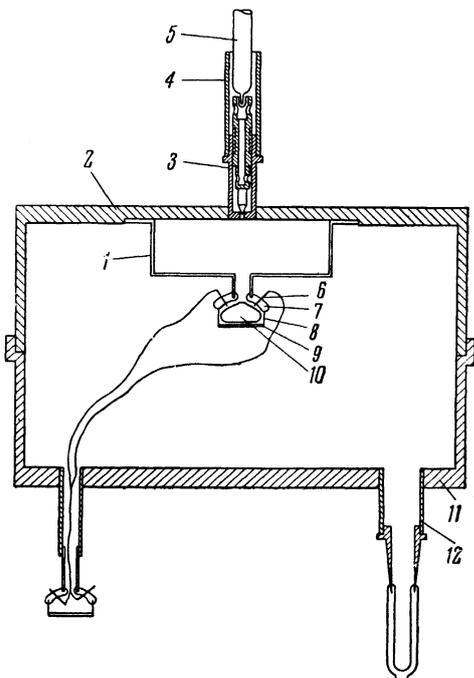
FIG. 1. Diagram of apparatus for thermal conductivity and diffusion measurements in He^3 — He^4 solutions.

sion of the thermal excitations is characterized by a thermal conductivity coefficient κ.

A theory of diffusion and thermal conductivity for weak He^3-He^4 solutions has been developed by Khalatnikov and Zharkov.[3] In the general case, the total heat flux $\mathbf{Q}$ is related to the temperature gradient by the following expression:

$$\mathbf{Q} = - [(\rho D m_3 \sigma_0^2/kc)(\rho_n/\rho_{n0})^2 + \varkappa] \nabla T, \qquad (3)$$

where D is the impurity diffusion coefficient, and ρ_{n0} is the fraction of the solution normal component density (ρ_n) contributed by the rotons and phonons.

The proportionality coefficient between the heat flux $\mathbf{Q}$ and ∇T is the effective thermal conductivity coefficient of the mixture

$$\varkappa_{eff} = \frac{\rho D m_3 \sigma_0^2}{kc}\left(\frac{\rho_n}{\rho_{n0}}\right)^2 + \varkappa = \varkappa_{M} + \varkappa. \qquad (4)$$

In this expression, the first term, κ_{M}, characterizes the heat transfer due to motion of the elementary excitations as a whole (i.e., motion of the normal component of the solution), and the second term, κ, is associated with the heat flow arising, as in the case of ordinary condensed bodies, from diffusion of the thermal excitations (the thermal conduction process).

Under stationary conditions, it is the effective thermal conductivity coefficient κ_{eff} which is determined experimentally. Only in the limiting case
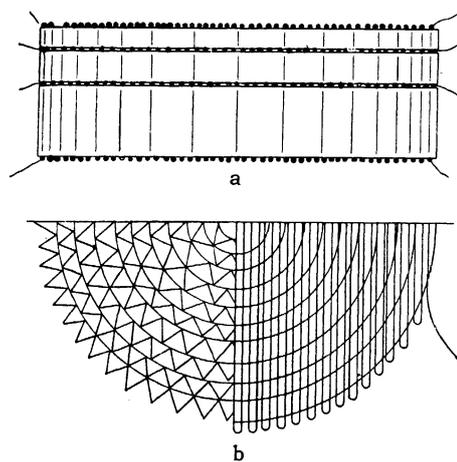

FIG. 2. Interior portion of apparatus: a — side view; b — top view.

$\kappa \gg \kappa_{M}$ is it possible to find separately the thermal conductivity coefficient κ, while under conditions for which $\kappa_{M} \gg \kappa$ the diffusion coefficient D can be determined.

METHOD OF MEASUREMENT

Several kinds of apparatus were tried out in the course of the present studies; two of these are represented in the results given here. Figure 1 illustrates the apparatus with which the preliminary experiments were conducted, and results obtained in the temperature region from the λ point to 1.5° K. In this case, low temperatures were attained by pumping He^4 vapor (a second system was used to produce ultra-low temperatures, with the aid of He^3).

