LIMITING VALUES OF THE $\pi^{\pm}$p SCATTERING AMPLITUDE

V. P. KANAVETS, I. I. LEVINTOV, and B. V. MOROZOV

Institute of Theoretical and Experimental Physics, Academy of Sciences, U.S.S.R.

Submitted to JETP editor December 28, 1960

J. Exptl. Theoret. Phys. (U.S.S.R.) **41**, 146-153 (July, 1961)

Dispersion relations in which subtraction is transferred to points located at infinity are derived on the basis of Pomeranchuk's assumptions regarding the asymptotic behavior of the scattering amplitude. In this form, the dispersion relations are most convenient for estimating the asymptotic behavior of the amplitude on the basis of the experimental data on $\pi^{\pm}$p scattering. A preliminary numerical estimate of the asymptotic behavior of the $\pi^{\pm}$p scattering amplitude is presented. The question whether the validity of the dispersion equations at high energies is consistent with the statistical theory is considered.

POMERANCHUK[1] has shown that if the existence of an energy-independent limited radius of interaction is assumed, the behavior of the complex scattering amplitude of particles and antiparticles for an arbitrary scatterer at an angle 0° should be described by a function which increases to infinity no faster than the first power of the energy. A consequence of this is the equality of the limiting values of the total interaction cross sections for particles and antiparticles:

$$\sigma_t^+(\infty) = \sigma_t^-(\infty). \quad (1)$$

Experimental studies of the total cross sections of $\pi^{\pm}$p interactions apparently confirm Pomeranchuk's conclusions that at high energies the cross sections for π^+ and π^- approach the same, almost constant value.

In this connection, it is of interest to estimate, within the framework of these theoretical assumptions, the asymptotic value of the real part of the $\pi^{\pm}$p scattering amplitude on the basis of the latest data on the behavior of the total cross sections at high energies. For this, it will be convenient to start from symmetric and antisymmetric combinations of the amplitudes $D_+(E) \pm D_-(E)$, where $D_{\pm}(E)$ are the real parts of the $\pi^{\pm}$p scattering amplitudes at 0°. For such combinations at sufficiently high energy E, according to Pomeranchuk's assumptions, the following relations should hold:

$$C(E) \equiv [D_+(E) - D_-(E)]/2E = C_\infty, \quad (2)$$

$$\tfrac{1}{2}[D_+(E) + D_-(E)] = Q(E), \quad (3)$$

where $Q(E)$ is a function that increases more slowly than the first power of the energy, an C_∞ is a constant.

For an estimate of quantities (2) and (3), we should let the energy go to infinity in the dispersion relations. For this, it is convenient not to fix the energy at which the second subtraction is made. Since many values of $D_{\pm}(E)$ are now known with good accuracy over a rather large energy interval, the method permits an essential increase in the accuracy of the determination of $D_{\pm}(\infty)$ in comparison with calculations by means of the dispersion relations in Goldberger's form, in which the second subtraction is fixed at $E = m$ (m is the meson mass). We note that the transfer of the subtraction to infinity leads [together with condition (2)] to definite restrictions on the way the cross sections approach their limiting values.

We shall start from the dispersion relations for the case of forward scattering of charged mesons in the form in which they were written by Bogolyubov, Medvedev, and Polivanov:[2]

$$C(E) - C(E_0) = \frac{1}{4\pi^2}(E^2 - E_0^2)\,\mathrm{P}\int_m^\infty \frac{k'[\sigma_t^+(E') - \sigma_t^-(E')]\,dE'}{(E'^2 - E_0^2)(E'^2 - E^2)} + 2f^2\frac{E^2 - E_0^2}{(E^2 - r^2)(E_0^2 - r^2)}, \quad (4)$$

$$Q(E) - Q(E_0) = \frac{1}{4\pi^2}(E^2 - E_0^2)\,\mathrm{P} \times \int_m^\infty \frac{k'E'[\sigma_t^+(E') + \sigma_t^-(E')]}{(E'^2 - E_0^2)(E'^2 - E^2)}\,dE' + 2f^2\frac{r(E^2 - E_0^2)}{(E^2 - r^2)(E_0^2 - r^2)}. \quad (5)$$

Here $E^2 = m^2 + k^2$, $r = m^2/2M$, $f^2 = 0.08$, M is the mass of the nucleon.

