ANALYSIS OF COSMIC-RAY SHOWERS PRODUCED BY HIGH ENERGY PRIMARY PARTICLES: BASED ON THE EXCITED NUCLEON MODEL

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High-energy ($E > 10^{11}$ ev) interactions of cosmic-ray nucleons in emulsion are analyzed on the basis of the excited nucleon model. The c.m.s. angular distribution of the excited nucleons is given. The anisotropy of the angular distribution of the shower particles and the multiplicity of the shower particles are examined in relation to the velocity and the emission angle of the excited nucleons in the c.m.s. The experimental results are compared with the prediction of the single-meson pole approximation.

THE two-center model,[1] without stipulating a specific mechanism for the interaction between the nucleons or a mechanism for their decay, presupposes that the two colliding nucleons are excited to certain isobaric states. The excited nucleons, which have large masses, rapidly decay independently of each other with formation of pions, and go to the ground state. In a frame fixed to the excited nucleon (radiating center), the pions are isotropically distributed, as confirmed by numerous experimental data obtained at high energies (see for example, [2]), and their angular distribution in the center of mass system (c.m.s.) depends on the velocity of motion of the radiating centers in this system, $\bar{v}$, or on the quantity $\bar{\gamma} = (1-\bar{v}^2)^{-1/2}$.*

If $\bar{\gamma}$ is close to unity, the angular distribution of the shower particles is almost isotropic in the c.m.s., and if $\bar{\gamma} > 1$, the distribution of the shower particles is anisotropic in the c.m.s. Thus, the model of excited nucleons describes various angular distributions, the form of which in the c.m.s. depends on the value of $\bar{\gamma}$. The value of $\bar{\gamma}$ can be determined from the angular distribution of the shower particles in the laboratory system (l.s.) and from the number of mesons n_1 and n_2 produced by the decay of each of the excited nucleons[2]:

$$\bar{\gamma} = (\gamma_1 + \gamma_2)/2\gamma_c.$$

In the experiments, γ_1 and γ_2 are determined from the angular distribution of the particles in the narrow and diffuse cones of the shower. In the determination of γ_1 and γ_2 we do not assume that the pions have a monoenergetic distribution in the system of the excited nucleon.

Making use of the experimental data of Boos et al,[3] who investigated showers with low anisotropy (corresponding to $\bar{\gamma} \approx 1$), we assume that the energy spectrum of the shower particles obeys in the system of the excited nucleon a power law:

$$N(E') \sim \frac{1}{E'^2}dE'$$

and therefore[4] *

$$\gamma_1 = 0.77\gamma_1', \qquad \lg\gamma_1' = -\frac{1}{n_1}\sum_{i=1}^{n_1}\lg\operatorname{tg}\theta_i;$$

$$\gamma_2 = 0.77\gamma_2', \qquad \lg\gamma_2' = -\frac{1}{n_2}\sum_{i=1}^{n_2}\lg\operatorname{tg}\theta_i;$$

$$n_s = n_1 + n_2.$$

Taking account of the fact that the excited nucleons can be emitted at arbitrary c.m.s. angles ϑ' to the primary direction, we must determine the parameter γ_c not from the formula $\gamma_c = \sqrt{\gamma_1\gamma_2}$, but from the expression[5]

$$\gamma_c = 2^{-1/2}[\gamma_1 + \gamma_2 + \tfrac{3}{2}E'(n_1\gamma_1 + n_2\gamma_2)]^{1/2},$$

where $E' = 0.4$ Bev is the average meson energy in the radiating-center system.

The emission angle ϑ is calculated, using the energy and momentum conservation law, from the values of γ_1, γ_2, and γ_c[6]:

$$\cos\vartheta' = (\gamma_1 - \gamma_2)/\sqrt{(\gamma_1 + \gamma_2)^2 - 4\gamma_c^2}.$$

The purpose of the present investigation was a study of the angular distribution of the excited nucleons in the c.m.s., its connection with the form of the angular distribution of the generated shower

*We use a system of units in which the velocity of light and the mass of the nucleon are equal to unity.