The reservoir 1, in which the mixture under study was held (inside diameter 19.52 mm, height 6.57 mm), was turned from a single piece of stainless steel and had a wall thickness of 0.2 mm. In order to provide good thermal contact with the helium bath, the cover 2 of the reservoir was made of copper. To the cover was soldered a small valve 3, communicating with a tube 4 through which the mixture was introduced into the volume 1. The valve was operated within the Dewar by means of a Stay-bright rod 5; the latter could be decoupled from the valve and raised, in order to prevent heat conduction along it to the reservoir.

Four resistance thermometers were assembled for measuring the temperature gradient. The support for the thermometers consisted of three flat disks (shown in cross-section in Fig. 2a) consisting of smooth and corrugated ribbons coiled together. Parchment was used as the material for the ribbons; in addition to its poor thermal conduc-

tivity, this substance possesses another essential property — its linear dimensions change little with temperature. Inasmuch as the thickness of each disk determines the distance between thermometers, which must be accurately known, the preparation of the disks presented certain difficulties. However, since the disks were prepared in precision-made forms, their dimensions can be guaranteed to an accuracy of 0.01 mm. The thermometers were made of 35-micron phosphor bronze wire, laid out in a flat zig-zag pattern across the surfaces of the disks, and fixed in place with BF cement (Fig. 2b).

After preparation of the thermometers the maximum error in the value of Δx (the distance between thermometers) was found to be 0.05 mm.

Current and voltage leads for the electrical circuit were carried out of the reservoir by means of a platinum glass seal 7 with platinum wires 6 bonded into the glass. The seal terminated in a platinum ring 8 and was closed off by a copper plate 9. A glass block 10 ground to match its interior dimensions was placed within the seal, making its volume negligibly small (it amounted to 1% of the reservoir volume).

The heat flow in the mixture under study was generated by a plane constantan heater. To avoid convection, the heater was placed at the bottom of the reservoir.

The experimental volume 1 was surrounded by a vacuum jacket formed by the cover 2 in conjunction with a cylindrical copper vessel 11. The tube 12 served for evacuating the jacket.

The experiment was carried out in the following manner. With the aid of a Toepler pump a known quantity of the He^3-He^4 mixture was condensed into the reservoir 1 through a small charcoal trap cooled by liquid nitrogen. Condensation proceeded at a temperature somewhat below that of the λ point. When the necessary quantity of the mixture had been condensed, the valve 3 was immediately closed. Thus any danger of variation in the concentration in the liquid due to He^3 enrichment of the gaseous phase was completely avoided. Any surplus of the mixture above the valve was then pumped off with the Toepler pump; the presence of a vacuum in the tube 4 was regulated by a thermometer lamp. The inner helium bath was brought to the specified temperature, which could be maintained constant to an accuracy of 10^{-5} °K with the aid of the stabilizing apparatus developed by Vetchinkin.[4] Temperature measurements were then carried out for various thermal fluxes from the heater.
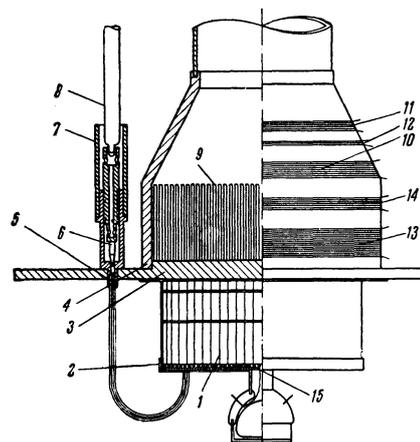

FIG. 3. Apparatus for ultra-low temperatures.

The magnitude of the thermal flux was determined from the known resistance of the heater and the current flowing through it. Since the reservoir 1 had thin stainless-steel walls, heat leakage along its walls was negligibly small, amounting to 0.5% of the total power supplied to the heater.

