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ROTATION OF THE PLANE OF POLARIZATION OF LIGHT IN THE CASE OF PARITY NONCONSERVATION

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Interactions which do not conserve parity are included in a treatment of the rotation of the plane of polarization of light by a system. A concrete calculation is carried out for the hydrogen atom.

THE rotation of the plane of polarization of light by an optically active molecule, i.e., a molecule whose mirror image is not congruent with the original molecule, was first treated from the quantum-mechanical point of view by Rosenfeld[1] (cf. also [2] and [3]). It has been pointed out by Zel'dovich[4] that in the case of parity nonconservation the effect of rotation of the plane of polarization is possible in a substance which does not contain optically active molecules, and an estimate of the effect was given.*

Let us consider this question more fully. We introduce the following notations: $\mathbf{r}$, $\mathbf{p}$ are the coordinate and momentum of the electron, $\mathbf{l}$ and $\boldsymbol{\sigma}/2$ are the orbital angular momentum and the spin, $\mathbf{m} = \mathbf{l} + \boldsymbol{\sigma}$, $\mathbf{A}(\mathbf{r})$ is the vector potential of the light wave, $\mathbf{k}$ and $\boldsymbol{\epsilon}$ are the wave vector and and the polarization, $|\mathbf{k}| = \omega/c = 1/\lambda\!\!\!^{-}$, ω and $\lambda\!\!\!^{-}$ are the frequency and wavelength of the light, m_i and m_f are the projections of the angular momentum on the z axis before and after the collision, and c_i and c_f are the remaining quantum numbers characterizing the states of the atom before and after the collision.

If the wavelength of the light is much larger than the distance between atoms, the substance can be regarded as continuous and can be characterized by an index of refraction n. In the case of parity conservation the indices of refraction of right and left circularly polarized light are the same for a substance that does not contain optically active molecules. In the case of parity nonconservation, which can be caused, for example, by a direct four-fermion interaction between electron and proton (neutron), or by the existence of an anapole moment of these particles,[6,7] the even states of the atom (or molecule) will contain a small admixture of odd states, and the index of refraction n_+ for right circularly polarized light will not be equal to the index n_- for left circularly polarized light. When plane polarized light passes through the medium its plane of polarization is rotated in unit length through the angle

$$\varphi = (n_+ - n_-)/2\lambda\!\!\!^{-}. \tag{1}$$

The index of refraction can be expressed either in terms of the dipole moment $\mathbf{d}$ induced by the electromagnetic field of the light wave, or by the amplitude for scattering of light by the atom through the angle 0°. In [1-3] an expression for the dipole moment has been found,

$$\mathbf{d} = \alpha'\mathbf{E} - \frac{\beta'}{c}\frac{\partial \mathbf{H}}{\partial t} + \gamma'\mathbf{H},$$

which is not invariant under a change of the sign of the time, and becomes invariant only when the last term is absent, i.e., for $\gamma' = 0$. If we express the index of refraction in terms of the scattering amplitude, then as is well known n is connected with the forward-scattering amplitude a_0, averaged over all orientations of the atom (or molecule), in the following way:[8]

$$n^2 = 1 + 4\pi\lambda\!\!\!^{-2} a_0 N_0,$$

where N_0 is the number of atoms per unit volume. In a theory which is invariant under time reversal we have

$$a(c_f, m_f, \mathbf{k}', \boldsymbol{\varepsilon}'; c_i, m_i, \mathbf{k}, \boldsymbol{\varepsilon}) = a(c_i, -m_i, -\mathbf{k}, -\boldsymbol{\varepsilon}; c_f, -m_f, -\mathbf{k}', -\boldsymbol{\varepsilon}'). \tag{3}$$

Using the Hermitian character of the Hamiltonian, we find in the first nonvanishing approximation

$$a(c_f, m_f, \mathbf{k}', \boldsymbol{\varepsilon}'; c_i, m_i, \mathbf{k}, \boldsymbol{\varepsilon})^* = a(c_i, m_i, \mathbf{k}, \boldsymbol{\varepsilon}; c_f, m_f, \mathbf{k}', \boldsymbol{\varepsilon}'). \tag{4}$$

In the general case the amplitude

$$a_0 = \frac{1}{2j_i + 1}\sum_{m_i} a(c_i, m_i, \mathbf{k}, \boldsymbol{\varepsilon}'; c_i, m_i, \mathbf{k}, \boldsymbol{\varepsilon})$$

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*There is also a similar estimate in a later paper by Baier and Khriplovich,[5] which deals with parity nonconservation in the interaction between electrons.

can be represented in the form

$$a_0 = \alpha(\varepsilon\varepsilon') + i\beta\frac{\mathbf{k}}{|\mathbf{k}|}[\varepsilon\varepsilon']. \qquad (5)^*$$

Using the Hermitian character of a_0, we find that α and β are real. We note that only the second term contributes to the rotation of the plane of polarization. The first term contributes to the polarizability of the atom (or molecule). Since $(\boldsymbol{\epsilon}\cdot\boldsymbol{\epsilon}')$ does not change sign under inversion and time reversal, and $\mathbf{k}\cdot[\boldsymbol{\epsilon}\times\boldsymbol{\epsilon}']$ changes sign under inversion but not under time reversal, it follows that the detection of a rotation of the plane of polarization will prove parity nonconservation and violation of charge-conjugation invariance, but can say nothing about conservation or nonconservation of the time parity.

