PRODUCTION OF TRITIUM IN COLLISIONS OF FAST PROTONS WITH HEAVY NUCLEI

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The probability of the production of tritons as a result of the indirect evaporation process when heavy nuclei are bombarded by protons of energy ~ 100 Mev is calculated.

1. INTRODUCTION

It is known that nucleons, deuterons d, tritons t, helium nuclei α, and other heavier particles are produced in collisions between high-energy nucleons and nuclei. As a rule, these particles are produced as a result of a direct interaction between the bombarding nucleon and the nucleus and the subsequent evaporation of the residual excited nucleus. However, at incident nucleon energies up to 100 Mev, the production reaction can take place through a compound state. For example, at an incident nucleon energy of 100 Mev (mean free path of a nucleon in the nucleus ~ 4 $\times 10^{-13}$ cm), the nucleon, in the case of a central collision with a heavy nucleus, can experience several collisions in the nucleus and lose a large part of its energy, as a result of which it is trapped in the nucleus and produces an excited compound nucleus. We shall consider such a mechanism of emission of particles.

It is usually assumed that entire compound particles (d, t, α) are evaporated from the compound nucleus, but the excitation can be removed by other channels, in particular, by the evaporation of individual nucleons (or various combinations of them) and their subsequent uniting into d, t, or α close to the boundary of the nucleus (indirect process).

Kikuchi[1] calculated the deuteron yield by means of the indirect process. The basic difference in comparison with ordinary evaporation is in an increase in the mean kinetic energy of the emitted deuterons and a change in the shape of the energy distribution. The probability of deuteron emission in the indirect process, at large excitation, exceeds the corresponding probability for ordinary evaporation.

In contrast to the case of deuterons, the indirect process for triton production can proceed via two channels: by the evaporation of two neutrons and one proton and their subsequent union and by the evaporation of a deuteron and neutron and their subsequent union. We calculated the triton yield via the first channel only and determined its energy spectrum. The calculations were based on the method of Kapur and Peierls[2] (see also [1] and [3]).

2. DETERMINATION OF THE REACTION MATRIX

We shall consider the reaction in which a triton is produced in a heavy nucleus by a high-energy proton p

$$p + A_Z \to C \to (A-2)_Z + t, \qquad (1)$$

where C denotes a compound nucleus, A_Z is the target nucleus of mass number A and charge Z. The production of a triton can take place through the union of combinations of two neutrons and one proton undergoing evaporation. In order to take into account all possibilities, we divide the configuration space of two neutrons (n_1 and n_2) and a proton p, which form the triton, into eight regions corresponding to different combinations of these particles with respect to a surface S of given radius r_0 (r_0 is the distance at which the potential energy between nucleons n_1, n_2, p and the residual nucleus $(A-2)_Z$ is sufficiently small; r_0 is a little bigger than the radius of the nucleus). The configuration space is divided in the following way:

N regions	1	2	3	4	5	6	7	8
Particles outside S	n_1, n_2, p	n_1, p	n_2, p	n_1, n_2	p	n_1	n_2	—
Particles inside S	—	n_2	n_1	p	n_1, n_2	n_2, p	n_1, p	n_1, n_2, p

In region 1, the particles n_1, n_2, and p unite into a triton. The transition from 8 to 1 can take place through one of the regions 5, 6, 7, and then, depending on the case, through one of the regions 2, 3, 4. In all, there are six ways of transition from 8 to 1. We shall consider below one of these transitions: 8-5-2-1.

The Hamiltonian of the entire system H is written in the form

$$H = H_0 + K_p + K_1 + K_2 + V_p + V_1 + V_2 + V_{p1} + V_{p2} + V_{12}, \quad (2)$$

where H_0 is the Hamiltonian of the residual nucleus $(A-2)_Z$; K_p, K_1, K_2 are the kinetic energy operators of the nucleons p, n_1, n_2; V_p, V_1, V_2 are the potential energies of interaction of nucleons p, n_1, n_2 with the residual nucleus $(A-2)_Z$; V_{p1}, V_{p2}, V_{12} are the potential energies of interaction of nucleons p and n_1, p and n_2, n_1 and n_2. It is convenient to express the matrix of the transition T by means of wave functions of the final state $\Psi_f^{(-)}$, which are solutions of Schrödinger's equation

$$H\Psi_f^{(-)} = E\Psi_f^{(-)}. \quad (3)$$

To do this, we consider the initial $(p + A_Z)$ and final $[(A-2)_Z + t]$ states of the system.