*lg = log, tg = tan.

particles in the l.s. and with their multiplicity, and also a check on the theory of peripheral interactions in the region of superhigh energies.

EXPERIMENTAL RESULTS

We chose for the analysis showers with $N_n \leq 2$ and $n_s \geq 6$, formed by singly-charged or neutral cosmic-ray particles (nucleons) with energy $E > 10^{11}$ ev interacting in emulsion. The total number of showers was forty-two.* We plotted the integral and differential angular distributions of the shower particles in coordinates $\log \tan \theta - \overline{\log \tan \theta}$, and determined the approximate number of particles radiated by each nucleon. If the integral-distribution curve split into two branches, we determined n_1 and n_2 as the number of particles in each branch, and if there was no split we assumed that the nucleons are similarly excited, i.e., their masses after collision are $m_1 = m_2 = m$, and consequently the same number of mesons $n_1 = n_2 = n_s/2$ is produced by their decay. We then calculated γ_1, γ_2, γ_c, $\bar{\gamma}$, and $\cos \vartheta'$, for all the analyzed showers.

The angular distribution of the excited nucleons in the c.m.s. is highly anisotropic (Fig. 1), with small emission angles predominating (25 — 30°), but the angle reaches 70° in some showers with low multiplicity. Considering that the influence of fluctuations is particularly great at small n, we disregarded showers with $n_s < 10$, and the form of the distribution of the excited nucleons was not changed thereby. For comparison we calculated the angular distribution by the theory of peripheral interactions in the single-meson pole approximation. In view of the fact that the multiplicities of the shower particles of the analyzed showers are small on the average, it becomes necessary to take account of the influence of fluctuations in the number of particles.

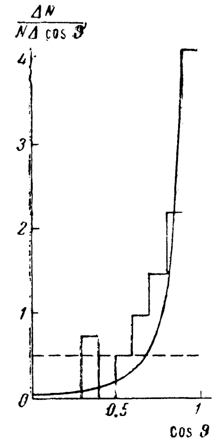

FIG. 1. Angular distribution of excited nucleons in the c.m.s. The curve was calculated with formula (5) (see below) and normalized in area with the histogram; the dashed line corresponds to isotropic distribution in the c.m.s.

*In many cases data on the showers were taken from published papers or obtained from other laboratories (Moscow, Leningrad, Prague, Berlin, Budapest, Krakow).

Podgoretskii et al[7] define the fluctuations in the number of particles as quantities proportional to the square root of the number of particles: $n_s \pm \alpha\sqrt{n_s}$. The proportionality coefficient α depends on the type of generated particles and is close to unity for the case of pions. We have calculated the fluctuations in the number of particles generated by each excited nucleon: $n_1 \pm \sqrt{n_1}$ and $n_2 \pm \sqrt{n_2}$, and then determined the corresponding γ_1 and γ_2 and evaluated the angles ϑ' for several showers with different multiplicities. As a result it became clear that an account of the fluctuations in the number of particles n_1 and n_2 produced by each excited nucleon changes the value of the angle ϑ' very little, and the change $\Delta \cos \vartheta' = 0.03 - 0.04$ is insignificant. Thus, fluctuations in the number of particles do not change the form of the c.m.s. angular distribution of the excited nucleons. The fluctuations in the angular distributions were not taken into account rigorously, but we assume that such an account would not change appreciably the angular distribution, for otherwise there would be no regular connection between the multiplicity n_s and $\cos \vartheta'$, and a much greater straggle in the points would be observed in Fig. 2.

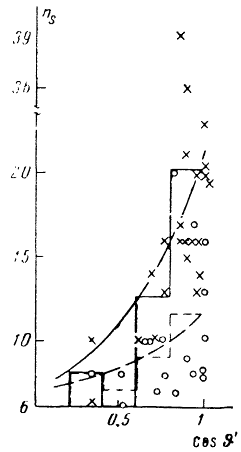

FIG. 2. Dependence of the multiplicity on the cosine of the angle of emission of excited nucleons in the c.m.s. The solid and dashed curves were calculated by the least-squares method. The points × and o pertain to showers with $\gamma_c \gtrsim 12$ and with $5 \lesssim \gamma_c < 12$, respectively.

