THE CONDUCTION-ELECTRON INTERACTION INDUCED BY SPIN WAVES IN A FERROMAGNETIC SUBSTANCE

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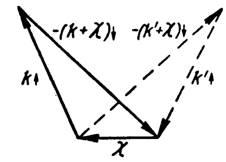
We use the s-d exchange model of a ferromagnetic metal and Bogolyubov's method in the theory of superconductivity to study the interaction between conduction electrons which is induced by their exchange coupling with the inner (3d and 4f) shells which determine the spontaneous magnetic moment of the ferromagnetic. We show that in contradistinction to the electron-electron interaction induced by phonons, this coupling is repulsive by nature and thus inhibits the establishment of a superconducting state in ferromagnetic metals.

1. It is well known that the exchange interaction between the conduction electrons and the electrons taking part in the atomic magnetic order in ferromagnetics leads to two factors which influence the establishment of superconductivity: 1) a shift of the Fermi sphere for conduction electrons with different spin directions, and 2) an extra interaction between the conduction electrons produced by the spin waves. The first of these factors* was shown earlier by us[1,2] to impede the establishment of superconductivity in a ferromagnetic. There are, however, two opposite opinions about the influence of the second factor.

On the one hand, Kasuya[3] has shown that in the rare earth metals the effective interaction between conduction electrons which is induced by the s-f exchange is repulsive in character and impedes thus the attraction caused by the phonons which leads to superconductivity. Kasuya did, however, not take into account the shift of the Fermi surface for conduction electrons with different spin directions and his conclusions refer therefore to antiferromagnetics, in which such a shift does not occur,[4] rather than to ferromagnetics. On the other hand, Akhiezer and Pomeranchuk[5] considered the interaction between a pair of conduction electrons $k\uparrow$, $-(k+\chi)\downarrow$ (see figure)† which was caused by the exchange of spin waves of a ferromagnetic and they came to the conclusion that this interaction



had the character of an additional attraction which thus assists the appearance of superconductivity. In view of such a difference in points of view about the nature of the interaction between conduction electrons induced by spin waves it is of interest to consider this problem using the method proposed by Bogolyubov,[6] and that will be the aim of the present paper.

2. The Hamiltonian of a system of conduction electrons which interact with the spin waves of a ferromagnetic can be written in the form[7]

$$H = U_0 + \sum_k \varepsilon_{k\uparrow} c^+_{k\uparrow} c_{k\uparrow} + \sum_k \varepsilon_{-(k+\chi)\downarrow} c^+_{-(k+\chi)\downarrow} c_{-(k+\chi)\downarrow} + \sum_g \omega_g b^+_g b_g - \frac{1}{\sqrt{N}} \sum_{k,k'} J c^+_{-(k+\chi)\downarrow} c_{k'\uparrow} b^+_{k'+k+\chi} + \text{c.c.}, \tag{1}$$

where

$$\varepsilon_{k\uparrow} = E_k + \tfrac{1}{2}\mu J - E_F, \qquad \varepsilon_{k\downarrow} = E_k - \tfrac{1}{2}\mu J - E_F$$

are the energies of the conduction electrons with momentum k with spins directed to the left and to the right, respectively, counted from the energy E_F of the Fermi surface; μ is the excess per crystal lattice site of d-electrons with the predominant spin orientation; N is the number of lattice sites; J is the s-d exchange parameter which we assume approximately to be independent of k (see reference 5); ω_g is the energy of a spin wave of momentum g; $c^+_{k\uparrow}$, $c^+_{k\downarrow}$ and $c_{k\uparrow}$, $c_{k\downarrow}$ are the electron Fermi-operators, and b^+_g and b_g the

*It is clear that the Meissner effect does not impede the influence of this factor as the exchange interaction is connected with an electrostatic and not with a magnetic interaction.