The splitting up of the Hamiltonian into an unperturbed part and a perturbation is different for the initial (i) and final (f) states. In the initial state

$$H = H_i + V_i, \qquad V_i = V_p + V_{p1} + V_{p2}. \quad (4)$$

In the final state

$$H = H_f + V_f, \qquad V_f = V_p + V_1 + V_2. \quad (5)$$

Here H_i is the unperturbed Hamiltonian of the initial state, V_i is its perturbation; H_f and V_f are the corresponding quantities in the final state.

We also introduce the wave functions Φ_i and Φ_f of the unperturbed Hamiltonians H_i and H_f, respectively:

$$H_i\Phi_i = E\Phi_i, \qquad H_f\Phi_f = E\Phi_f. \quad (6)$$

Then the matrix T is written in the form:[4,5]

$$T = \langle \Psi_f^{(-)} | V_i | \Phi_i \rangle, \quad (7)$$

$$\Psi_f^{(-)} = [1 + (E - i\varepsilon - H)^{-1} V_f]\, \Phi_f. \quad (8)$$

We are interested in those transitions which go through the compound state. It is therefore necessary to exclude from matrix (7) those processes of triton production in which the incident proton p is not inside the region bounded by the surface S and interacts with the tails of the wave functions of neutrons n_1 and n_2 (direct capture). For this purpose, we split the perturbation V_i occurring in (7) into two parts corresponding to the positions of the proton p in the initial state outside and inside the nucleus:

$$V_i = V_i^{(5)} + V_i^{(8)}. \quad (9)$$

We now introduce the Hamiltonian $H^{(2)}$ describing the decaying intermediate nucleus after p and n_1 have left it:

$$H^{(2)} = H_0 + K_2 + V_2,$$

the Hamiltonian $H^{(5)}$ describing the decaying intermediate nucleus after the proton has left it:

$$H^{(5)} = H_0 + K_1 + K_2 + V_1 + V_2 + V_{12}$$

and the Hamiltonian of the compound nucleus $H^{(8)}$:

$$H^{(8)} = H_i + V_i^{(8)}.$$

We also introduce the quantities $E^{(2)} = E - \hbar^2(k_p^2 + k_1^2)/2M$, $E^{(5)} = E - \hbar^2 k_p^2/2M$, where $\hbar k_p$ and $\hbar k_1$ are the momenta of p and n_1, and M is the mass of the nucleon.

We shall consider the eigenfunctions $\varphi_\nu^{(2)}$, $\varphi_\mu^{(5)}$, $\varphi_\lambda^{(8)}$ of the Hamiltonians $H^{(2)}$, $H^{(5)}$, $H^{(8)}$, respectively, which have complex eigenvalues $W_\nu^{(2)}$, $W_\mu^{(5)}$, $W_\lambda^{(8)}$ (the subscripts ν, μ, λ indicate the level number). The function φ satisfies on the surface S the boundary conditions

$$(\partial/\partial r_2 - f(k_2 r_2))\,\varphi_\nu^{(2)}|_{r_2=r_0} = 0, \quad (\partial/\partial r_1 - f(k_1 r_1))\,\varphi_\mu^{(5)}|_{r_1=r_0} = 0,$$

$$(\partial/\partial r_p - f(k_p r_p))\,\varphi_\lambda^{(8)}|_{r_p=r_0} = 0, \quad (10)$$

where the function f is given by Kapur and Peierls.[2] The functions $\varphi_\nu^{(2)\prime}$, $\varphi_\mu^{(5)\prime}$, $\varphi_\lambda^{(8)\prime}$ corresponding to the complex conjugate eigenvalues $W_\nu^{(2)*}$, $W_\mu^{(5)*}$, $W_\lambda^{(8)*}$ satisfy the complex conjugate boundary conditions.

The functions φ are normalized in the following way:

$$\sum_\nu |\varphi_\nu^{(2)}\rangle \frac{1}{N_\nu^{(2)}} \langle \varphi_\nu^{(2)\prime}| = 1, \qquad \langle \varphi_\nu^{(2)\prime} | \varphi_\mu^{(2)} \rangle = N_\nu^{(2)}\,\delta_{\nu\mu}, \quad (11)$$

$|N_\nu^{(2)}| = 1$, and similarly for $\varphi^{(5)}$ and $\varphi^{(8)}$. The complex factor N occurs when $\varphi^{(2)}$ is replaced by $\varphi^{(2)\prime}$ in the usual normalization conditions

$$\sum_\nu |\varphi_\nu^{(2)}\rangle \langle \varphi_\nu^{(2)}| = 1, \qquad \langle \varphi_\nu^{(2)} | \varphi_\mu^{(2)} \rangle = \delta_{\nu\mu}.$$