The thermometer resistances were measured with an ordinary potentiometer. For convenience in measuring the resistance, a high degree of stability in the currents (to 10^{-5} of their net value) in both the potentiometer and measuring circuits was insured by use of the weak-current stabilizer described by Vetchinkin.[5]

So far, we have described the apparatus used to obtain data in the temperature region from the λ point down to 1.15° K. Temperatures below 1.15° K were attained by pumping He^3 vapor. All experiments with He^3 were carried out in an ultra-low temperature cryogenic system constructed under the supervision of Peshkov.[6]

The apparatus employed for the measurements in this temperature range is illustrated schematically in Fig. 3. The chamber for the mixture to be studied was a thin-walled stainless steel reservoir having an inside diameter of 20 mm and a height of 6.67 mm. The copper cap 2 served as the bottom of the chamber. The reservoir was in contact with the He^3 bath 4 through the copper cover 3. The volume was filled with the mixture by means of the capillary 5 (of 0.4 mm i.d.), the valve 6, and the thin-walled tube 7. The rod 8 by which the valve was operated could be disengaged. Since liquid He^3 has extremely poor thermal conductivity, an additional copper latticework 9 was silver-soldered to the bottom of the copper-walled bath 4. This significantly improved the thermal contact between the warm lower and cooler upper layers of the liquid.

The temperature of the cold surface 3 was held constant with the aid of the electronic stabilizer mentioned above.[4] The phosphor bronze thermometer 10, the signals from which, amplified and transformed, were applied to the heater 11, served as a temperature sensing device. The thermometer 10 was incorporated into a bridge circuit; the ballast resistance 12, placed in helium, served as its other branch. This arrangement made it possible to maintain the temperature of the He³ bath constant to an accuracy of 10^{-4}°K. The He³ bath, together with the mixture reservoir, was surrounded by a vacuum jacket.

Four resistance thermometers of 35-micron phosphor bronze wire (such thermometers possess a linear characteristic from 2°K to 0.7°K) were prepared, in the manner described above (Fig. 2), for measurement of the temperature gradient in this apparatus. The thermometers were calibrated against the He³ vapor pressure, which was measured with a MacLeod gauge. In addition, to provide standardization of the calibration curves, two resistance thermometers, made of 50-micron aluminum (13) and 53-micron cadmium wire (14) were wound onto the outside of the He³ bath. The temperatures of the transitions of aluminum and cadmium into the superconducting state served as reference points.

The heat flow was generated by the heater 15.

In the work described here, measurements were carried out for solutions having three different concentrations. The purity of the He³ used in preparation of the mixtures was determined mass-spectrometrically,* and was not less than 99.9995%. The relative error in concentration of a mixture was 1% for the lowest concentration and 0.5% for the higher ones.

Before passing on to a consideration of the results obtained with the systems thus described, it is necessary to explain why it was required that the interior volume of the reservoir containing the mixture be divided into the cells formed by coiling the corrugated ribbons into flat disks. In the initial experiments the thermometers were wound onto flat rings of ivory. The coils in each of them were strung through specially-cut notches in the rings in such a manner that they lay in one plane. Joined together, the rings formed a cylinder 18.2 mm in inside diameter. The thermal current was generated by a plane heater. The framework containing the thermometers and the

heater was placed within the apparatus illustrated in Fig. 1, with the heater, to avoid convection, situated at the bottom of the reservoir 1. The experiment was carried out in the same fashion as that described above.