Setting $E^2 \ll \epsilon^2 \ll E_0^2$, we can rewrite (4) in the form

$$p(\varepsilon) = C(E) - \frac{1}{4\pi^2} \mathrm{P} \int_m^{\varepsilon} \frac{k'(\sigma_t^+ - \sigma_t^-)}{E'^2 - E^2} dE' + 2f^2 \frac{1}{E^2 - r^2}$$

$$= C(E_0) - \frac{1}{4\pi^2} \mathrm{P} \int_{\varepsilon}^{\infty} \frac{E_0^2(\sigma_t^+ - \sigma_t^-)}{E'(E'^2 - E_0^2)} dE', \quad (6)$$

where $p(\epsilon)$ is obviously a function of ϵ only.

We now let E_0 go to infinity for any fixed value of ϵ. According to (2), the function $C(E_0)$ should then go over into C_∞. Since $p(\epsilon)$ is constant, the integral with E_0 in (6) also should go over to a constant as $E_0 \rightarrow \infty$. The necessary condition for this is the existence of the integral*

$$\int_{\varepsilon}^{\infty} dE' (\sigma_t^+ - \sigma_t^-)/E' = A(\varepsilon).$$

We then have the condition

$$\lim_{E_0 \to \infty} \left(-\frac{1}{4\pi^2} \mathrm{P} \int_{\varepsilon}^{\infty} \frac{E_0^2(\sigma_t^+ - \sigma_t^-)}{E'(E'^2 - E_0^2)} dE' \right) = \frac{1}{4\pi^2} \int_{\varepsilon}^{\infty} \frac{\sigma_t^+ - \sigma_t^-}{E'} dE'. \quad (7)$$

From (6) and (7) it follows that

$$C(E) - \frac{1}{4\pi^2} \mathrm{P} \int_m^{\varepsilon} \frac{k'(\sigma_t^+ - \sigma_t^-)}{E'^2 - E^2} dE' - \frac{1}{4\pi^2} \int_{\varepsilon}^{\infty} \frac{\sigma_t^+ - \sigma_t^-}{E'} dE'$$

$$+ 2f^2 \frac{1}{E^2 - r^2} = C(E) - \frac{1}{4\pi^2} \mathrm{P} \int_m^{\infty} \frac{k'(\sigma_t^+ - \sigma_t^-)}{E'^2 - E^2} dE'$$

$$+ 2f^2 \frac{1}{E^2 - r^2} = C_\infty. \quad (8)$$

Formula (8) represents the dispersion relation for the difference in amplitudes in which the subtraction is transferred to points located at infinity. Setting $C_\infty = 0$ and $E = m$ in (8), we arrive at the summation rule of Goldberger et al.[4]

We do not know whether the function $Q(E)$ remains bounded as $E \rightarrow \infty$. The asymptotic behavior of $Q(E)$ is determined by how fast $\sigma_t^+(E) + \sigma_t^-(E)$ approaches the limiting value $2\sigma_\infty$. If $\sigma_t^\pm$ decreases monotonically beginning at some energy, then it can be shown (see Appendix) that, for sufficiently large E, a necessary condition for the relation

$$Q(E) = Q_\infty, \quad (9)$$

where Q_∞ is a bounded quantity, is the existence of the integral

$$\int_{\varepsilon}^{\infty} [\sigma_t^+(E') + \sigma_t^-(E') - 2\sigma_\infty] dE' = B(\varepsilon) < \infty. \quad (10)$$

This means that the difference $\sigma_t^+ + \sigma_t^- - 2\sigma_\infty$ should, under condition (9), tend to zero as $E \rightarrow \infty$ faster, on the average, than $(E \ln E)^{-1}$. At present, however, we cannot give definite theoretical arguments in favor of assumptions (9) or (10),* and the transferring of the substraction points $\pm E_0$ to infinity in relation (5) does not lead to definite results.