We now separate out of the matrix T given by formula (7) the part $T^{(c)}$ associated with the transition through the compound nucleus:

$$T^{(c)} = \langle F_f^{(-)} | V_i^{(8)} | \Phi_i \rangle. \quad (12)$$

Here[5]

$$F_f^{(-)} = \Omega_3\Omega_2\Omega_1\Phi_f,$$
$$\Omega_1 = 1 + [E - i\varepsilon - H^{(5)} - K_p - (V_{p1} + V_{p2})^{(8)} - L_{12} + V_1 + V_2]^{-1}L_{12},$$
$$\Omega_2 = 1 + (E - i\varepsilon - H^{(5)} - K_p)^{-1}[V_1 + V_2 - (V_{p1} + V_{p2})^{(8)} - L_{12}],$$
$$\Omega_3 = 1 + (E - i\varepsilon - H^{(5)} - K_p - V_i^{(8)})^{-1}V_i^{(8)}, \qquad (13)$$

where L_{12} is an arbitrary interaction potential, which it is convenient to choose in the form of a potential of a rigid sphere.

We now expand the matrix $T^{(c)}$ in the wave functions $\varphi^{(2)}$, $\varphi^{(5)}$, $\varphi^{(8)}$. However, the function $F_f^{(-)}$ in (12) does not satisfy on the surface S the boundary conditions of the type (12) corresponding to the decay. Kapur and Peierls[2] showed that the function $F_f^{(-)} - \chi$ can be expanded in terms of φ if χ is chosen so that the difference satisfies the boundary conditions. The quantity χ is taken in such a way that all final expressions containing it vanish.

It is also convenient to introduce the potential energy[3] L_p [in analogy to the quantity L_{12} introduced in (13)] in order to separate out of $T^{(c)}$ terms responsible for potential scattering. The matrix $T^{(c)}$ then takes the form

$$T^{(c)} = \langle \chi_f^{(-)} | L_p | \Phi_i \rangle + \langle F_f^{(-)} | V_i^{(8)} - L_p | \chi_i^{(+)} \rangle; \qquad (14)$$
$$\chi_i^{(+)} = [1 + (E + i\varepsilon - H^{(5)} - K_p - L_p)^{-1} L_p]\,\Phi_i,$$
$$\chi_f^{(-)} = [1 + (E - i\varepsilon - H^{(5)} - K_p - L_p)^{-1} L_p] \times [1 + (E - i\varepsilon - H^{(5)} - K_p)^{-1} \times (V_1 + V_2 - V_{p1} - V_{p2})]\,\Phi_f. \qquad (15)$$

The first term in (14) vanishes, since it describes the elastic scattering of the proton, and its interaction with nucleons n_1 and n_2 does not lead to the production of a triton because of the endothermic character of this transition.

For the expansion of $T^{(c)}$ in terms of $\varphi^{(2)}$, we note that the function $\Omega_1\Phi_f$ in (12) satisfies the equation

$$(E - H^{(2)} - K_p - K_1)\,\Omega_1\Phi_f = [(V_{p1} + V_{p2})^{(8)} + V_{12} + L_{12} - V_2]\,\Omega_1\Phi_f. \qquad (16)$$

We multiply (16) on the left by the wave function $\langle \mathbf{k}_1\mathbf{k}_p |$ of the free neutron n_1 (momentum $\hbar\mathbf{k}_1$) and the proton p ($\hbar\mathbf{k}_p$); we take the transposed matrix (we shall denote it with the tilde) and act with it on the function $\varphi_\nu^{(2)}$. Then, using the identity

$$\frac{1}{(E^{(2)} - W_\nu^{(2)})\,N_\nu^{(2)}} \langle \varphi_\nu^{(2)'} \mathbf{k}_1\mathbf{k}_p | = \frac{1}{N_\nu^{(2)}} \langle \varphi_\nu^{(2)'} \mathbf{k}_1\mathbf{k}_p | \frac{1}{E + i\varepsilon - \widetilde{H}^{(2)} - \widetilde{K}_p - \widetilde{K}_1}$$

and carrying out the summation over ν, $\mathbf{k}_1$, and $\mathbf{k}_p$, we obtain

$$\sum_{\nu\mathbf{k}_1\mathbf{k}_p} \Delta^{(2)}(\nu;\, \mathbf{k}_2)\,[N_\nu^{(2)}(E^{(2)} - W_\nu^{(2)})]^{-1} \langle \varphi_\nu^{(2)'} \mathbf{k}_1\mathbf{k}_p | = \langle \{1 + (E - i\varepsilon - H^{(2)} - K_p - K_1)^{-1} [V_2 - L_{12} - V_{12} - (V_{p1} + V_{p2})^{(8)}]\}\,\Omega_1\Phi_f |, \qquad (17)$$

$$\Delta^{(2)}(\nu;\, \mathbf{k}_2) = \langle \Omega_1\Phi_f | \widetilde{H}^{(2)} - H^{(2)} | \mathbf{k}_1\mathbf{k}_p\varphi_\nu^{(2)} \rangle. \qquad (18)$$

