LEPTONIC DECAY OF THE Λ HYPERON AND THE PROBABILITY OF THE $K_{\mu 2}$ AND K_{e3} PROCESSES

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A new postulate is introduced for the divergence of the vector part of the strangeness-violating baryon current. The Gell-Mann and Levy[1] postulate is taken for the divergence of the axial vector part, but with the π-meson operator replaced by the K-meson operator. As a result it is found to be possible to relate the probability for the leptonic decay of the Λ particle to the probability for the $K_{\mu 2}$ and K_{e3} processes. The theory leads to the conclusion that the vector variant plays a most important role in the leptonic decay of the Λ particle.

INTRODUCTION

RECENTLY Gell-Mann and Levy[1] introduced a certain postulate for the divergence of the axial vector part of the strong current and succeeded in finding a relation between the decay probability of the charged π meson and the β interaction coupling constant. The lifetime of the charged pion obtained by them coincides with that obtained by Goldberger and Treiman,[2] who made use of dispersion relation techniques with only certain intermediate states taken into account, and is in good agreement with experiment.

The Gell-Mann and Levy hypothesis refers essentially to the baryon current only, not including strange particle, responsible for β decay, μ capture and the decay of the charged pion. When these results are generalized to the case of strong interactions including strange particles, it is therefore necessary to satisfy both the condition of parity conservation and strangeness conservation.

To generalize the ideas of Gell-Mann and Levy[1] to the case of the baryon current including strange particles ($K_{\mu 2}$, K_{e2}, $K_{\mu 3}$ and K_{e3} decays and β decay of the Λ particle) one can introduce an additional postulate referring to the divergence of the vector part of the baryon current, which now no longer vanishes. The simplest form of the postulates follows from strangeness and parity conservation considerations, and we assume that the K meson is pseudoscalar. Thus we assume that

$$\partial_\alpha P_\alpha = ia\varphi_K, \qquad (1)$$

$$\partial_\alpha V_\alpha = ib\varphi_\pi\varphi_K. \qquad (2)$$

The first equation, giving the divergence of the axial vector current, is the same as assumed by Gell-Mann and Levy[1] but with the π-meson field operator replaced by the K-meson operator.[3] The divergence of the vector current is assumed to be equal to the product of the π- and K-field operators.

In Sec. 1 we discuss the consequences of such a hypothesis. It turns out that the $K_{\mu 2}$ and K_{e2} decay probabilities can be expressed in terms of the constant a, and the $K_{\mu 3}$ and K_{e3} decay probabilities in terms of the constant b. In Sec. 2 the probability for the leptonic decay $\Lambda \rightarrow p + e + \nu$ is obtained and found to depend on the constants a and b.

The new postulate, Eq. (2), here introduced means in fact that the vector part of the leptonic Λ decay is mainly connected with the intermediate state K meson + π meson.

1. PROBABILITY OF THE $K_{\mu 2}$ AND K_{e3} DECAYS

The matrix element for the $K_{\mu 2}$ decay is given by

$$\langle \mu\nu | K \rangle = 2^{-1/2} G \langle 0 | P_\lambda | K \rangle \bar{u}_\mu \gamma_\lambda (1 + \gamma_5) u_\nu \delta (p_K - p_\mu - p_\nu). \qquad (3)$$

On the basis of Eq. (1) we find, as in reference 1, that

$$\langle 0 | P_\lambda (x) | K \rangle = -a \sqrt{Z_{3K}} (p_{K\lambda}/m_K^2) \langle 0 | \varphi_{Kr} (x) | K \rangle; \qquad (4)$$

the matrix element of the renormalized $\varphi_{Kr}(x)$ between a meson state and vacuum is the same as that of the free field between the states of a free particle and the free vacuum.

It is now easy to express the $K_{\mu 2}$ probability in terms of $a\sqrt{Z_{3K}}/m_K^2$. An elementary comparison with experimental data (see, for example, Okun's review,[4] p. 469) yields

$$a \sqrt{Z_{3K}} / m_K^2 = 0.0375\ m, \qquad (5)$$

where m is the nucleon mass and m_K is the K-meson mass.

