REFLECTION OF ELECTROMAGNETIC WAVES IN GYROTROPIC MEDIA FROM A MAGNETIC-FIELD WAVE

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We consider reflection of electromagnetic waves in a ferrite or in an infinite homogeneous plasma from a magnetic-field wave (moving magnetic "mirror"), and also reflection from a moving plasma and from an ionization wave produced in a stationary plasma. The calculations are made in the geometric optics approximation, and a more exact solution is found near the point at which this approximation becomes invalid. It is shown that in all cases considered the frequency increases upon reflection. The wave amplitude and total energy of the wave packet also increase after reflection; an exception is reflection from an ionization wave, for which the amplitude of the reflected wave is equal to the amplitude of the incident wave and the total energy of the wave packet decreases upon reflection.

INTRODUCTION

THE increase in frequency and amplitude, occurring when electromagnetic waves are reflected from a plasma moving in a medium with dielectric constant $\epsilon > 1$, were discussed by Lampert[1] and by Feĭnberg and Tkalich.[2] Zagorodnov et al[3] reported an experiment in which an increase in frequency was observed when electromagnetic waves were reflected from a plasma moving inside a helical transmission line.

An analogous effect can be obtained by using the dependence of the effective dielectric constant or permeability of certain types of electromagnetic waves, propagating in a plasma or in a ferrite, on the magnetic field intensity. For example, the effective dielectric constant for a right-hand polarized plane wave, propagating in a plasma along the direction of the magnetic field, is

$$\varepsilon_{\text{eff}} = \varepsilon[\omega^2 - (\omega_{\text{pl}}^2 + \omega\omega_H)]/\omega(\omega - \omega_H). \qquad (1)$$

Here $\omega_{\text{pl}}^2 = 4\pi e^2 N/\epsilon m$ is the square of the plasma frequency, $\omega_H = eH_0/cm$ the Larmor frequency, N the electron density, H_0 the intensity of the magnetic field, e the absolute value of the electron charge, and ϵ the dielectric constant of the medium in which the plasma is situated. The expression for the effective dielectric constant of waves with left-hand polarization differs in the sign of ω_H from expression (1) for right-hand polarized waves.

It is seen from (1) that ϵ_{eff} can be negative not only in a region with sufficiently high electron density, but also in a region with sufficient magnetic field intensity. Consequently, such a region, like a region with large electron concentration, can serve as a mirror for an electromagnetic wave with right-hand polarization; a "mirror" for left-hand polarized waves will be a region where the magnetic field intensity H_0 decreases.

It is also easy to verify that a region with increasing magnetization field intensity can serve as a mirror for several types of waves propagating in a ferrite. For example, quasi-transverse waves in a two-dimensional waveguide (Fig. 1)

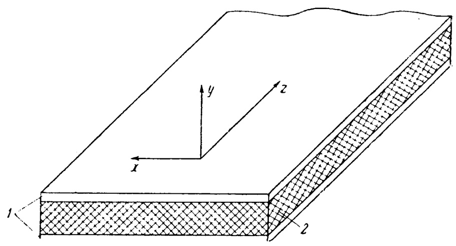

FIG. 1. Two-dimensional waveguide; 1 – perfectly conducting plates, 2 – ferrite.

filled with ferrite and right-hand polarized waves in a homogeneous unbounded ferrite will be reflected from the region where

$$0 < \omega - 4\pi M\gamma < \gamma H_0,$$

for the effective magnetic permeability for these waves will be negative in this region[4] (here γ is the absolute value of the gyromagnetic ratio for the electron spin, and M is the saturation magnetization of the ferrite).

By producing fields that vary in space and in time, it is possible to obtain "mirrors" moving with high velocities, and consequently increase the frequency greatly.* However, reflection from such a moving "mirror" has certain features distinguishing it from reflection from regions where the magnetic field is constant in time. We consider in the present paper the reflection of quasi-transverse waves in a two-dimensional waveguide filled with ferrite, and of right-polarized waves in an unbounded plasma, from the traveling wave of a longitudinal magnetic field. For comparison, we consider the reflection of electromagnetic waves from a plasma moving in a medium with $\epsilon > 1$, and from an ionization wave produced in a stationary plasma.