Multiplying (17) by

$$1 + (E - i\varepsilon - H^{(5)} - K_p)^{-1}(V_1 + V_2),$$

we obtain $\langle \Omega_2\Omega_1\Phi_f |$ in the right-hand part. After inserting this expression into (14), $T^{(c)}$ takes the form

$$T^{(c)} = \sum_{\nu\mathbf{k}_1\mathbf{k}_p} \Delta^{(2)}(\nu;\, \mathbf{k}_2)\,[N_\nu^{(2)}(E^{(2)} - W_\nu^{(2)})]^{-1} \times \langle \mathbf{k}_1\mathbf{k}_p\varphi_\nu^{(2)'} [1 + (\widetilde{V}_1 + \widetilde{V}_2) \times (E + i\varepsilon - \widetilde{H}^{(5)} - \widetilde{K}_p)^{-1}]\,\widetilde{\Omega}_3 | V_i^{(8)} - L_p | \chi_i^{+} \rangle. \qquad (19)$$

The quantity $\Delta^{(2)}$ occurring here characterizes the probability of the evaporation of the neutron n_2 from the intermediate state of the nucleus $(A-1)_Z$ and the subsequent formation of a triton.

Carrying out the expansion of matrix (19) in terms of $\varphi^{(5)}$ and $\varphi^{(8)}$, we obtain, by similar arguments,

$$T_1^c = \sum_{\substack{\nu\mu\lambda \\ \mathbf{k}_1\mathbf{k}_p}} \Delta^{(2)}(\nu;\, \mathbf{k}_2)\,[N_\nu^{(2)}(E^{(2)} - W_\nu^{(2)})]^{-1}\Delta_{(2)}^{(5)}(\mu;\, \nu\mathbf{k}_1) \times [N_\mu^{(5)}(E^{(5)} - W_\mu^{(5)})]^{-1}\Delta_{(5)}^{(8)}(\lambda;\, \mu\mathbf{k}_p)\,[N_\lambda^{(8)}(E - W_\lambda^{(8)})]^{-1}\, y_{\lambda p}. \qquad (20)$$

Here

$$\Delta_{(2)}^{(5)}(\mu;\, \nu\mathbf{k}_1) = \langle \mathbf{k}_1\varphi_\nu^{(2)'} | \widetilde{H}^{(5)} - H^{(5)} | \varphi_\mu^{(5)} \rangle,$$
$$\Delta_{(5)}^{(8)}(\lambda;\, \mu\mathbf{k}_p) = \langle \mathbf{k}_p\varphi_\mu^{(5)'} | \widetilde{H} - H | \varphi_\lambda^{(8)} \rangle \qquad (21)$$

characterize the probabilities of transition of the system from region 5 to region 2 (with the emission of the neutron n_1) and from region 8 to region 5 (with the emission of the proton). The quantity

$$y_{\lambda p} = \langle \varphi_\lambda^{(8)'} | H - \widetilde{H} \mid \chi_i^{(+)} \rangle \qquad (22)$$

characterizes the probability of the formation of a compound nucleus in the state λ when the target nucleus is bombarded by protons.

The subscript 1 of the matrix $T_1^{(c)}$ denotes a given channel for the reaction; we did not indicate this earlier for the sake of brevity. The matrices for processes taking place by other channels are written in a similar way. The complete matrix element is, of course, equal to their sum

$$T^{(c)} = \sum_{j=1}^{6} T_j^{(c)}. \tag{23}$$

3. CALCULATION OF THE CROSS SECTION FOR THE PRODUCTION OF TRITONS

We shall calculate the cross section $\sigma(p, t)$ for the process corresponding to the matrix $T^{(c)}$ (23). It is convenient to write it in the form of a dispersion formula in which terms characterizing the formation of a compound nucleus are separated:

$$\sigma(p, t) = \frac{\pi}{k_p^2}\left|\sum_\lambda \frac{(\Gamma_t \Gamma_{\lambda p})^{1/2}}{N_\lambda^{(8)}(E - W_\lambda^{(8)})}\right|^2. \tag{24}$$

Here $\Gamma_{\lambda p} = |y_{\lambda p}|^2$ is the width for the formation of a compound nucleus by the incident proton and the nucleus A_Z; Γ_t is the width corresponding to the formation of a triton; $W_\lambda^{(8)} = E_\lambda - \frac{1}{2}i\Gamma_\lambda$, where E_λ is the energy level of the compound nucleus in the state λ, and Γ_λ is the width of this level.