Let us consider now the K_{e3} process. Because of the pseudoscalarity of the K meson only the vector current contributes to the matrix element of this process

$$\langle e\nu\pi | K\rangle = G2^{-1/2}\langle\pi|V_\alpha|K\rangle \bar{u}_e\gamma_\alpha (1+\gamma_5) \times u_\nu \delta(p_K - p_e - p_\nu - p_\pi). \quad (6)$$

Because $V_\alpha \sim \varphi_\pi\varphi_K$ [according to Eq. (2)] it follows from invariance considerations that

$$\langle\pi|V_\alpha|K\rangle = [c_1(p_\pi + p_K)_\alpha + c_2(p_\pi - p_K)_\alpha]\langle\pi|\varphi_\pi\varphi_K|K\rangle, \quad (7)$$

but, in addition,

$$\partial_\alpha V_\alpha - ib\varphi_\pi\varphi_K, \quad (8)$$

i.e., $(p_\pi - p_K)_\alpha V_\alpha = b\varphi_\pi\varphi_K$, where b is a constant.

There are grounds for expecting that c_1 and c_2 depend on k^2 only weakly. We shall consider them to be constant. Then it follows immediately from Eqs. (7) and (8) that

$$c_2 = 0, \qquad c_1 = b/(m_K^2 - m_\pi^2). \quad (9)$$

The equality $c_2 = 0$ means that the probabilities for $K_{\mu3}$ and K_{e3} are in a definite ratio: $W_{\mu3}/W_{e3} = 0.6875$. This results from the data given by Okun'[4] (p. 473), since in our case $f_1 = f_2$, $g = 2f$ (in the notation of the above-mentioned review), which is not in bad agreement with the experimental value 0.765.[5]

Thus,

$$\langle\pi|V_\alpha|K\rangle = [b/(m_K^2 - m_\pi^2)]\sqrt{Z_{3K}}\sqrt{Z_{3\pi}}\,(p_K + p_\pi)_\alpha \times\langle\pi|\varphi_{\pi r}\varphi_{Kr}|K\rangle. \quad (10)$$

The last matrix element is of the same order of magnitude as the free-field matrix element. This is valid in the case when $K\pi$ scattering processes contribute insignificantly to this matrix element. Therefore Eq. (11) below is valid only approximately. By making use of Eq. (10) one can find the probability for K_{e3} decay. Comparing the resultant formula with the experimental value we obtain

$$b\sqrt{Z_{3K}Z_{3\pi}}/(m_K^2 - m_\pi^2) \approx 0.16. \quad (11)$$

2. LEPTONIC DECAY OF THE Λ PARTICLE

The amplitude for this decay is

$$2^{-1/2}G\bar{u}_e\gamma_\alpha(1+\gamma_5)u_\nu[\langle p|V_\alpha|\Lambda\rangle + \langle p|P_\alpha|\Lambda\rangle]. \quad (12)$$

On the basis of Eqs. (1) and (2) we have

$$\langle p|k_\alpha P_\alpha|\Lambda\rangle = a\sqrt{Z_{3K}}\langle p|\varphi_{Kr}|\Lambda\rangle, \quad (13a)$$

$$\langle p|k_\alpha V_\alpha|\Lambda\rangle = b\sqrt{Z_{3K}}\sqrt{Z_{3\pi}}\langle p|\varphi_{\pi r}\varphi_{Kr}|\Lambda\rangle; \quad (13b)$$

$$k = p_\Lambda - p, \quad \langle p|\varphi_{Kr}|\Lambda\rangle = i(k^2 + m_K^2)^{-1} g_\Lambda \bar{u}_p\gamma_5 u_\Lambda L(k^2) \quad (14)$$

Here g_Λ is the strong interaction coupling constant, k is the Λ-nucleon momentum transfer, and $L(k^2)$ is the form factor of the vertex part.

It follows from relativistic invariance considerations that

$$\langle p|V_\alpha|\Lambda\rangle = \bar{u}_p\{\gamma_\alpha f_1 + f_2(\gamma_\alpha\hat{k} - \hat{k}\gamma_\alpha) + f_3 k_\alpha\}u_\Lambda,$$

$$\langle p|P_\alpha|\Lambda\rangle = \bar{u}_p\{\gamma_\alpha h_1 + h_2(\gamma_\alpha\hat{k} - \hat{k}\gamma_\alpha) + h_3 k_\alpha\}\gamma_5 u_\Lambda. \quad (15)$$

