DOUBLE DISPERSION RELATIONS AND THE PHOTOPRODUCTION OF PIONS. II

N. F. NELIPA and V. A. TSAREV

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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An approximate estimate of the contribution of the annihilation of a nucleon pair into a π meson and a γ quantum to the π-meson photoproduction amplitude is made by using previously derived[1] integral equations.

THE aim of the present paper is the approximate evaluation of the contribution of the process of annihilation of a nucleon pair into a meson and photon to the π-meson photoproduction amplitude. A system of integral equations derived previously by one of the authors[1] will be utilized.

If one assumes that there exists a resonance in the π-π interaction with angular momentum $J = 1$ and isospin $T = 1$ and that this resonance is sufficiently narrow, one can write according to Eq. (5.6) of reference 1 (we assume that only the isoscalar part of the amplitude will contribute to the two-meson approximation)*

$$\Delta E_{0+}= (\Lambda/e)\Big\{C_1[r_1^1I_1+r_1^2I_5-4kqr_1^3] - C_2[r_2^1I_2+r_2^2I_6]+\tfrac{1}{2}qC_2\Big[r_4^1I_3-\tfrac{8}{3}\alpha kqr_4^1\Big]\Big\}, \quad (1.1)$$

$$\Delta E_{1-} = (\Lambda/e)\Big\{C_1[r_1^1I_2+r_1^2I_6]-C_2[r_2^1I_1+r_2^2I_5-4kqr_2^3] - qC_1\Big[r_3^1I_3-\tfrac{8}{3}\alpha kqr_3^1\Big]+qC_2r_4^1I_3\Big\}, \quad (1.2)$$

$$\Delta M_{1+} = (\Lambda/2e)\Big\{C_1[r_1^1I_2+r_1^2I_6]-C_2r_2^1I_4 - \tfrac{1}{2}qC_1\Big[r_3^1I_3-\tfrac{8}{3}\alpha kqr_3^1\Big]\Big\}, \quad (1.3)$$

$$\Delta M_{1-} = (\Lambda/e)\Big\{-C_1[r_1^1I_2+r_1^2I_6] + C_2[r_2^1I_1+r_2^2I_5-4kqr_2^3]+\tfrac{1}{2}qC_1\Big[r_3^1I_3-\tfrac{8}{3}\alpha kqr_3^1\Big]\Big\}. \quad (1.4)$$

We have used the following abbreviations:

$$I_1=\ln\frac{a+1}{a-1}\equiv\beta,\quad I_2=-2+a\beta,\quad I_3=2a+(1-a^2)\beta,$$

$$I_4=-6a+(3a^2-1)\beta,\qquad I_5=4kq(2E_\mu k-1),$$

$$I_6=-\tfrac{1}{3}8q^2k^2,$$

$$C_1=-\frac{1}{4kq}\,\frac{W-M}{8\pi W}\sqrt{(E_k+M)(E_M+M)},$$

$$C_2=-\frac{1}{4k}\,\frac{W-M}{8\pi W}\sqrt{\frac{E_k+M}{E_M+M}};$$

$a = (2E_\mu k-1)/2qk$, Λ is the photon-meson coupling constant.

The values of r_k^i are:

$$r_1^1=Q_1+\frac{2(W-M)^2-1}{2(W-M)}Q_3,$$

$$r_2^1=-Q_1+\frac{2(W+M)^2-1}{2(W+M)}Q_3,$$

$$r_3^1=(W-M)Q_2-Q_3,\qquad r_4^1=-(W+M)Q_2-Q_3,$$

$$r_1^2=Q_3\alpha,\qquad r_2^2=Q_3\alpha/2(W+M),$$

$$r_1^3=r_1^1\alpha+Q_3/2(W-M),\qquad r_2^3=r_2^1\alpha+Q_3/2(W+M).$$

Here

$$Q_1\approx-0.55\xi,\qquad Q_2=-Q_1/t_r,\qquad Q_3=\xi/2\sqrt{t_r};$$

$$\xi=\gamma t_r^2/(t_r+\gamma)\sqrt{t_r-1}.$$

In deriving the above formulae, we used the following expressions for the amplitudes of photoproduction of π mesons on mesons and of the annihilation of nucleon pairs into two mesons, which have to be inserted into Eqs. (5.6) and (A.5) of reference 1. The form of the π-π resonance was chosen[2] as

$$f_{\pi\pi}=\gamma/(t_r-t-i\gamma q^3),$$

where $q = (t/4-1)^{1/2}$ is the meson momentum in the intermediate state, t_r is the energy of the meson-meson scattering resonance, and γ is a parameter characterizing the widths of the resonance.

The expression for the amplitudes $T_\pm$ of the process of annihilation of a nucleon pair into two mesons* is written as[2,3]

$$T_\pm=f_{\pi\pi}\int_{-\infty}^{a}\frac{\operatorname{Im}f_\pm(t')dt'}{f_{\pi\pi}(t')(t'-t-i\varepsilon)}=f_{\pi\pi}N_\pm(t).$$

In obtaining $N_\pm(t)$ we have utilized the connection between $T_\pm(t)$ and the isovector parts of the nucleon formfactors.[4]

*We shall use the notation of reference 1 throughout.

*We use here the following connection between the partial amplitudes $G_{l\pm1,1}$ [see Eq. (A.15) of reference 1] and the coefficients in the expansion of $T_\pm^1$ in helicity states:[3] $G_{l+1,1}=\tfrac{1}{3}3^{-1/2}(T_+^1+\sqrt{2}T_-^1)$; $G_{l-1,1}=6^{-1/2}(-T_+^1+2^{-1/2}T_-^1)$.

In the determination of the π-meson photoproduction amplitude on π mesons we have made use* of the solution of Gourdin and Martin:[5]

$$F_{\gamma\pi} = \frac{1}{8}\Lambda f_\pi \left[\frac{(t-4)(t-1)}{t}\right]^{1/2} \frac{2t_r - t - 1}{2t_r + t - 3},$$

$$f_\pi = \frac{t_r + \gamma}{t_r - t - i\gamma q^3}$$

(f_π is the meson formfactor).

It should be noted that the expression (1) has been obtained without an expansion in powers of 1/M.

With the presently available experimental data it is not possible to determine uniquely the parameters which enter the formulae (1). However, the limits within which these parameters are determined are not too large: the parameter t_r lies between ~15 and ~25, and $1.7 \leq q_r^3 \leq 3$.[2] Therefore we have carried through an evaluation[6] in the region close to threshold ($E_\gamma \leq 250$ Mev in the laboratory system) for a few choices of the parameters, remaining within the indicated limits.

From these evaluations it can be seen that the additions to the multipoles do not exceed a few percent of the results obtained by simple dispersion relations.[7] The additions to the coefficients A and B, which characterize the angular distribution of the photoproduction are, a few percent, and those to the coefficient C are on the order 10 — 20%. In both cases it was assumed that $\Lambda/e = 1$. These results are close to analogous results obtained by somewhat other methods.[8]

In conclusion we thank P. N. Komolov and A. T. Matachun for numerical computations.

*We note that our choice of the form of the solution of $F_{\gamma\pi}$, which differs from the solution of Gourdin and Martin,[5] influences in the present calculation only the choice of the constant A.

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