The expression for the cross section (24) must be averaged over the initial state and summed over all final states. To do this, we consider the average of $\sigma(p, t)$ with respect to the energy of the bombarding beam, where we neglect the contribution from cross terms:

$$\sigma(p, t) = \frac{\pi}{k_p^2}\frac{1}{D(E_c)}\int \frac{\Gamma_t \Gamma_{\lambda p}}{(E - E_\lambda)^2 + \Gamma_\lambda^2/4} dE = \frac{2\pi}{k_p^2}\frac{\Gamma_t \Gamma_{\lambda p}}{\Gamma_\lambda D(E_c)}; \tag{25}$$

here $D(E_c)$ is the average distance between the compound nucleus levels for an excitation energy $E_c \equiv E_\lambda$.

Taking into account the fact that the density of the final states of the triton is

$$2\cdot 4\pi K_t^2 dK_t/(2\pi)^3\, dE_t = 3MK_t/\pi^2\hbar^2$$

where K_t is the wave number of the emitted triton, $E = \hbar^2 K_t^2/6M$ is the kinetic energy of the triton, we obtain the expression for the width Γ_t:

$$\Gamma_t = \Sigma_f \frac{3}{\pi}\frac{MK_t}{\hbar}|B|^2, \tag{26}$$

$$B = \sum_{\mu\nu \mathbf{k}_1 \mathbf{k}_p} \Delta^{(2)}(\nu; \mathbf{k}_2)\,[N_\nu^{(2)}(E^{(2)} - W_\nu^{(2)})]^{-1}\,\Delta_{(2)}^{(5)}(\mu; \nu\mathbf{k}_1)$$
$$\times [N_\mu^{(5)}(E^{(5)} - W_\mu^{(5)})]^{-1}\,\Delta_{(5)}^{(8)}(\lambda, \mu\mathbf{k}_p)$$
$$+ \text{5 similar terms from (23)} \tag{27}$$

Here the symbol Σ_f denotes summation over all final states.

We estimate the first term B_1 in the sum (27). Introducing the notation

$$g(\mathbf{r}_2, \mathbf{R}; \mathbf{k}_1, \mathbf{k}_p) = \langle \mathbf{k}_1\mathbf{k}_p | \Phi_f\rangle \approx \langle \mathbf{k}_1\mathbf{k}_p | \Omega_1\Phi_f\rangle, \tag{28}$$

we rewrite (18) in the form

$$\Delta^{(2)}(\nu; \mathbf{k}_2) \approx \langle g | \widetilde{H}^{(2)} - H^{(2)} | \varphi_\nu^{(2)}\rangle. \tag{29}$$

In (29), we neglected the effect of L_{12} on the wave function Φ_f. The error in doing this is small, since, by choosing L_{12} in the form of a potential of a rigid sphere, we extrapolate the given form of g (28) to the small volume bounded by the surface S.

In (28), $\mathbf{R}$ denotes the coordinates of all the particles except n_1, n_2, p. Then

$$g = \frac{1}{(2\pi)^3}$$
$$\times \int d\mathbf{r}_1 d\mathbf{r}_p e^{-i\mathbf{k}_1\mathbf{r}_1} e^{-i\mathbf{k}_p\mathbf{r}_p}\,\Phi_t(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_p)\, e^{i\mathbf{K}_t(\mathbf{r}_1+\mathbf{r}_2+\mathbf{r}_p)/3}\,\Psi_f(\mathbf{R}). \tag{30}$$

Here $\Psi_f(\mathbf{R})$ is the wave function of the residual nucleus $(A-2)_Z$; $\Phi_t(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_p)$ is the wave function of the internal motion of the triton; for our problem it is convenient to choose it in the form

$$\Phi_t = \frac{\alpha}{2\sqrt{7}\pi}\left[\frac{\exp\{-\alpha(\rho_1+\rho_2)\}}{\rho_1\rho_2} + \frac{\exp\{-\alpha(\rho_2+\rho_3)\}}{\rho_2\rho_3}\right.$$
$$\left. + \frac{\exp\{-\alpha(\rho_3+\rho_1)\}}{\rho_3\rho_1}\right]. \tag{31}$$