In the two-center model the multiplicity of shower particles is determined by the momentum transfer, and depends on the degree of excitation of the colliding nucleons, i.e., on their mass m after collision. We find from the experimental data that the masses of the excited nucleons (or the multiplicity of the shower mesons) depend on the angle at which they are emitted in the c.m.s., and the smaller the angle ϑ', the larger the multiplicity for a given primary-particle energy (for specified γ_c). As can be seen from Fig. 2, where the experimental values of the multiplicity n_s and of $\cos \vartheta'$ were plotted for two energy intervals and the corresponding curves then calculated by the method of least squares, the multiplicity increases with decreasing angle ϑ'. We excluded from consideration showers with $n_s < 10$, but these do not change the general character of this dependence.

We next investigated the dependence of the form of the angular distribution of the shower particles on the velocity of motion and angle of emission of the excited nucleons in the c.m.s. We chose as a characteristic of the form of the angular distribution of the shower particles the mean-square deviation[4]

$$\sigma = \left[\frac{1}{n_s} \sum_{i=1}^{n_s} (\lg \operatorname{tg} \theta_i - \overline{\lg \operatorname{tg} \theta})^2 \right]^{1/2},$$

and the measure of departure from isotropy was the ratio σ/σ_{is}, where $\sigma_{is} = 0.31$ corresponds to isotropic angular distribution of the particles and to a power-law energy spectrum.

Figure 3 shows the values of σ/σ_{is} as a function of $\bar{\gamma}$. The parameter σ/σ_{is} characterizes the degree of anisotropy and increases (on the average) with increasing $\bar{\gamma}$, i.e., the velocity of motion of the excited nucleons in the c.m.s. The angle at which the excited nucleons are emitted in the c.m.s. is also connected with the form of the angular distribution of the shower particles, i.e., with the anisotropic parameter σ/σ_{is}. It is seen in Fig. 4 that the greater ϑ', the closer the angular distribution is to isotropic; at small angles the distribution is in most cases anisotropic.

For many cases (see Figs. 3 and 4) the values of the parameter σ/σ_{is} were found to be less than unity. Were the model to be confined to an examination of the angles $\vartheta' = 0$, this fact would remain unexplained. The admission of large ϑ' into the model provides us with the following explanation:

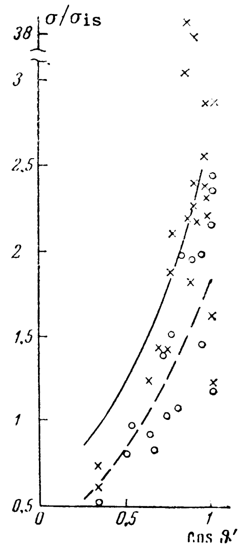

FIG. 4. Dependence of σ/σ_{is} on the cosine of the angle of emission of the excited nucleons in the c.m.s. The curves (solid and dashed) were calculated by the method of least squares. The points × and o pertain to showers with $\gamma_c \geq 12$ and with $5 \leq \gamma_c < 12$, respectively.

the measure of the anisotropy σ/σ_{is} depends on two parameters $\bar{\gamma}$ and ϑ'; if $\bar{\gamma} \approx 1$, then the distribution is isotropic for all emission angles and $\sigma/\sigma_{is} \approx 1$; on the other hand, if $\gamma > 1$, then the distribution should be anisotropic but the anisotropy parameter σ/σ_{is}, calculated from angles obtained in the l.s., is less than unity at large emission angles ϑ'.

The experimental data cited are comparable with the deductions of the theory of peripheral interactions. This comparison is essentially qualitative, since the results shown in Figs. 1 and 2 are, first, summary material for a rather extensive interval of values of γ_c ($5 \leq \gamma_c \leq 70$), and second, they are subject to unaccountable errors. Furthermore, it must be noted that apparently not all forty-two interactions correspond to the case of nucleon excitation, and a certain fraction of the showers may correspond to a virtual $\pi\pi$ interaction.[8] However, in the comparison of experiment with theory we shall assume that all the analyzed showers correspond to excitation of nucleons.