The results achieved in the measurements with thermometers of this construction were, briefly, as follows. With no power applied to the heater, no temperature differences were observed in the experimental volume. With heat flow present, a temperature gradient ∇T did appear, but this temperature distribution varied from experiment to experiment in such a way that for a given initial bath temperature and heat-flux the results varied by a factor of ten. The results of the measurements were so chaotic that it was difficult to distinguish any regularity in them, save only that a dependence of ∇T on power was observed; the greater the power supplied to the heater, the smaller the temperature difference as compared with that to be expected for the given heat-flux. The impression was formed that some sort of secondary convection currents were established in the reservoir, which led to a reduction in the concentration and temperature gradients. Inasmuch as for the solutions under investigation, below the λ point, the density falls with decreasing temperature[7] — i.e., the denser layers of the liquid were located further down in the reservoir — these could not be currents due to natural convection. To avoid these secondary convection currents, therefore, the volume of the reservoir was divided into cells, of average cross-section area 0.6 mm².
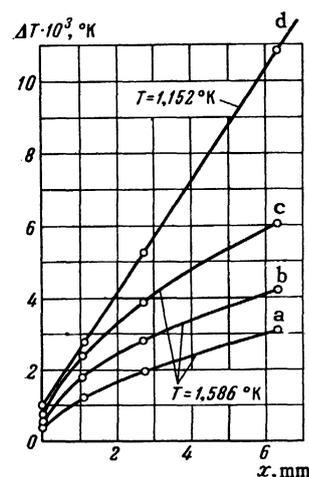

FIG. 4. Temperature distribution in reservoir for solutions of concentration $\varepsilon = 1.32 \times 10^{-3}$ of He³ in He⁴: a — for Q = 147.2; b — for Q = 212.1; c, d — for Q = 289.2 erg/cm² sec.

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*The He³ mass-spectrometer analysis was carried out in the laboratory of N. E. Alekseevskiĭ (Institute for Physics Problems, Academy of Sciences, U.S.S.R.).

	ε = 1,39·10⁻⁴					ε = 1,32·10⁻³					ε = 1,36·10⁻²	
T, °K	$10^{-4}\,\kappa_{eff}$, ergs/cm-sec-degree	$10^{-4}\,\kappa$, ergs/cm-sec-degree	K(T), ergs/cm-sec-degree	$10^4 D$, cm²/sec	T, °K	$10^{-4}\,\kappa_{eff}$, ergs/cm-sec-degree	$10^{-4}\,\kappa$, ergs/cm-sec-degree	K(T), ergs/cm-sec-degree	$10^4 D$, cm²/sec	T, °K	$10^{-4}\,\kappa_{eff}$, ergs/cm-sec-degree	$10^{-4}\,\kappa$, ergs/cm-sec-degree
1.918	0.58	—	60	2.1	2.150	9.4	—	93	0.94	2.061	115	—
1.65	0.42	—	44	6.8	2.060	8.6	—	85	1.16	1.960	105	—
1.501	0.21	—	22	10.4	1.951	7.1	—	70	1.84	1.815	91	—
1.360	0.126	—	13	22.0	1.920	6.8	—	67	2.3	1.664	77	—
1.296	0.097	—	10.8	27.0	1.804	5.8	—	57	3.4	1.615	72	—
1.245	0.070	—	7.3	35.0	1.694	4.7	—	47	5.7	1.484	65	—
1.145	0.058	—	6.0	—	1.615	3.8	—	37	7.7	1.354	68	68
1.055	0.078	0.078	—	—	1.586	3.4	—	33	9.8	1.330	66	66
1.022	0.074	0.074	—	—	1.501	2.7	—	—	—	1.265	72	72
0.901	0.130	0.130	—	—	1.483	2.7	—	—	—	1.222	75	75
0.896	0.121	0.121	—	—	1.465	2.6	—	—	—	1.202	81	81
0.850	0.180	0.180	—	—	1.374	2.3	—	—	—	1.165	78	78
0.773	0.31	0.31	—	—	1.314	2.1	—	—	—	1.160	78	78
0.761	0.35	0.35	—	—	1.265	2.1	1.15	—	—	1.129	91	91
0.670	0.59	0.59	—	—	1.152	1.94	1.94	—	—	0.890	122	122
					1.141	1.87	1.87	—	—	0.794	145	145
					1.090	2.1	2.1	—	—	0.667	163	163
					1.073	2.4	2.4	—	—			
					0.948	4.2	4.2	—	—			
					0.826	7.9	7.9	—	—			
					0.773	9.8	9.8	—	—			
					0.662	14.7	14.7	—	—			
					0.607	17.7	17.7	—	—			

RESULTS OF THE MEASUREMENTS

Preliminary results of the present work have already been published.[8] In this article a more detailed treatment of these results will be carried out, and data obtained for a new mixture of concentration $\epsilon = 1.39 \times 10^{-4}$ He will also be presented.