REFLECTION OF QUASI TRANSVERSE WAVES IN A TWO DIMENSIONAL WAVEGUIDE FILLED WITH A FERRITE, FROM THE WAVE OF THE MAGNETIZATION FIELD

It follows from the results of Suhl and Walker[4] that when a two-dimensional waveguide filled with a ferrite magnetized to saturation is sufficiently thin, the components of the field of the quasi-transverse wave can be regarded as being independent of the transverse coordinates, except when the frequency is close to a certain critical value:†

$$\omega_{cr} = \gamma \sqrt{H_0 (H_0 + 4\pi M)}.$$

This, obviously, remains valid also when the magnetization field H_0 varies sufficiently slowly in time and in the coordinate z along which the wave propagates. In this approximation, the problem of the propagation of quasi-transverse waves is one-dimensional, and Maxwell's equations as well as the equation for the magnetization vector can be readily reduced to the form

$$\frac{\partial^2 h_x}{\partial z^2} = \frac{\varepsilon}{c^2}\left[\frac{\partial^2 h_x}{\partial t^2} - \omega_M \omega_B h_x + \omega_0 \omega_B m_x + \frac{1}{\omega_B}\frac{\partial \omega_B}{\partial t}\frac{\partial m_x}{\partial t}\right],$$
$$h_y = - m_y, \qquad e_x = e_z = 0, \qquad \frac{\partial e_y}{\partial t} = \frac{c}{\varepsilon}\frac{\partial h_x}{\partial z};$$
$$\frac{\partial m_x}{\partial t} = - \omega_B m_y, \qquad \frac{\partial m_y}{\partial t} = - \omega_M h_x + \omega_0 m_x. \tag{2}$$

It is assumed here that the dissipative term in the equation for the magnetization vector vanishes, the magnetization field $\mathbf{H}_0(t, z)$ has only a longitudinal component $H_{0z} = H_0(t, z)$, and the y axis is perpendicular to the walls of the two-dimensional waveguide (see Fig. 1). In addition, the following notation is used

$$m_x = 4\pi M_x, \quad m_y = 4\pi M_y, \quad \omega_0 = \gamma H_0,$$
$$\omega_M = \gamma 4\pi M, \quad \omega_B = \omega_0 + \omega_M;$$

M is the magnetization vector.

If $H_0(t, z)$ is a sufficiently slow function, the propagation of quasi-transverse waves can be investigated by the method of geometric optics.[8-10] We seek the solution in the form

$$h_x = (h_0 + h_1 + \ldots) e^{i\psi}, \qquad \mathbf{m} = (\mathbf{m}_0 + \mathbf{m}_1 + \ldots) e^{i\psi},$$
$$e_y = (e_0 + e_1 + \ldots) e^{i\psi}, \tag{3}$$

where $h_0 \gg h_1 \gg \ldots$; $|\mathbf{m}_0| \gg |\mathbf{m}_1| \gg \ldots$; $e_0 \gg e_1 \gg \ldots$ and $\partial\psi/\partial t$ and $\partial\psi/\partial z$ are slow functions compared with the eikonal ψ.

The equations obtained from (2) and (3) for the eikonal ψ and for the successive approximations of the field amplitudes can be readily integrated in quadratures, if the magnetization field has the form of a wave traveling at a constant velocity V along the z axis, i.e., if $H_0(t, z)$ depends only on the quantity $\xi = Vt - z$. In this case the field amplitude depends only on ξ, and we have, in particular, in the zeroth approximation

$$h_0 = h_0(-\infty)\left[\psi_z + \beta\frac{\sqrt{\varepsilon}}{c}\psi_t\left(1 + \frac{\omega_0}{\omega_M}\chi^2\right)\right]_{-\infty}^{1/2}$$
$$\times\left[\psi_z + \beta\frac{\sqrt{\varepsilon}}{c}\psi_t\left(1 + \frac{\omega_0}{\omega_M}\chi^2\right)\right]_{\xi}^{-1/2}$$
$$\times\exp\left\{\frac{1}{2}\beta\frac{\sqrt{\varepsilon}}{c}\int_{-\infty}^{\xi}\psi_t\frac{\omega_M(\omega_B^2 + \psi_t^2)}{(\psi_t^2 - \omega_0\omega_B)^2}\right.$$
$$\left.\times\left[\psi_z + \beta\frac{\sqrt{\varepsilon}}{c}\psi_t\left(1 + \frac{\omega_0}{\omega_M}\chi^2\right)\right]^{-1}\frac{d\omega_B}{d\xi}d\xi\right\},$$
$$\beta = V\sqrt{\varepsilon}/c, \quad \chi = m_{x0}/h_0 = -\omega_M\omega_B/(\psi_t^2 - \omega_0\omega_B). \tag{4}$$