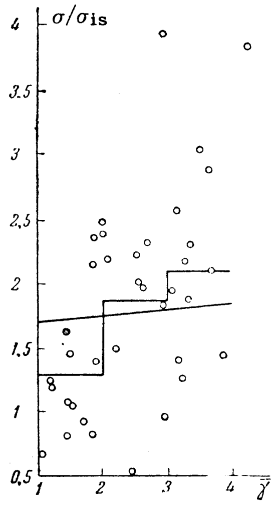

FIG. 3. Dependence of the anisotropy parameter σ/σ_{is} on $\bar{\gamma}$ for the analyzed nucleon-nucleon showers. The histogram was constructed from the average values of σ/σ_{is} in each chosen interval of $\bar{\gamma}$, while the straight line was calculated by the method of least squares.

COMPARISON OF THE RESULTS OF THE THEORY OF PERIPHERAL INTERACTIONS WITH EXPERIMENT

A method based on perturbation theory has been developed in a series of papers[9-11] for the description of peripheral interaction of high-energy nucleons. In the single-meson pole approximation, the results of this method are in satisfactory agreement with experiments on nucleon-nucleon interactions at energies of 9 Bev[9,10] and 200 — 300 Bev,[11,12] indicating that the peripheral interactions make the principal contribution at these energies. It is interesting to compare the theory in this approximation with the experimental data at higher energies. In view of the fact that the experimental material at our disposal pertains to a broad energy interval, a detailed comparison between theory and experiment is impossible. We can, however, present an analysis that pertains to an important characteristic of the interaction, namely the 4-momentum of the intermediate pion, exchanged by the interacting nucleons.

The single-meson interaction of these nucleons, which leads to their excitation and subsequent decay, is described by the Feynman diagram shown in Fig. 5. For the left node of this diagram, the 4-momentum conservation law has the form

$$k = p_0 - p_1.$$

Squaring both halves of this expression we have

$$k^2 = p_0^2 + p_1^2 + 2\varepsilon_0\varepsilon_1 - 2\,|\mathbf{p}_0|\,|\mathbf{p}_1|\cos\vartheta, \qquad (1)$$

where $k^2 = \mathbf{k}^2 - k_0^2$ is the square of the 4-momentum of the intermediate pion, ϵ_0 is the nucleon energy prior to interaction, ϵ_1 the energy of the excited nucleon, and ϑ the angle between the directions of motion of the nucleon before and after the interaction.

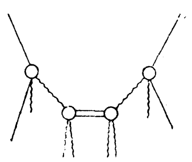

FIG. 5

Denoting by m_1 and m_2 the masses of the excited nucleons, we can rewrite (1) as

$$k^2 = -1 - m_1^2 + 2\varepsilon_0\varepsilon_1 - 2\sqrt{(\varepsilon_0^2 - 1)(\varepsilon_1^2 - m_1^2)}\cos\vartheta. \qquad (2)$$

For the c.m.s. of the colliding nucleons, the energy-momentum conservation law is given by the equations

$$\varepsilon_1 + \varepsilon_2 = 2\gamma_c, \qquad \varepsilon_1^2 - m_1^2 = \varepsilon_2^2 - m_2^2. \qquad (3)$$

Hence

$$\varepsilon_1 = (4\gamma_c^2 + m_1^2 - m_2^2)/4\gamma_c \qquad (4)$$

and we obtain finally for the momentum of the intermediate pion

$$k^2 = -1 - m_1^2 + \frac{4\gamma_c^2 + m_1^2 - m_2^2}{2} - 2\cos\vartheta'\sqrt{\gamma_c^2 - 1} \times \left[\left(\frac{4\gamma_c^2 + m_1^2 - m_2^2}{4\gamma_c}\right)^2 - m_1^2\right]^{1/2}. \qquad (5)$$

In the case of symmetrical excitation of the nucleons we have $m_1 = m_2 = m$; for k^2 we obtain from (5)

$$k^2 = -1 - m^2 + 2\gamma_c^2 - 2\sqrt{(\gamma_c^2 - 1)(\gamma_c^2 - m^2)}\cos\vartheta'. \qquad (6)$$

Using (6), we have calculated for the analyzed interaction cases the distribution over k^2, shown in Fig. 6.