As already mentioned, several thermometers were prepared for measurements of the temperature in the experimental volume. This made it possible to clarify the nature of the temperature distribution along the direction of the heat flow. At high temperatures, for which heat transport takes place principally through the motion of the normal component of the liquid ($\kappa_M > \kappa$), the temperature varies along the direction of the heat flow in accordance with the formula[3]

$$T - T_0 = \frac{c_0 dkT}{x_0 m_3 \sigma_0} \frac{e^{-x/x_0} - 1}{1 - e^{d/x_0}}, \qquad x_0 = \frac{kTK(T)}{m_3 Q \sigma_0}, \qquad (5)$$

where d is the height, T_0 the temperature of the cold wall of the reservoir (at which x = 0), and K(T) is a function independent of concentration for which

$$\kappa_M = K(T)/c. \qquad (6)$$

Figure 4 shows several typical curves (a,b,c) obtained in this temperature region for various values of the power supplied to the heater (the y coordinate is the temperature difference ΔT indicated by the thermometers with and without the thermal current present, while the x coordinate is the distance of the thermometers from the cold wall). For comparison, curve d in the same figure shows the function ΔT(x) characteristic of the opposite limiting case, in which heat is transported chiefly by a mechanism analogous to ordinary thermal conductivity: for a power input the same as that for which curve c was obtained, a linear variation of ΔT with x is now observed.

For control purposes, several experiments with pure He⁴ were carried out using each apparatus. In these cases no temperature gradient was observed when the power was turned on: the thermometers all indicated a uniform rise in the temperature of the helium within the reservoir. This temperature rise resulted from the Kapitza discontinuity at the boundary between the liquid helium and the reservoir cover. For a given thermal flux the difference between the final and initial (power-off) temperature of the helium in the reservoir increased as the bath temperature was lowered.
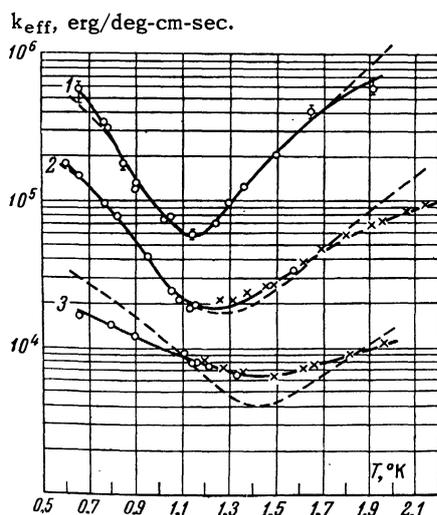
FIG. 5. Variation of the effective thermal conductivity of solutions of He³ in He⁴ with temperature and concentration: curves 1 − $\varepsilon = 1.39 \times 10^{-4}$; curves 2 − $\varepsilon = 1.32 \times 10^{-3}$; curves 3 − $\varepsilon = 1.36 \times 10^{-2}$; dashed curves are theoretical.

The time required for establishment of the steady-state condition never exceeded $10 - 12$ minutes for any of the temperatures and concentrations employed in the experiments. For a given mixture concentration and thermal flux this time decreased as the initial temperature in the reservoir was lowered. Below $1°$ K, equilibrium was reached, practically speaking, in $1 - 2$ min. For a given temperature, the equilibrium time also decreased with decreasing solution concentration.