We note that here Φ_t is normalized, the parameter α characterizes the dimensions of the nucleus and the binding energy of the triton, and

$$\boldsymbol{\rho}_1 = \mathbf{r}_1 - \mathbf{r}_2, \qquad \boldsymbol{\rho}_2 = \mathbf{r}_2 - \mathbf{r}_p, \qquad \boldsymbol{\rho}_3 = \mathbf{r}_p - \mathbf{r}_1.$$

The calculation of g (28) gives

$$g \approx \xi(\mathbf{k}_1, \mathbf{k}_p, K_t)\,\Psi_f(\mathbf{R})\exp(i\mathbf{q}\mathbf{r}_2), \tag{32}$$

$$\xi = \frac{\alpha}{\pi^2\sqrt{7}}\left[\frac{1}{(\alpha^2+q_1^2)(\alpha^2+q_2^2)} + \frac{1}{(\alpha^2+q_2^2)(\alpha^2+q_3^2)}\right.$$
$$\left. + \frac{1}{(\alpha^2+q_3^2)(\alpha^2+q_1^2)}\right],$$

$$\mathbf{q} = \mathbf{K}_t - \mathbf{k}_1 - \mathbf{k}_p, \quad \mathbf{q}_1 = \tfrac{2}{3}\mathbf{K}_t - \mathbf{k}_1 - \mathbf{k}_p,$$

$$\mathbf{q}_2 = \tfrac{1}{3}\mathbf{K}_t - \mathbf{k}_1, \quad \mathbf{q}_3 = \tfrac{1}{3}\mathbf{K}_t - \mathbf{k}_p. \tag{33}$$

For greater accuracy in the estimate (28), we take into account the spin components of all particles, except n_1 and p. After lengthy calculations, in which we neglect the recoil of the nucleus (so that $\mathbf{q} = \mathbf{K}_t - \mathbf{k}_1 - \mathbf{k}_p \approx \mathbf{k}_2$), and going over to the integral over the surface,[6] we obtain for the average value of (29)

$$|\Delta^{(2)}(\nu;\mathbf{k}_2)|^2_{av} \approx (\pi\hbar^2/8Mk_2)\,\xi^2\Gamma_{2\nu}(k_2),\quad \Gamma_{2\nu}(k_2)=|\gamma_2(\nu)|^2, \tag{34}$$

where, according to reference 1,

$$\gamma_2(\nu) = -\left(\frac{\hbar^2 k_2}{M}\right)^{1/2} \times \int_S \left\langle \Psi_f(\mathbf{R})\, Y_{JMl_2\sigma}\,|\,\varphi_\nu^{(2)}\left(\frac{\partial}{\partial r_2} - f(k_2 r_2)\right) j_{l_2}(k_2 r_2)\right\rangle dS \tag{35}$$

is the amplitude of the width of the emission of nucleon n_2 with a momentum $\hbar k_2$ from the intermediate nucleus $(A-1)_Z$ in the state ν. In (35), $j_{l_2}(k_2 r_2)$ is the spherical Bessel function of the first kind and of order l_2, and

$$Y_{JMl_2\sigma} = \sum_{m_2\mu\mu_2\mu_0} (l_2\sigma m_2\mu\,|\,JM)\,(\sigma_2\sigma_0\mu_2\mu_0\,|\,\sigma\mu) \times \chi_{\mu_2}(\sigma_2)\,\chi_{\mu_0}(\sigma_0)\,Y_{l_2 m_2}(\Omega_2),$$

where $\chi_{\mu_2}(\sigma_2)$ and $\chi_{\mu_0}(\sigma_0)$ are the spin functions of the neutron n_2 and the residual nucleus $(A-2)_Z$, respectively; $(\ldots.|\ldots)$ are the Clebsch-Gordan coefficients; $Y_{l_2 m_2}$ are spherical functions.

The calculation of (21) gives

$$\begin{aligned} |\Delta^{(5)}_{(2)}(\mu;\nu\mathbf{k}_1)|^2_{av} &\approx (\pi\hbar^2/Mk_1)\,\Gamma_{1\mu}(k_1);\\ |\Delta^{(8)}_{(5)}(\lambda;\mu\mathbf{k}_p)|^2_{av} &\approx (\pi\hbar^2/Mk_p)\,\Gamma_{p\lambda}(k_p), \end{aligned} \tag{36}$$

where $\Gamma_{1\mu}(k_1)$ is the width for the emission of neutron n_1 with momentum $\hbar k_1$ from the intermediate nucleus A_Z in the state μ; $\Gamma_{p\lambda}(k_p)$ is the width for the emission of the proton with momentum $\hbar k_p$ from the compound nucleus in the state λ.