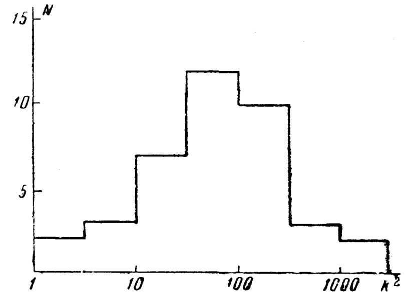

FIG. 6. Distribution of showers with respect to the value of k^2 calculated from formula (6).

The result obtained can be compared with the data of Dremin and Chernavskii,[11] who found that the total cross section $\sigma_{NN}(E)$ of the single-meson interaction of two nucleons agrees at energies $E \sim 200$ Bev with the corresponding experimental value only if it is assumed that the total cross section of the interaction of a virtual pion with the nucleon, $\sigma_{\pi N}(k^2)$, is a smooth function of k^2 when $k^2 \leq (7\mu)^2$, and decreases sharply with further increase in k^2. In our case, however, $k^2 \ll (7\mu)^2$ for all showers (Fig. 6). This indicates that the showers which we are considering cannot be described by perturbation theory in the single-meson approximation.

It was noted in several papers[9-11] that in the single-meson approximation the c.m.s. angular distribution of the excited nucleons should be strongly anisotropic; the degree of anisotropy should increase with increasing γ_c. In addition, the width of the angular distribution of the excited nucleons is determined by the cross section $\sigma_{\pi N}(k^2)$. If this cross section decreases sharply, starting with $k^2 \sim (7\mu)^2$, then this is equivalent to the existence of a maximum angle ϑ'_{max}, which should be considerably smaller than $\pi/2$. Actually, if we determine ϑ'_{max} from the formula

$$\cos\vartheta' = (2\gamma_c^2 - 1 - m^2 - k^2)/2\sqrt{(\gamma_c^2 - 1)(\gamma_c^2 - m^2)},$$

which follows from (6), we have $\vartheta'_{max} = 6°\,6'$ when $\gamma_c = 10$, $m = 1.32$ [mass of the isobar ($3/2$, $3/2$)] and $k^2 = (7\mu)^2$, whereas $\vartheta_{max} = 2°\,54'$ when $\gamma_c = 20$ and $m = 1.32$.

Greater values of m correspond to even smaller limiting angles. Thus, the observed angular distribution of the excited nucleons (Fig. 1) goes far beyond the limits permissible in our theory. However, if no limitations are imposed on the virtuality, we can readily obtain the theoretical angular distribution for fixed γ_c and m by using the results of Dremin and Chernavskii.[9] The curve shown in Fig. 1 is calculated for average experimental values of γ_c and m ($\gamma_c = 20$, $m = 4.5$).

In spite of the fact that the assumption of the large virtualities is unfounded, the curve fits the observed angular distribution quite satisfactorily.

In addition to the features of the angular distribution, the theory predicts a reduction (for γ_c = const) in the mass of the excited nucleons (or in the multiplicity of the shower particles) with increasing angle ϑ'. The prediction of such a tendency does not contradict experiment (Fig. 2). However, inasmuch as the experimental angular distribution is much broader than that admitted by the theory, it is impossible to carry out a complete graphic comparison.

CONCLUSIONS

1. The mass of the excited nucleons, and consequently the multiplicity of the generated mesons, depends on the direction of motion of the excited nucleons in the c.m.s.

2. The form of the angular distribution of the shower particles in the laboratory system depends on the velocity and direction of motion of the excited nucleons in the c.m.s.

3. The transfer of 4-momentum, occurring upon interaction of the nucleons, is in all cases large compared with $(7\mu)^2$. Therefore, if the centers of emission of the shower mesons are actually the excited nucleons and if the directions of their motion coincide with the axes of the cones, then the mechanism of excitation of the nucleons should be appreciably different from the single-meson interaction.

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