†In the figure we have indicated by dotted lines the pairs after the exchange of a spin wave.

spin wave Bose-operators; U_0 is a constant. The extra momenta χ of the conduction electrons with their spins to the left in (1) are chosen in agreement with Akhiezer and Pomeranchuk's data (see reference 5) in such a way that

$$|k+\chi| \gtreqless k_F\downarrow \text{ when } |k| \gtreqless k_F\uparrow, \tag{2}$$

where $k_{F\uparrow}$ and $k_{F\downarrow}$ are respectively the radii of the Fermi spheres for electrons with "left-handed" and "right-handed" spins in k space. The extra momenta χ are thus, generally speaking, different for different momenta.

It is clear that we can not use Bogolyubov's general canonical transformation,[6] as in the case under consideration we study the interaction of pairs whose total momentum χ is not equal to zero. All the same, it turns out to be possible to carry out a similar transformation applicable both when the Fermi sphere is shifted and when the total momentum of the electron pairs is non-vanishing. To do this we change over from the operators $c_{k\uparrow}$, $c_{(k+\chi)\downarrow}$ to new Fermi operators α_{k0} and α_{k1} through the transformation

$$c_{k\uparrow} = u_{k\uparrow}\alpha_{k0} + v_{k\uparrow}\overset{+}{\alpha}_{k1}, \qquad c_{(k+\chi)\downarrow} = u_{k\uparrow}\alpha_{-k1} - v_{k\uparrow}\overset{+}{\alpha}_{-k0}, \tag{3}$$

where $u_{k\uparrow}$ and $v_{k\uparrow}$ are real numbers satisfying the relations

$$u_{k\uparrow}^2 + v_{k\uparrow}^2 = 1, \qquad u_{k\uparrow} = u_{-k\uparrow}, \qquad v_{k\uparrow} = -v_{-k\uparrow}. \tag{4}$$

Using (3) and (4) one obtains easily the inverse transformation

$$\alpha_{k0} = u_{k\uparrow}c_{k\uparrow} - v_{k\uparrow}\overset{+}{c}_{-(k+\chi)\downarrow}, \qquad \alpha_{k1} = u_{k1}c_{-(k+\chi)\downarrow} + v_{k\uparrow}\overset{+}{c}_{k\uparrow} \tag{5}$$

and one can easily show that α_{k0} and α_{k1} satisfy all commutation relations of Fermi-operators.

In the ground state

$$\begin{aligned} u_{k\uparrow} &= 1, \quad v_{k\uparrow} = 0 \quad \text{when } |k| > k_F\uparrow, \\ u_{k\uparrow} &= 0, \quad v_{k\uparrow} = 1 \quad \text{when } |k| < k_F\uparrow. \end{aligned} \tag{6}$$

It follows from (5) that

$\alpha_{k0} = c_{k\uparrow}$, $\alpha_{k1} = c_{-(k+\chi)\downarrow}$ when $|k| > k_F\uparrow$ and $|k+\chi| > k_F\downarrow$;

$$\alpha_{k0} = -\overset{+}{c}_{-(k+\chi)\downarrow}, \qquad \alpha_{k1} = \overset{+}{c}_{k\uparrow} \quad \text{when } |k| < k_F\uparrow \text{ and } |k+\chi| < k_F\downarrow, \tag{7}$$

so that when $|k| > k_{F\uparrow}$ the operator α_{k0} describes the annihiliation of an electron with right-hand spin above the Fermi sphere of radius $k_{F\uparrow}$, and the operator α_{k1} the annihilation of an electron with left-hand spin above the Fermi sphere with radius $k_{F\downarrow}$. When $|k| < k_{F\uparrow}$ the operator α_{k0} describes the annihilation of a left-hand hole under the Fermi-sphere of radius $k_{F\downarrow}$ and α_{k1} the annihilation of a right-hand hole under the Fermi sphere of radius $k_{F\uparrow}$. In the general case when $u_{k\uparrow}v_{k\uparrow} \neq 0$ the operator α_{k0} (or α_{k1}) describes for $|k| > k_{F\uparrow}$ the superposition of a left-hand electron (or a left-hand hole) above the Fermi-sphere of radius $k_{F\downarrow}$ and a right-hand hole (or a right-hand electron) under the Fermi-sphere of radius $k_{F\uparrow}$. A similar superposition of electrons and holes also occurs when $|k| < k_{F\uparrow}$. The transformation (3), and (5) by itself thus does not yet solve the problem of whether bound pairs can be formed. To solve this problem one must find the equation for $u_{k\uparrow}v_{k\uparrow}$ and study the possibility that that product is different from zero.