We insert (34) and (36) into (27) and make the substitution for the widths in accordance with Weisskopf's formulas:[7]

$$\Gamma_{1\mu} = \frac{1}{w(E^{(5)})}\frac{2ME_1}{\pi^2\hbar^2}\sigma^{(1)}(E_1),$$
$$\Gamma_{2\nu} = \frac{1}{w(E^{(2)})}\frac{2ME_2}{\pi^2\hbar^2}\sigma^{(2)}(E_2),\qquad \Gamma_{p\lambda} = \frac{1}{w(E_c)}\frac{2ME_p}{\pi^2\hbar^2}\sigma^{(p)}(E_p), \tag{37}$$

here $w(E^{(5)})$, $w(E^{(2)})$, and $w(E_C)$ are the level densities of the intermediate nuclei A_Z, $(A-1)_Z$, and the compound nucleus C, respectively, with excitation energies $E^{(5)}$, $E^{(2)}$, and E_C; $\sigma^{(1)}(E_1)$, $\sigma^{(2)}(E_2)$, and $\sigma^{(p)}(E_p)$ are the cross sections for the formation of intermediate nuclei A_Z, $(A-1)_Z$, and the compound nucleus C by the incident nucleons n_1, n_2, and p of energy E_1, E_2, and E_p, where these cross sections are slowly varying functions of the energy, and E_1, E_2, and E_p are eigenvalues of the operators K_1, K_2, K_p, respectively. After integration, we obtain the quantity B_j (j = 1, 2, 3, 4, 5, 6):

$$B_j \approx 10^{-2}\left(\frac{M}{\hbar^2}\right)^{1/4} \alpha\,[w(E_c)\,\sigma^{(1)}\sigma^{(2)}\sigma^{(p)}]^{1/2} \times \exp\left[-\frac{3S_1+E_t}{2\Theta_c}\right] f(E_t)\,E_t^{7/4},$$
$$f(E_t) = \left\{\left[\ln\frac{\hbar^2\alpha^2/M + 8E_t/3}{\hbar^2\alpha^2/M}\right]^2 + \frac{1}{2}\ln\frac{\hbar^2\alpha^2/M + 8E_t/3}{\hbar^2\alpha^2/M}\ln\frac{\hbar^2\alpha^2/M + 32E_t/3}{\hbar^2\alpha^2/M}\right\}. \tag{38}$$

In the calculation of (38), we took into account the fact that[8]

$$w(E^{(2)}) \approx w(E_c)\exp\left(-\frac{E_1+E_p+S_1+S_p}{\Theta_c}\right),$$
$$w(E^{(5)}) = w(E_c)\exp\left(-\frac{E_p+S_p}{\Theta_c}\right),$$

where S_p and S_1 are the binding energies of the nucleons p and n_1 in nuclei A_Z and $(A-1)_Z$, respectively, Θ_C is the temperature of the compound nucleus C; we also took into account the fact that, according to reference 9, $S_1 \approx S_2 \approx S_p$ in the case of heavy nuclei.

Inserting the value of B_j (38) into (26) and carrying out the summation over all final states (after going over from summation to integration):

$$\Sigma_f \to \int \frac{dE_f}{D(E_f)} = \int_0^{E_c - S_t} w(E_c)\exp\left(-\frac{E_t+S_t}{\Theta_c}\right) dE_t$$

(S_t is the binding energy of a triton in the residual nucleus $(A-2)_Z$, $E_f = E_C - E_t - S_t$), we obtain the width for the triton production:

$$\Gamma_t \approx 1.4\cdot 10^{-3}\,(M\alpha/\hbar^2)^2\,\sigma^{(1)}\sigma^{(2)}\sigma^{(p)}\exp\left[-(3S_1+S_t)/\Theta_c\right] \times \int f^2(E_t)\,E_t^4\exp(-2E_t/\Theta_c)\,dE_t. \tag{39}$$

If we neglect the slowly varying logarithmic term, the integrand gives the energy spectrum $dw(E_t)$ of the emitted tritons:

$$dw(E_t) \sim E_t^4\exp(-2E_t/\Theta_c)\,dE_t. \tag{40}$$

Calculation of the integral in (39) by the method of steepest descent leads to

$$\Gamma_t \approx 1.1\cdot 10^{-3}\left(\frac{M\alpha}{\hbar^2}\right)^2 \sigma^{(1)}\sigma^{(2)}\sigma^{(p)}\,w^2(E_c) \times \exp\left[-\left(\frac{3S_1+S_t}{\Theta_c}\right)\right] f^2(2\Theta_c)\,\Theta_c^5. \tag{41}$$