Nevertheless, to investigate the behavior of $Q(E)$ at high energies, it is useful to introduce the function $q(\epsilon)$, which is determined by experimentally measured quantities:

$$q(\varepsilon) = Q(E) - \frac{1}{4\pi^2} \mathrm{P} \int_m^{\varepsilon} \frac{E'k'(\sigma_t^+ + \sigma_t^-)}{E'^2 - E^2} dE'$$

$$+ \frac{\varepsilon}{4\pi^2} [\sigma_t^+(\varepsilon) + \sigma_t^-(\varepsilon)] + 2f^2 \frac{r}{E^2 - r^2}, \quad (11)$$

$$\varepsilon^2 \gg E^2.$$

In the Appendix, it is shown that $q(\epsilon)$ has the following properties: 1) it does not depend on E under the condition that $\epsilon^2 \gg E^2$; 2) if (9) and (10) are fulfilled, then $\lim q(\epsilon) = \bar{Q}_\infty$ as $\epsilon \rightarrow \infty$; 3) if (9) and (10) are not fulfilled, then the absolute value of $q(\epsilon)$ increases without limit as ϵ increases.

COMPARISON WITH EXPERIMENT

Experimental information concerning $\sigma_t^\pm$ and $D_\pm(E)$ is available only up to a certain energy. For a number of reasons, the energy ϵ to which information on $\sigma_t^\pm$ is available is considerably higher than the energy E to which $D_\pm(E)$ is known. This is in accord with the conditions of separation $\epsilon^2 \gg E^2$. Hence, for comparison with experiment it is convenient to separate out of (8), along with C_∞, that part of the integral in the limits between ϵ and ∞ for which $\sigma_t^\pm$ is not known at present.

Adding and subtracting (6) and (11), we arrive at a system of two equations valid for $E^2 \ll \epsilon^2$:

$$q(\varepsilon) \pm Ep(\varepsilon) = D_\pm(E)$$

$$- \frac{1}{4\pi^2} \mathrm{P} \int_m^{\varepsilon} \frac{E'k'(\sigma_t^+ + \sigma_t^-) \pm Ek'(\sigma_t^+ - \sigma_t^-)}{E'^2 - E^2} dE'$$

$$+ \frac{\varepsilon}{4\pi^2} [\sigma_t^+(\varepsilon) + \sigma_t^-(\varepsilon)] \pm \frac{2f^2}{r \mp E}, \quad (12)$$

where

$$p(\varepsilon) = C_\infty + \frac{1}{4\pi^2} \int_{\varepsilon}^{\infty} \frac{\sigma_t^+ - \sigma_t^-}{E'} dE'. \quad (13)$$

If, in fixing ϵ, we subtract from the experimental data on $D_\pm(E)$ and $\sigma_t^\pm$ the right-hand part of (12) and plot the result as a function of E, we consequently obtain a straight line whose parameters determine $q(\epsilon)$ and $p(\epsilon)$. From (12) it follows that any error in the information on the total cross

*The condition under which (2) is fulfilled was obtained earlier by Amati, Fierz, and Glaser.[3]

*For this reason, the conclusions on the behavior of the cross sections made by Lomsadze, Lend'el and Érnst[5] on the basis of assumption (9) cannot be considered well-founded.

sections at energies much higher than E does not disturb the linear dependence of the right-hand part of (12) on E, but changes the values of q (ε) and p (ε) by the quantities δq (ε) and δp (ε):

$$\delta q(\varepsilon) = \int \frac{[\delta\sigma_t^+(E') + \delta\sigma_t^-(E')]\,dE'}{4\pi^2},$$

$$\delta p(\varepsilon) = \int \frac{\delta\sigma_t^+ - \delta\sigma_t^-}{4\pi^2 E'}\,dE'. \qquad (14)$$

Therefore the errors in the cross sections at high energies have a far greater effect on the determination of q (ε) than on p (ε).

To determine the parameters p (ε) and q (ε), we used the experimental data[6-9] on the values of $D_{\pm}(E)$. In the calculations, we used the values of $D_{\pm}(E)$ in the laboratory system. For the calculation of the integrals in the right-hand part of (12), we took all the values for $\sigma_t^{\pm}$ up to 5.2 Bev published by the middle of 1960, except for the data of Devlin et al,[10] since they are in poor agreement with the results of the measurements of other authors in this energy interval.[11-13]

The energy interval in which the total cross sections are known were divided into 14 subintervals for σ_t^- and 13 subintervals for σ_t^+. In each subinterval, the cross section was approximated by a quadratic parabola by the method of least squares. The values of the right-hand part of (12) as a function of E are shown in the figure in units of 10^{-13} cm.