Knowing the temperature gradient and thermal flux, one can obtain the effective solution thermal conductivity coefficient κ_{eff}. Values of κ_{eff} for various temperatures and concentrations are given in the table.*

The computations of κ_{eff} were carried out in the following manner: at high temperatures, for which, with power applied to the heater, the steady state temperature distribution along the x coordinate had an exponential form (for example, Fig. 4 a, b, c), use was made of Eq. (5). In this case, the experimentally-determined temperature differences and thermal fluxes were used to derive values of the function $K(T)$, from which it was then possible to compute the value of κ_{eff} for each given solution concentration. The values of the function $K(T)$ are given in the table.

For the case of a linear dependence of ΔT upon x (Fig. 4, d) the usual relation

$$Q = - \varkappa_{eff} \, \Delta T/d \qquad (7)$$

was employed.

*All data presented in the table are referred to the 1958 temperature scale.[9]

The variation of the effective thermal conductivity coefficient κ_{eff} with temperature is shown in Fig. 5. The values of κ_{eff} presented in the table and shown in the figure are averages of the values of κ_{eff} computed for various heater input powers. The coefficients κ_{eff} were independent of the magnitude of the thermal flux. The statistical scatter in the values of κ_{eff} computed for various powers at a given temperature did not exceed 10%.

For control, the measurements were performed using both types of apparatus described above and illustrated in Figs. 1 and 2. The results obtained from each (represented by the circles and crosses) agreed well with one another.

It is evident from Fig. 5 that the $\kappa_{eff}(T)$ curves possess a minimum which, as the solution concentration is reduced, shifts toward lower temperatures. This behavior in the curves confirms the hypothesis of the dual nature of heat transfer in a weak He³-He⁴ solution. Specifically, in the high temperature region lying to the right of the temperature of the minimum (T_{min}) the effective thermal conductivity coefficient κ_{eff} falls with decreasing temperature. This implies that in this region the predominant heat transport mechanism is that of motion of the normal component of the solution as a whole toward the cold end of the reservoir (i.e., the case $\kappa_M \gg \kappa$).

At temperatures to the left of T_{min}, the coefficient increases with decreasing temperature. This means that the further the temperature lies below T_{min}, the greater is the part played in the process of heat transfer from the heated to the cold end by a mechanism analogous to ordinary heat conduction (in which the thermal energy is transferred by the diffusion of thermal excitations), and, since the number of thermal excitations falls with decreasing temperature − i.e., their mean free path increases − the thermal conductivity coefficient and, in consequence, the effective thermal conductivity coefficient ($\kappa_{eff} = \kappa$) must grow (the case for which $\kappa \gg \kappa_M$).

In the vicinity of the minimum, clearly, both mechanisms make equal contributions to the heat transfer process in the solution ($\kappa_M \sim \kappa$).

Values of the thermal conductivity coefficient for the case $\kappa = \kappa_{eff}$ are given in the table. From comparison of the κ's obtained for various concentrations it is evident that at low temperatures the impurities cause a fundamental change in the magnitude of κ: for a given temperature the value of the coefficient κ becomes smaller as the He³ concentration of the solution increases. At temperatures below $1.1°$ K the principal role in the
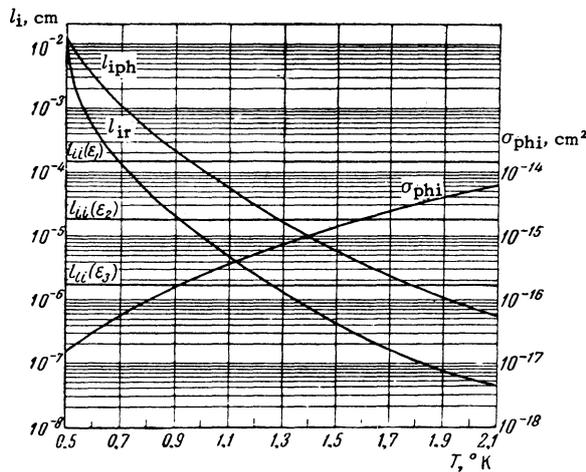
FIG. 6. Impurity mean free paths ($\varepsilon_1 = 1.39 \times 10^{-4}$; $\varepsilon_2 = 1.32 \times 10^{-3}$; $\varepsilon_3 = 1.36 \times 10^{-2}$). Cross section for scattering of phonons by impurities as a function of temperature.

heat conduction process is played by the scattering of phonons by impurities, which leads to a reduction in the phonon mean free path.