In the nonrelativistic approximation the Hamiltonian of an atom (or molecule) including a part that does not conserve parity can contain terms of the type $\Sigma\mathbf{p}_i\mathbf{b}_i$, where $\mathbf{b}_i$ is an operator which contains no derivatives. These terms give an additional interaction with the electromagnetic field, $(e/c)\Sigma\mathbf{A}(\mathbf{r}_i)\mathbf{b}_i$, but it can be shown that this interaction does not contribute to the rotation of the plane of polarization.

In the dipole approximation we get

$$\alpha = \frac{2e^2\omega^2}{3c^2}\sum_n \frac{\mathbf{r}_{in}\,\mathbf{r}_{ni}(E_n - E_i)}{(E_i - E_n)^2 - (\hbar\omega)^2},$$

$$\beta = \frac{2e^2}{3\hbar c}\frac{(\hbar\omega)^3}{mc^2}\sum_n \frac{\operatorname{Im}\mathbf{r}_{in}\,\mathbf{m}_{ni}}{(E_i - E_n)^2 - (\hbar\omega)^2}. \qquad (6)$$

Let us consider the hydrogen atom. Suppose there exists an anapole moment of the electron, or else a weak four-fermion interaction between the electron and the proton. Then in nonrelativistic approximation, neglecting the spin of the proton in the latter case, we have

$$\mathscr{H}_0 = \frac{p^2}{2m} - \frac{e^2}{r} + \frac{f}{2m}[\boldsymbol{\sigma}_e\mathbf{p}\,\delta(\mathbf{r}) + \delta(\mathbf{r})\,\boldsymbol{\sigma}_e\mathbf{p}]. \qquad (7)$$

If it is the proton that has the anapole moment, we must replace σ_e by σ_p in this formula.

The only terms of importance for the calculation are the admixture of the $nP_{1/2}$ states in the ground $1S_{1/2}$ state and the admixture of the $1S_{1/2}$ state in the $nP_{1/2}$ state:

$$\psi_i = |1S_{1/2}\rangle + i\sum_n \delta_n |nP_{1/2}\rangle,$$

$$\psi_n = |nP_{1/2}\rangle + i\delta_n|1S_{1/2}\rangle + \ldots,$$

$$\delta_n = -i\frac{\langle 1S_{1/2}|V|nP_{1/2}\rangle}{E_n - E_1} = -\frac{2e^4m^2f}{\pi}\frac{1}{\sqrt{1-n^{-2}}}\frac{1}{n^{3/2}},$$

$$V = (f/2m)\,[\boldsymbol{\sigma}_e\mathbf{p}\,\delta(\mathbf{r}) + \delta(\mathbf{r})\,\boldsymbol{\sigma}_e\mathbf{p}]. \qquad (8)$$

After some simple calculations we get

$$\varphi \approx \frac{8\pi}{9}\frac{\rho}{n_0}\left(\frac{a}{\lambda\!\!\!^-}\right)^2\frac{1}{a}\sum_n \delta_n r_{n1}\frac{2(E_{nP_{3/2}} - E_{nP_{1/2}})(E_n - E_1)e^4a^{-2}}{[(E_n - E_1)^2 - (\hbar\omega)^2]^2},$$

$$r_{n1} = \frac{1}{a}\int R_{n1}(r)\,R_{10}(r)\,r^3dr, \quad \rho = N_0a^3, \qquad f = bG, \qquad (9)$$

where $a = \hbar^2/me^2$ is the Bohr radius, $G = 10^{-5}M_n^{-2}$ is the weak-interaction constant, and $n_0 = (n_+ + n_-)/2$ is the mean index of refraction.

Substituting for r_{n1} the expression

$$r_{n1} = \frac{16n^4\sqrt{n}(n-1)^{n-3}}{(n+1)^{n+3}}\sqrt{1-\frac{1}{n^2}} \qquad (10)$$

(cf., e.g., [9]), we find

$$\varphi \approx 2\cdot 10^{-19}\frac{b\rho}{n_0}\frac{1}{a}\left(\frac{a}{\lambda\!\!\!^-}\right)^2\sum_n\left(\frac{2}{n}\right)^4\left(\frac{n-1}{n+1}\right)^n\left(1-\frac{1}{n^2}\right)^{-6}. \qquad (11)$$

For $b \approx \rho \approx n_0 \approx 1$, $(a/\lambda\!\!\!^-) \sim 10^{-3}$, we have $\varphi \approx 10^{-17}$ rad/cm.

Since this expression $\sim 1/Z$ for $\hbar\omega \ll E_n - E_1$, the rotation of the plane of polarization must be largest in hydrogen. It can be seen from Eq. (9) that if we do not take the fine structure into account we get zero rotation of the plane of polarization. This leads to a decrease of the rotation of the plane of polarization by a factor of 10^{-4} in comparison with the value that could be expected from general considerations. Neither inclusion of the electron spin nor inclusion of the proton spin changes this result. The rotation of the plane of polarization of light near resonance, and also the conversion of plane polarized light into elliptically polarized light (the Cotton-Mouton effect) can evidently not be observed on account of the strong absorption. A more exact formula for φ can be obtained by calculating with relativistic functions.

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*$\varepsilon\varepsilon' = \varepsilon\cdot\varepsilon'$; $[\varepsilon\varepsilon'] = \varepsilon\times\varepsilon'$.

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