The parameters of the line for ε = 5.2 Bev determined by the method of least squares are:

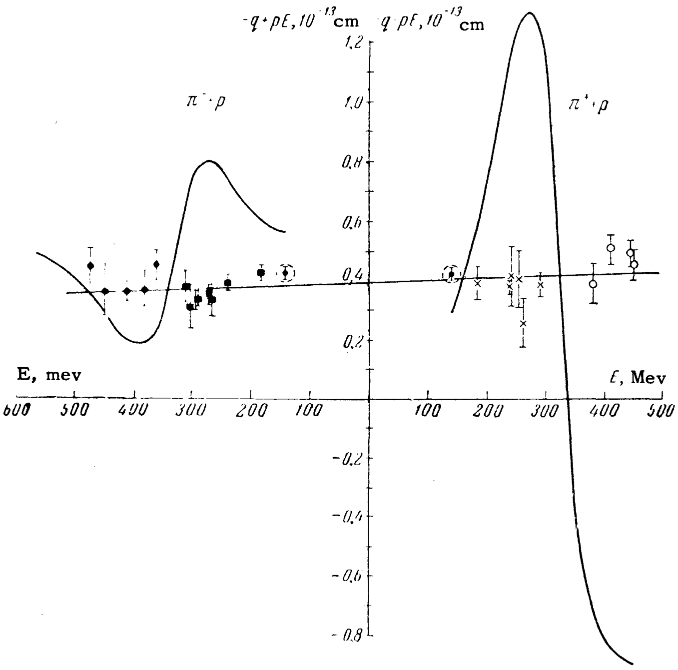
$$p(\varepsilon) = (-0.069 \pm 0.03)\cdot 10^{-13}\ \text{cm/Bev},$$

$$q(\varepsilon) = (-0.395 \pm 0.01)\cdot 10^{-13}\ \text{cm}.$$

In the calculation, we took $f^2 = 0.08$.

Data on $D_{\pm}(m)$, shown in the dotted circles in the figure, were not included in the calculation of p (ε) and q (ε), since they were calculated from the experimental results used for the determination of $D_{\pm}(\epsilon)$ at low energies, and have already been included in the calculations. The errors given for p (ε) and q (ε) were determined by the statistical errors in $D_{\pm}(E)$ and do not take into account uncertainties connected with the experimental data on the total cross sections and the approximation of the cross sections, or the uncertainty in f^2.

The parameter q (ε), because of (14), is very sensitive to the behavior of the cross section in the high-energy region, and since the behavior of σ_t^- in the energy interval 1.9 — 4.15 Bev is not known, the actual error in the value of q (ε) can be several times the value cited. Therefore we refrain from analyzing the variation of q (ε) with energy.



Values of the right-hand part of (12) for ε = 5.2 Bev based on the following data for $D_{\pm}(E)$: ● – reference 6; ■ – reference 7; × – reference 8; ○ – reference 9. The parameters of the line $q(\varepsilon) \pm Ep(\varepsilon)$ are: $q = (-0.395 \pm 0.010) \times 10^{-13}$ cm, $p = (-0.069 \pm 0.030) \times 10^{-13}$ cm/Bev. The curves represent the right-hand part of expression (12) after subtraction of the term $D_{\pm}(E)$.

It should be stressed that, although the quantity p (ε) is not very sensitive [as follows from (14)] to the measurement errors and the approximation of the total cross sections in the high-energy region, an error in the approximation in the low-energy region can lead to a shift in the calculated points, and consequently, to important errors in p (ε). This is connected with the fact that the magnitudes of the principal values of the integrals in (12) are very sensitive to the behavior of the cross section in the region of resonances, and the contribution of the integral terms is very large. (The right-hand part of expression (12), after subtraction of the term $D_{\pm}(E)$, is shown in the figure as a solid line.)

The only objective criterion for errors of this kind in the estimate of p (ε) is the degree to which the data shown in the figure are consistent with a linear dependence. In our case, the degree of agreement is characterized by the value $\chi^2 = 26.3$ for 21 degrees of freedom, which corresponds to $P(\chi^2) = 15.5\%$. In this connection, we note the following. It can be shown that a linear dependence of the right-hand part of (12) on E is equivalent to a linear dependence of the right-hand part of Schnitzer and Salzman's[14] formula (1±) on the energy.*

Hence, experimental confirmation of a linear dependence of (12) reflects the validity of the dispersion relations in the form given by Goldberger et al.[4] Moreover, a check of these dispersion relations, particularly that by Zinov et al.[15] leads to excellent agreement with experiment. We therefore believe that more accurate values of $p(\epsilon)$ can be obtained on the basis of recent experimental data after a more correct approximation of the total cross sections by means of resonance curves, as was done, for example in [15] and [16]