It is clear from (6) that $u_{k\uparrow}$ and $v_{k\uparrow}$ characterize the occupation of k space by right-hand holes and electrons respectively. The fact that one then succeeds in obtaining also information about the distribution of left-hand holes and electrons is due to the fact that according to (2) knowledge of the position of k relative to the Fermi surface of radius $k_{F\uparrow}$ gives us information about the position of $k+\chi$ relative to the Fermi-sphere of radius $k_{F\downarrow}$. It thus turns out to be sufficient here to restrict ourselves to only the first two parameters (for which we shall drop in the following the index $\uparrow$) of the four parameters[1] $u_{k\uparrow}$, $v_{k\uparrow}$, $u_{k\downarrow}$, and $v_{k\downarrow}$.

Substituting (3) into (1) one can transform the Hamiltonian to the form

$$H = U_1 + H_0 + H_1 + H_2 + H_3, \tag{8}$$

where

$$U_1 = U_0 + \sum_k [\varepsilon_{k\uparrow} + \varepsilon_{k+\chi,\downarrow}]\, v_k^2, \tag{9}$$

$$H_0 = \sum_k [(\varepsilon_k u_k^2 - \varepsilon_{k+\chi,\downarrow}v_k^2)\overset{+}{\alpha}_{k0}\alpha_{k0} + (\varepsilon_{k+\chi,\downarrow}u_k^2 - \varepsilon_{k\uparrow}v_k^2)\overset{+}{\alpha}_{k1}\alpha_{k1}] + \sum_g \omega_g \overset{+}{b}_g b_g, \tag{10}$$

$$H_1 = -\frac{1}{\sqrt{N}}\sum_{k,k'} J\,(u_k v_{k'}\overset{+}{\alpha}_{k1}\overset{+}{\alpha}_{k'1} - u_{k'}v_k\alpha_{k0}\alpha_{k'0})\,\overset{+}{b}_{k'+k+\chi} + \text{c.c.}, \tag{11}$$

$$H_2 = -\frac{1}{\sqrt{N}}\sum_{k,k'} J\,(u_k u_{k'}\overset{+}{\alpha}_{k1}\alpha_{k'0} - v_k v_{k'}\alpha_{k0}\overset{+}{\alpha}_{k'1})\,\overset{+}{b}_{k'+k+\chi} + \text{c.c.}, \tag{12}$$

$$H_3 = \sum_k (\varepsilon_{k\uparrow} + \varepsilon_{k+\chi,\downarrow})\, u_k v_k\, (\overset{+}{\alpha}_{k0}\overset{+}{\alpha}_{k1} + \alpha_{k1}\alpha_{k0}). \tag{13}$$

It follows from (11) to (13) that the terms containing the operators $\alpha_{k0}^+\alpha_{k1}^+$ describing the creation of pairs occur in H_3 and can also be obtained if H_1 and H_2 operate simultaneously. If we put the coefficients of the terms which contain the operators $\alpha_{k0}^+\alpha_{k1}^+$ (since in the ground state the creation of excited pairs is forbidden[6]), we get in second approximation the compensation equation

$$\xi_k u_k v_k = -\frac{1}{2N}(u_k^2 - v_k^2)\sum J^2 u_{k'}v_{k'}\,[\omega_{g=k'+k+\chi} + \varepsilon_{k+\chi,\downarrow}u_k^2 - \varepsilon_{k\uparrow}v_k^2 + \varepsilon_{k'+\chi,\downarrow}u_{k'}^2 - \varepsilon_{k'\uparrow}v_{k'}^2]^{-1}, \tag{14}$$

where

$$2\xi_k = \varepsilon_{k\uparrow} + \varepsilon_{k+\chi,\downarrow} - \frac{1}{N}\sum_{k'} J^2 (u_{k'}^2 - v_{k'}^2) \times [\omega_{g=k'+k+\chi} + \varepsilon_{k+\chi,\downarrow} u_k^2 - \varepsilon_{k\uparrow} v_k^2 + \varepsilon_{k'+\chi,\downarrow} u_{k'}^2 - \varepsilon_{k'\uparrow} v_{k'}^2]^{-1}. \quad (15)$$