Finally, we compare the width (41) with the width for the evaporation of a triton Γ_{tc} obtained from the ordinary theory of evaporation. For this, we introduce[1] the integral width for the evaporation of nucleons n_1 and p and a triton by the compound nucleus:

$$\Gamma_{ic} = \int \Gamma_{ic}(E_i)\, dE_i \qquad (i = 1, p, t)$$
$$\Gamma_{1c} = (2M/\pi^2\hbar^2)\, \sigma^{(1)}\, \Theta_c^2 \exp(-S_1/\Theta_c),$$
$$\Gamma_{pc} = (2M/\pi^2\hbar^2)\, \sigma^{(p)}\, \Theta_c^2 \exp(-S_p/\Theta_c),$$
$$\Gamma_{tc} = (6M/\pi^2\hbar^2)\, \sigma^{(t)}\, \Theta_c^2 \exp(-S_t/\Theta_c), \qquad (42)$$

where $\sigma^{(1)}$, $\sigma^{(p)}$, and $\sigma^{(t)}$ are the cross sections for the formation of a compound nucleus by the incident nucleons n_1 and p and the triton. We note that formulas (42) are readily derived from (37). As a result, we obtain

$$\Gamma_t/\Gamma_{tc} \approx 4.5 \cdot 10^{-2} \frac{\Gamma_{1c}\Gamma_{pc}}{D^2(E_c)} \frac{\sigma^{(2)}}{\sigma^{(t)}} f^2(2\Theta_c) \frac{\hbar^2\alpha^2}{M\Theta_c} \exp(-S_1/\Theta_c). \qquad (43)$$

The quantity $\Gamma_{1c}\Gamma_{pc}/D^2(E_c)$ is of the order of unity, since the excitation energy is large.[8] The ratio $\sigma^{(2)}/\sigma^{(t)}$ is also of the order of magnitude of unity; the term $\hbar^2\alpha^2/M$ is equal to 0.3 Mev.* The binding energy S_1 can be taken equal to 6 Mev (heavy nuclei). Then

$$\Gamma_t/\Gamma_{tc} \approx 1.4 \cdot 10^{-2}\, \Theta_c^{-1} f^2(2\Theta_c) \exp(-6/\Theta_c) \qquad (\Theta_c \text{ in Mev}). \qquad (44)$$

For $\Theta_c \approx 2.5$ Mev, which corresponds to an excitation energy of 100 Mev for heavy nuclei with A > 200, we have

$$\Gamma_t/\Gamma_{tc} \sim 0.3. \qquad (45)$$

Hence the indirect evaporation process associated with the evaporation of the three nucleons n_1, n_2, and p and their subsequent uniting makes an appreciable contribution to the cross section for the production of tritons.

*The quantity α is estimated from the value of the triton radius $R_t = 2.24 \times 10^{-13}$ cm determined from a comparison of the binding energies of the mirror nuclei t and He_2^3:

$$R_t = r_1 - \frac{r_1 + r_2 + r_3}{3} = \frac{1}{3}(\rho_1 - \rho_3),$$
$$R_t = \frac{1}{3}\int |\rho_1 - \rho_3|\, |\Phi_t|^2\, \delta(\rho_1 + \rho_2 + \rho_3)\, d\rho_1\, d\rho_2\, d\rho_3.$$

Hence $\alpha = 0.85 \times 10^{12}$ cm^{-1} and $\hbar^2\alpha^2/M = 0.3$ Mev.

We note that this result does not take into account the probability that the particles cross the Coulomb barrier. If this is taken into account, the ratio (45) will be somewhat larger.

[1] K. Kikuchi, Progr. Theoret. Phys. (Kyoto) **18**, 503 (1957).

[2] P. Kapur and R. Peierls, Proc. Roy. Soc. (London) **A166**, 277 (1938).

[3] H. Ui, Progr. Theoret. Phys. (Kyoto) **16**, 299 (1956).

[4] B. Lippman and J. Schwinger, Phys. Rev. **79**, 469 (1950).

[5] M. Gell-Mann and M. Goldberger, Phys. Rev. **91**, 398 (1953).

[6] V. I. Smirnov, Kurs vysshei matematiki (Course in Higher Mathematics), Gostekhizdat, 1951, vol. 2, Sec. 193.

[7] V. F. Weisskopf, Phys. Rev. **52**, 295 (1937).

[8] A. Akhiezer and I. Pomeranchuk, Nekotorye voprosy teorii yadra (Some Problems on the Theory of the Nucleus), Gostekhizdat, 1950.

[9] J. M. Blatt and V. F. Weisskopf, Theoretical Nuclear Physics, John Wiley and Sons, New York, 1952, Chap. 1.

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