In conclusion, we note that the method by which we determined the parameters $p(\epsilon)$ and $q(\epsilon)$ is of full statistical value since it permits the utilization of all the experimental data concerning $D_{\pm}(E)$. The statistical error in the value of $p(\epsilon)$ in the determination of the latter by $D_{\pm}(m)$ only, i.e., by means of the formulas of Goldberger et al[4] [in this case, the straight line in the figure passes only through the points $D_{\pm}(m)$] is five times the statistical error obtained in this work. Another important advantage of the method is the fact that uncertainty in f^2 has comparatively little effect, since the contribution of the term with f^2 to (12) decreases with the energy as $(m/E)^2$.

DISCUSSION

From the principle of isotopic invariance, it follows that the charge-exchange cross section at 0° is[17]

$$k^{-2}\sigma_{ex}(0) = 2\,[C(E)]^2 + 2\,(4\pi)^{-2}\,(\sigma_t^{+} - \sigma_t^{-})^2. \quad (15)$$

If the statistical view is correct, then, as $E \to \infty$, the total cross section for charge-exchange $(\sigma_{ex})_t$ should drop very rapidly to zero (as $e^{-\sqrt{E}}$). This means that the right-hand part of (15) tends to zero as $E \to \infty$, and, consequently,†

$$C_\infty = 0. \quad (16)$$

We have estimated $p(\epsilon)$ for $\epsilon = 590$ and 930 Mev, corresponding to the intergration up to the second and third maxima in σ_t^- and obtained the following values (in units of cm/Bev):

$$p\,(590\,\text{Mev}) = -0.29\cdot 10^{-13}, \quad p\,(930\,\text{Mev}) = -0.17\cdot 10^{-13},$$
$$p\,(5.2\,\text{Bev}) = -0.069\cdot 10^{-13}.$$

Hence, the absolute value of $p(\epsilon)$ decreases rapidly. The value of $(\sigma_{ex})_t$ corresponding to p (5.2 Bev) estimated by means of (15) is $\sim 2\times 10^{-29}$ cm^2. Shalamov and Shebanov[19] obtained the value $(0.2 \pm 0.25)\times 10^{-27}$ cm^2 at 2.8 Bev. All this is in good agreement with the statistical view.

In view of this and also in view of the already-mentioned uncertainties in $p(\epsilon)$ connected with the approximation of the cross sections, we do not believe that the statistically significant deviation of our value of p (5.2 Bev) from zero definitely contradicts condition (16), even though in the estimate of p (5.2 Bev) it was assumed that $\sigma_t^+ = \sigma_t^-$ at E = 5.2 Bev.

It is of interest to estimate, starting from the relation

$$p\,(5.2\text{ Bev}) = C_\infty + \frac{1}{4\pi^2}\int_{5.2\text{Bev}}^{\infty}\frac{\sigma_t^{\pm} - \sigma_t^{\mp}}{E'}\,dE'$$
$$= 0.069\cdot 10^{-13}\text{ cm/Bev}, \quad (17)$$

how the cross sections should behave at E > 5.2 Bev under the assumption that $C_\infty = 0$. It follows from (17) that the difference $\sigma_t^- - \sigma_t^+$ should be $\sim 2\times 10^{-27}$ cm^2 for E > 5.2 Bev in the energy region of ~ 50 Bev.

The absence of information on the behavior of the cross section σ_t^- in the 1.9 — 4.15 Bev region can also lead to a deviation of p (5.2 Bev) from zero. We obtain the value p (5.2 Bev) = 0 if it is assumed that σ_t^- has a maximum of area ~ 8 Bev-mb in the region 1.9 — 4.15 Bev, while we obtain $\sigma_t^+ = \sigma_t^-$ for E ≥ 5.2 Bev.

From the above, it is clear that a more accurate estimate of the value of $p(\epsilon)$ is important.

We express our deep gratitude to E. M. Landis for investigating the passing to the limit in expressions (5) and (6) and I. Ya. Pomeranchuk for constant interest in this work and for helpful discussions, and also I. M. Shmushkevich for his critical remarks.