If we take into account that $E_{k_F\downarrow} - E_{k_F\uparrow} = \mu J$ and that relative to the Fermi surface E_F the relations

$$\varepsilon_{k+\chi,\downarrow} - \varepsilon_{k\uparrow} = E_{k+\chi} - E_k - \mu J \approx E_{k_F\downarrow} - E_{k_F\uparrow} - \mu J = 0$$

hold, we can in the limits of the approximation which we have used put

$$\varepsilon_{k+\chi,\downarrow} u_k^2 - \varepsilon_{k\uparrow} v_k^2 = \xi_k (u_k^2 - v_k^2) = \tilde{\varepsilon}_k. \quad (16)$$

We then get for Eq. (14)

$$\xi_k u_k v_k = -\tfrac{1}{2}(u_k^2 - v_k^2)\, c_k, \quad (17)$$

where

$$c_k = \frac{1}{N}\sum_{k'} J^2 u_{k'} v_{k'} [\omega_{g=k'+k+\chi} + \tilde{\varepsilon}_k + \tilde{\varepsilon}_{k'}]^{-1}. \quad (18)$$

From (17) and (4) it follows that

$$u_k^2 = \tfrac{1}{2}[1 + \xi_k(\xi_k^2 + c_k^2)^{-1}], \quad v_k^2 = \tfrac{1}{2}[1 - \xi_k(\xi_k^2 + c_k^2)^{-1}], \quad (19)$$

$$u_k v_k = -\tfrac{1}{2} c_k (\xi_k^2 + c_k^2)^{-1}, \qquad \tilde{\varepsilon}_k = \xi_k^2 (\xi_k^2 + c_k^2)^{-1}; \quad (20)$$

these relations differ formally from the usual relations[6] by the sign of the product $u_k v_k$. Substituting this product from (20) into (18) we get an equation for c_k

$$c_k = -\frac{1}{2N}\sum_{k'} J^2 (\omega_{g=k'+k+\chi} + \tilde{\varepsilon}_k + \tilde{\varepsilon}_{k'})^{-1} c_{k'} (\xi_{k'}^2 + c_{k'}^2)^{-1}. \quad (21)$$

Equation (21) differs also by the sign of the right-hand side from the analogous equation for the case of an interaction produced by phonons which occurs in the work by Bogolyubov, Tolmachev, and Shirkov (see reference 6). The quantity

$$J^2(\omega_{g=k'+k+\chi} + \tilde{\varepsilon}_k + \tilde{\varepsilon}_{k'})^{-1}$$

is thus positive and Eq. (21) has only the trivial solution: $c_k = 0$. Equation (19) goes then over into (6) which is satisfied in the ground state.

We have thus shown that the interaction between the conduction electrons caused by the exchange of spin waves in a ferromagnetic is repulsive by nature and can not assist the formation of a superconducting state.

3. The contrasting character of the phonon and the ferromagnon (spin-wave) effective interaction between conduction electrons can also be seen directly from comparing the expression

$$\hbar\omega_g |M_g|^2[(\varepsilon_k - \varepsilon_{k+g})^2 - (\hbar\omega_g)^2]^{-1} c_{k'-g,\downarrow}^{+} c_{k'\downarrow} c_{k+g,\uparrow}^{+} c_{k\uparrow}, \quad (22)$$

which occurs in second perturbation theory approximation for the interaction between two electrons with spins in opposite directions which is produced by the exchange of a virtual longitudinal phonon[8] with the analogous expression*