In the table are also given the values of the diffusion coefficients D, derived from the κ_{eff} in the alternative limiting case $\kappa_M \gg \kappa$. For the mixtures having He^3 concentrations ϵ equal to 1.32×10^{-3} and 1.39×10^{-4}, it was found possible to determine the coefficients D over the temperature interval from 2.15 down to 1.2° K. In this region diffusion is governed by the scattering of impurities by rotons, and here the diffusion coefficient increases exponentially with decreasing temperature. Inasmuch as the values of the coefficient D obtained for solutions of differing concentration agree well with one another, one may conclude that for weak solutions the diffusion coefficients are independent of concentration. The absolute values of D are in good agreement with the data of Beenakker et al.[10] as well as with the results of Garvin and Reich,[11] who used a spin-echo method for determination of D.

The results of these experiments on the processes of heat conduction and diffusion permit the calculation of several quantities characterizing the interaction and scattering laws for elementary excitations. The following values were used here for the parameters of the roton energy spectrum: $\Delta_r/k = 8.65°$ K, $p_0/\hbar = 1.92 \times 10^8$ cm^{-1}, and $\mu_r = 1.08 \times 10^{-24}$ g.

The value $\mu = 8.5$ m_1 (m_1 is the mass of the proton) was used for the effective mass of an impurity. The cross section σ_{ir} for scattering of impurities by rotons was calculated. The characteristic time for impurity-roton scattering, required for this purpose, was computed from the

experimental value of κ_{eff}, taken at T = 1.5° K for a mixture of concentration $\epsilon = 1.39 \times 10^{-4}$. The absolute value of σ_{ir} is 2.2×10^{-14} cm^2.

According to the theory of Khalatnikov and Zharkov[3] the thermal conductivity coefficient κ is made up of roton, phonon and impurity components: $\kappa = \kappa_r + \kappa_{ph} + \kappa_i$. For the solutions used in these experiments below 1.1° K, the phonon component of the thermal conductivity is much greater than the sum of the roton and impurity components of κ. From the values of the thermal conductivity coefficient κ in this temperature region one can compute the effective cross section $\sigma_{ph\,i}$ for scattering of a phonon by an impurity. In the present work the value

$$\sigma_{ph\,i} = 2.7 \cdot 10^{-17}\, \delta T^5 \text{ cm}^2$$

is obtained for $\sigma_{ph\,i}$, where δ is a certain function of the parameters characterizing the impurity energy spectrum which, as follows from reference 3, has the form $\delta = A + B/T$. Inasmuch as the function $\partial\Delta_i/\partial\rho$ (Δ_i is the zero-point energy of an impurity excitation) was not known, Khalatnikov and Zharkov[3] assumed δ to be of order unity. For the function δ, however, as determined from the experimental values obtained in the present work for the phonon contribution to the thermal conductivity, it is necessary to have $\delta = 10/T$ (A ≪ B).

An article has just appeared, by Staas, Taconis and Fokkens,[12] in which data are presented on the viscosity of solutions. They have found for a certain quantity $\bar{\delta}$ the value 1.86. As follows from Zharkov's theory of viscosity,[13] $\bar{\delta}$ is a function of the temperature and of the parameters determining the impurity energy spectrum. Since, however, the dependence of Δ_i upon ρ was unknown, it was assumed in this theory that $\bar{\delta} = 1$. It was pointed out in reference 13 that the δ in the thermal conductivity theory should differ slightly from the $\bar{\delta}$ appearing in the viscosity theory. It is difficult, however, to make a comparison of these quantities, as calculated from the experimental data on viscosity and thermal conductivity, since in the paper by Staas, et al.[12] it is not stated at what temperatures $\bar{\delta}$ was determined, nor what values were assumed for the theoretical parameters.