APPENDIX

E. M. Landis has shown that if, as $x \to \infty$, the function $f(x)$ tends to zero, so that, beginning with some value of x, it is positive and monotonic, then the following equality holds:

$$\lim_{N\to\infty}\frac{N^2}{I_N}\int_{\varepsilon}^{\infty}\frac{f(x)\,dx}{x^2 - N^2} = 1, \quad I_N = \int_{\varepsilon}^{N} f(x)\,dx. \quad (A.1)$$

Here, by lim we understand the low side of the limits over all possible sequences of N which go

*This statement means that, although the parameters of a straight line in formula (1 ±) of reference 14 and formulas (12) of the present work are quite different quantities [C(m) and Q(m) in reference 14 and p(ε) and q(ε) in the present work], the same errors in the total cross sections and the values of $D_{\pm}(E)$ lead in both cases to the same deviation of the calculated points from a straight line.

†Amati, Fierz, and Glasser[3] maintain that it follows from assumption (2) that $C_\infty = 0$. However, they did not actually prove this statement (in this connection, see Chew's article).[18]

to infinity. If, in particular, the limit of the numerator in (A.1) exists and is equal to A, then the integral I_∞ exists and is equal to $-A$.

To elucidate the properties of the function $q(\epsilon)$ given in (11) and $Q(E_0)$ of the form (9) and (10), we set in (5)

$$E^2 \ll \varepsilon^2 \ll E_0^2. \qquad \text{(A.2)}$$

Then (5) can be represented in the form

$$Q(E) - \frac{1}{4\pi^2} \mathrm{P} \int_m^{\varepsilon} \frac{E'k'(\sigma_t^+ + \sigma_t^-)}{E'^2 - E^2} dE' + \frac{\varepsilon}{4\pi^2} [\sigma_t^+(\varepsilon) + \sigma_t^-(\varepsilon)]$$

$$+ 2f^2 \frac{r}{E^2 - r^2} = Q(E_0) - \frac{1}{4\pi^2} \mathrm{P} \int_{\varepsilon}^{\infty} \frac{E_0^2(\sigma_t^+ + \sigma_t^-)}{E'^2 - E_0^2} dE'$$

$$+ \frac{\varepsilon}{4\pi^2} [\sigma_t^+(\varepsilon) + \sigma_t^-(\varepsilon)] = q(\varepsilon). \qquad \text{(A.3)}$$

If (A.2) is fulfilled, then (A.3) is fulfilled for any arbitrary values of E and E_0, from which it follows that $q(\epsilon)$ is independent of E in (11).

We shall now show the validity of (9) and (10). Using the auxiliary identity valid for $\epsilon^2 \ll E_0^2$,

$$\varepsilon = \mathrm{P} \int_{\varepsilon}^{\infty} \frac{E_0^2 dE'}{E'^2 - E_0^2}, \qquad \text{(A.4)}$$

we can represent $Q(E_0)$ in (A.3) in the form

$$Q(E_0) = \frac{1}{4\pi^2} \mathrm{P} \int_{\varepsilon}^{\infty} \frac{f(E') E_0^2 dE'}{E'^2 - E_0^2} - \frac{\varepsilon}{4\pi^2} f(\varepsilon) + q(\varepsilon), \qquad \text{(A.5)}$$

where $f(x) = \sigma_t^+(x) + \sigma_t^-(x) - 2\sigma_\infty$. If we allow E_0 to go to infinity in (A.5), we then have, owing to (A.1),

$$Q(E_0) \text{ bound from above } I_\infty < \infty,$$
$$Q(E_0) = \infty \text{ for } I_\infty = \infty,$$

i.e., we obtain (9) and (10).

Using (A.5) and (A.4), we can also investigate the behavior of $q(\epsilon)$ as $\epsilon \to \infty$. If $\overline{Q}_\infty$ is the upper limit of Q_∞, then from (A.5) and (A.1) we find

$$\lim_{\varepsilon \to \infty} q(\varepsilon) = \lim_{\varepsilon \to \infty} \Big[\overline{Q}_\infty + \lim_{E_0 \to \infty} \Big(-\frac{1}{4\pi^2} \mathrm{P} \int_{\varepsilon}^{\infty} \frac{f(E') E_0^2}{E'^2 - E_0^2} dE' \Big)$$

$$+ \frac{\varepsilon}{4\pi^2} f(\varepsilon) \Big] = \lim_{\varepsilon \to \infty} \Big(\overline{Q}_\infty - \frac{1}{4\pi^2} \int_{\varepsilon}^{\infty} f(E')\, dE'$$

$$+ \frac{\varepsilon}{4\pi^2} f(\varepsilon) \Big) = \overline{Q}_\infty,$$

since $\lim \epsilon f(\epsilon) = 0$ as $\epsilon \to \infty$. If conditions (9) and (10) are not fulfilled, then $q(\epsilon) \to \infty$, when $\epsilon \to \infty$, since the integral in the left-hand part of (A.3) diverges.