$$\Omega^{-1} a^3 \Delta^2 [\varepsilon_{k\uparrow} - \varepsilon_{k+g,\downarrow} - \theta(ag)^2]^{-1} c_{k'-g,\uparrow}^{+} c_{k'\downarrow} c_{k+g,\downarrow}^{+} c_{k\uparrow}, \quad (23)$$

which occurs in the same approximation for the interaction produced by the exchange of ferromagnons.[5] Indeed, introducing the creation and annihilation operators for a $(k\uparrow, k'\downarrow)$ pair:

$$b_{kk'}^{+} = c_{k\uparrow}^{+} c_{k'\downarrow}^{+}, \qquad b_{kk'} = c_{k'\downarrow} c_{k\uparrow} \quad (24)$$

(unlike reference 8 we do not assume here that $k' = -k$) and using the commutation relations for the Fermi-operators $c_{k\uparrow}$ and $c_{k\downarrow}$ we can transform (22) and (23) to

$$\hbar\omega_g |M_g|^2[(\varepsilon_k - \varepsilon_{k+g})^2 - (\hbar\omega)^2]^{-1} b_{k+g,k'-g}^{+} b_{kk'}, \quad (22')$$

$$-\Omega^{-1} a^3 \Delta^2 [\varepsilon_{k\uparrow} - \varepsilon_{k+g,\downarrow} - \theta(ag)^2] b_{k'-g,k+g}^{+} b_{kk'}. \quad (23')$$

The difference in the signs of (22′) and (23′) shows† that in the first case the "phonon interaction" is attractive near the Fermi surface, while in the second case the "ferromagnon interaction" is repulsive.‡ The physical cause of this difference is** that in the first case transitions of electrons are not accompanied by a change in their spin orientation while such a change does take place in the second case.

Indeed, if, for instance, there are initially two electrons in the states $k\uparrow$ and $k\downarrow$ with opposite spins, absorption of a phonon may lead to a transi-

*The notation in (22) and (23) is the same as in references 5 and 8; we note merely that in reference 5 expression (23) was written down without taking into account in the numerator the operator which is essential for grouping the electrons in pairs and thus for the determination of the sign of the matrix element. See also the text of reference 8 after Eq. (2.7) for this problem.

†In order to avoid misunderstandings we note that it is clear from (24) that $b_{k+g,k'-g} \neq -b_{k'-g,k+g}$ so that (22′) and (23′) indeed have opposite signs. Interchanging the indices of $b_{k+g,k'-g}^{+}$ in (22′) and (23′) means only that in the first case the pair $(k\uparrow, k'\downarrow)$ goes over into the pair $(k+g\uparrow; k'-g\downarrow)$ and in the second case into the pair $(k'-g\uparrow; k+g\downarrow)$.

‡We note that Cooper[10] also considered pairs with $\chi \neq -0$, and that the binding energy of the pair turned out to be energetically preferred only in the case of a negative matrix element of the interaction energy. The difference in sign of the matrix elements of the interactions caused on the one hand by longitudinal phonons and on the other hand by spin waves which we obtained means thus that the latter interaction will not lead to the Cooper effect.

**See also the concluding remark in a note by the present authors.[9]

tion k↑ → k + g↑ while the absorption of a ferromagnon must lead to the transition k↓ → k + g↑. Two electrons which before the transition are in different states k↑ and k↓ must after the above mentioned transitions turn out to be in the same state k + g↑ which is forbidden by the Pauli principle. It follows thus that the exchange of a spin wave inhibits the effect caused by the exchange of a longitudinal phonon.

What we have said so far allows us also to conclude that the opposition to the establishment of the superconducting state is not a basic property of the exchange of a spin wave, but will occur generally whenever a quasi-particle with unit spin is exchanged. In particular, there are some grounds for assuming that transverse phonons have unit spin. In metals (not necessarily ferromagnetics) the exchange of transverse phonons as well as the well-known Coulomb repulsion can thus inhibit the establishment of superconductivity. This problem we propose to consider in more detail at a later time.

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