From here it follows that

$$\langle p|k_\alpha V_\alpha|\Lambda\rangle = \bar{u}_p\{f_1\hat{k} + k^2 f_3\}u_\Lambda, \quad (16a)$$

$$\langle p|k_\alpha P_\alpha|\Lambda\rangle = \bar{u}_p\{h_1\hat{k} + k^2 h_3\}\gamma_5 u_\Lambda. \quad (16b)$$

Making use of Eqs. (13a) and (14) and comparing them with Eq. (16b) for* $k^2 \to 0$, we obtain

$$-h_1(m_\Lambda + m) = a\sqrt{Z_{3K}}\,g_\Lambda L(0)/m_K^2, \quad (17)$$

whereas using Eq. (5) we obtain

$$-h = 0.0375\,g_\Lambda L(0)m/(m_\Lambda + m). \quad (18)$$

The here obtained value of h_1 agrees with the results of Albright,[6] who made use of the dispersion technique of Goldberger and Treiman.[2]

If we assume $g_\Lambda^2/4\pi = 2.6^6$ and $L(0) \approx 1$, then $-h_1 = 0.1$.

Let us consider next the vector matrix element

$$\langle p|k_\alpha V_\alpha|\Lambda\rangle = b\sqrt{Z_{3K}}\sqrt{Z_{3\pi}}\langle p|\varphi_{\pi r}\varphi_{Kr}|\Lambda\rangle.$$

The matrix element $\langle p|\varphi_{\pi r}\varphi_{Kr}|\Lambda\rangle$ is connected with the process shown in the Figure. For small momentum transfers k it follows from invariance considerations that

$$\langle p|\varphi_{\pi r}\varphi_{Kr}|\Lambda\rangle = i\alpha(g_\Lambda g/m)\bar{u}_p u_\Lambda. \quad (19)$$

Making use of Eqs. (19) and (11) and comparing with Eq. (16a) we obtain

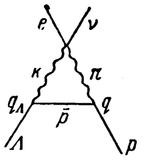
$$(m_\Lambda - m)f_1 = b\sqrt{Z_{3K}}\sqrt{Z_{3\pi}}\,g_\Lambda g\alpha/m \approx 0.16(m_K^2 - m_\pi^2)g_\Lambda g\alpha/m, \quad (20)$$

whence

$$\left|\frac{f_1}{h_1}\right| \approx \frac{0.16}{0.0375}\frac{\alpha g}{L(0)}\frac{m_K^2 - m_\pi^2}{m^2}\frac{m_\Lambda + m}{m_\Lambda - m}. \quad (21)$$

An exact evaluation of α presents a difficult problem. It is necessary to remember that there are in fact two identical diagrams, one with the intermediate state $K^-\pi^0$, the other with $K^0\pi^+$. In the second case the matrix element is twice as large. Therefore the Feynman integral of the diagram (see the figure) calculated in perturbation theory must be multiplied by 3. A perturbation

*Generally speaking the quantity k^2 is of the order of m_π^2 (the pion mass) in the decay $\Lambda \to p + e^- + \nu$. For the form factor of this decay, however, the characteristic mass is m_K^2, and the ratio m_π^2/m_K^2 may be ignored so that the limit $k^2 \to 0$ can be taken.



theory calculation, the details of which will be left out, leads to the value $\alpha = 0.016$.

If we assume $L(0) \approx 1$, $g^2/4\pi = 15$, then we get $f_1/h_1 \approx 2.76$. For the product of the leptonic decay probability by the lifetime τ of the Λ particle we then obtain (ignoring the form factor h_2)

$$W_\Lambda \tau = 0.4 \cdot 10^{-3}.$$

The experimental value[7] is approximately 1.3×10^{-3}.

At this time the experimental data on the decay $\Lambda \rightarrow p + e + \nu$ are most meager. The constant α also cannot be precisely calculated. However even in the roughest form of perturbation theory one obtains a not very large deviation from experiment.

The result obtained here indicates that in the Λ leptonic decay the main contribution is most likely to come from the vector part of the interaction. This can be checked experimentally most conveniently by utilizing the fact that the Λ particle is almost completely polarized; it then follows that for the axial vector covariant a strong asymmetry of the electron emission with respect to the Λ spin direction should be observed, whereas for the vector covariant no such asymmetry should be seen (this is correct to the extent that the proton recoil may be ignored).

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