The temperature dependence of the phonon-impurity scattering cross section under the condition $\delta = 10/T$ is shown graphically in Fig. 6. Using this expression for δ, we obtain for the derivative $\partial\Delta_i/\partial\rho$

$$\partial\Delta_i/\partial\rho = 4.6\, k/\rho$$

(k is the Boltzmann constant).

Using the cross section σ_{iph} obtained from the experimental results and the value $\delta = 10/T$, and making use of the relations (6.7) and (7.9) of Khalatnikov and Zharkov's paper,[3] one may compute theoretical curves for the temperature dependence of the effective solution thermal conductivity. These curves are shown dashed in Fig. 5 over the regions where they deviate from the experimental curves. It is evident from Fig. 5 that for curves 1 and 2 the experimental and theoretical values agree well, within their limits of accuracy, except in the temperature range above 1.7°K. Above 1.7°K the curves begin to diverge. It should be remarked that the theory developed in reference 3 is correct only for the case in which the rotons constitute an ideal gas. At temperatures above 1.7°K it is possible to make only a qualitative comparison with the theory. A greater divergence between the experimental and theoretical values is found for the mixture of concentration $\epsilon = 1.36 \times 10^{-2}$ He^3. The experimental curve has a less pronounced minimum than would be expected, and deviates from the theoretical curve over the whole temperature range (except at the points of intersection). This solution can evidently no longer be regarded as a weak one.

The impurity mean free paths were calculated from the cross sections for the scattering of the impurities by phonons and rotons obtained from the experimental data. The temperature dependence of the mean free paths for collisions of impurities with rotons (l_{ir}) and with phonons (l_{iph}) is shown in Fig. 6. For comparison, the same figure also shows the path lengths for impurity-impurity scattering (l_{ii}). It is clear from the figure that in the low temperature limit the mean free paths exceed by hundreds of thousands of times the mean interatomic distances. Such large values for the mean free paths cannot of course be explained, if helium is regarded as an ordinary liquid, since if it is assumed that the He^3 atoms are scattered by atoms of He^4, such path lengths could be obtained only for a He^4 density of 8×10^{-6} g/cm^3. These results again bear out the Landau-Pomeranchuk viewpoint that impurities in liquid helium below the λ point interact with the excitation quanta — the rotons and phonons.

[1] L. D. Landau and I. Ya. Pomeranchuk, Doklady Akad. Nauk SSSR **69**, 165 (1949).

[2] D. H. N. Wansink and K. W. Taconis, Physica **23**, 125 (1957).

[3] I. M. Khalatnikov and V. N. Zharkov, JETP **32**, 1108 (1957), Soviet Phys. JETP **5**, 905 (1957).

[4] A. N. Vetchinkin, Приборы и техника эксперимента (Instrum. and Exptl. Techniques) 1 (1961).

[5] A. N. Vetchinkin, ibid. **3**, 97 (1960).

[6] Peshkov, Zinov'eva, and Filimonov, JETP **36**, 1034 (1959).

[7] T. P. Ptukha, JETP **34**, 33 (1958), Soviet Phys. JETP **34**, 22 (1958).

[8] T. P. Ptukha, JETP **39**, 896 (1960), Soviet Phys. JETP **12**, 621 (1961).

[9] H. van Dijk and M. Durieux, Physica **24**, 920 (1958).

[10] Beenakker, Taconis, Lynton, Dokoupil, and Soest, Physica **18**, 433 (1952).

[11] R. L. Garvin and H. A. Reich, Phys. Rev. **115**, 1478 (1959).

[12] Staas, Taconis, and Fokkens, Physica **26**, 669 (1960).

[13] V. N. Zharkov, JETP **33**, 929 (1957), Soviet Phys. JETP **6**, 714 (1958).