In conclusion, we give the dispersion relation for $Q(E)$ with the subtracted points transferred to infinity. This relation holds when (9) and (10) are fulfilled:

$$Q_\infty = Q(E) - \frac{1}{4\pi^2} \int_m^{\varepsilon_1} \frac{E'k'(\sigma_t^+ + \sigma_t^-)}{E'^2 - E^2} dE'$$

$$+ \frac{\varepsilon_1}{2\pi^2} \sigma_\infty + 2f^2 \frac{r}{E^2 - r^2};$$

for $E \geq \epsilon_1$ we have $\sigma_t^{\pm} = \sigma_\infty$.

Note added in proof (June 9, 1961): We recently calculated $p(\varepsilon)$ with the aid of the data of Klepikov et al[16] in which the total cross sections were approximated by resonance curves. The new result is p (5.2 Bev) = -0.085 ± 0.03 cm/Bev. On the other hand, according to data obtained at the Joint Institute for Nuclear Research and kindly communicated to us by A. L. Lyubimov, σ_t for $\pi^- p$ has no maximum in the 2–7 Bev energy region. Hence our result can signify the existence of the difference $\sigma_t^- - \sigma_t^+$ in a broad energy interval of over 5 Bev if C_∞ is not different from zero.

[1] I. Ya. Pomeranchuk, JETP **34**, 725 (1958), Soviet Phys. JETP **7**, 499 (1958).

[2] Bogolyubov, Medvedev, and Polivanov, Voprosy teorii dispersionnykh sootnoshenii (Problems in the Theory of Dispersion Relations), Gostekhizdat, 1958.

[3] Amati, Fierz, and Glaser, Phys. Rev. Lett. **4**, 89 (1960).

[4] Goldberger, Miyazawa, and Oehme, Phys. Rev. **99**, 986 (1955).

[5] Lomsadze, Lend'el, and Érnst, JETP **39**, 1154 (1960), Soviet Phys. JETP **12**, 803 (1961).

[6] Zinov, Korenchenko, and Tentyukova, JETP **38**, 1407 (1960), Soviet Phys. JETP **11**, 1016 (1960).

[7] Zinov, Konin, Korenchenko, and Pontecorvo, JETP **38**, 1708 (1960), Soviet Phys. JETP **11**, 1233 (1960).

[8] H. P. Noyes and D. N. Edwards, Phys. Rev. **118**, 1409 (1960).

[9] G. Puppi and A. Stanghellini, Nuovo cimento **5**, 1305 (1957).

[10] Devlin, Barish, Hess, Perez-Mendez, and Solomon, Phys. Rev. Lett. **4**, 242 (1960).

[11] Burrowes, Caldwell, Frisch, Hill, Ritson, Schluter, and Wahlig, Phys. Rev. Lett. **2**, 119 (1959).

[12] Brisson, Detoeuf, Falk-Vairant, van Rossum, Valladas, and Yuan, Phys. Rev. Lett. **3**, 561 (1959).

[13] Longo, Helland, Hess, Moyer, and Perez-Mendez, Phys. Rev. **3**, 568 (1959).

[14] H. J. Schnitzer and G. Salzman, Phys. Rev. **113**, 1153 (1958).

[15] Zinov, Konin, Korenchenko, and Pontecorvo, JETP **38**, 1708 (1960), Soviet Phys. JETP **11**, 1233 (1960).

[16] Klepikov, Sokolov, and Meshcheryakov, Preprint, Joint Institute for Nuclear Research.

[17] L. B. Okun' and I. Ya. Pomeranchuk, JETP **30**, 424 (1956), Soviet Phys. JETP **3**, 307 (1956).

[18] H. Chew, Nuovo cimento **17**, 619 (1960).

[19] Ya. Ya. Shalamov and V. A. Shebanov, JETP **39**, 1232 (1960), Soviet Phys. JETP **12**, 859 (1961).

Translated by E. Marquit
32