The wave number ψ_z and the frequency ψ_t also depend only on ξ, and are determined from the equations

$$\psi_z^2 = \frac{\varepsilon}{c^2}\psi_t^2\frac{\psi_t^2 - \omega_B^2}{\psi_t^2 - \omega_0\omega_B}, \qquad \psi_z = \frac{\psi_{t0} - \psi_t}{V}, \tag{5}$$

while the eikonal ψ is equal to

$$\psi = -\int_{\xi_0}^{\xi}\psi_z(\xi)\,d\xi + \psi_{t0}t. \tag{6}$$

Here ψ_{t0} is an arbitrary integration constant, equal to the frequency at the point where $\psi_z = 0$; ξ_0 is an arbitrary constant.

Let us examine the case when an electromagnetic wave with initial frequency $\psi_{t;1}(-\infty)$, satisfying the condition $\psi_{t;1}(-\infty) > \omega_B(-\infty)$, propagates in a direction opposite to the wave of the

*It can be shown that in a two-dimensional waveguide and in an unbounded ferrite, such a 'mirror' can be a shock electromagnetic wave[5,6] with front duration longer than the period of the incident wave. If the duration of the front of the shock wave is shorter than the period of the incident weak electromagnetic wave, no reflection can take place.[7]

†It should be noted that [4] states erroneously that $\omega_{cr} = \gamma H_0$.

magnetization field, which is a monotonically increasing function of ξ. Analyzing the properties of the solutions of (5), we readily see that the frequency $\psi_{t;1}$ increases with increasing ξ, i.e., with increasing magnetization field, while the wave number $\psi_{z;1}$ decreases, vanishing at a point ξ_1 determined from the condition $\psi_{t;1}(\xi_1) = \psi_{t0} = \omega_B(\xi_1)$.

When $\beta = 0$, i.e., when the magnetization field is independent of the time, the geometric optics is no longer valid in the vicinity of this point. However, if $\beta > 0$ and $H_0(\xi)$ is a sufficiently slow and smooth function, the first approximation in the vicinity of ξ_1 remains much smaller than the zeroth approximation, since $h_1/h_0 \sim (dH_0/d\xi)^2$, $d^2H_0/d\xi^2$, and the proportionality coefficients remain finite in ξ_1 when $\beta > 0$. Consequently, geometric optics still applies at this point,* although the sign of the wave number, and consequently the direction of the phase and group velocities, does change (Fig. 2). The group velocity, however, is smaller than the V. The electromagnetic wave can therefore again be regarded as incident relative to the magnetization-field wave.

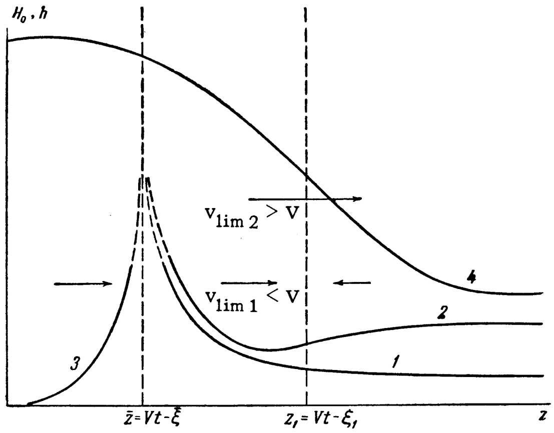

FIG. 2. Amplitudes of the magnetic fields of the incident (1), reflected (2), and refracted (3) waves and of the magnetization field H_0 (4), as functions of z at the instant t. The arrows show the direction of the energy, averaged over the period of the flux, in various regions of space.

When $0 < \beta < 1$, geometric optics does not hold for an arbitrarily slow variation of $H_0(\xi)$ in the vicinity of the point $\bar{\xi}$ where $d\psi_t/d\xi = \infty$, and the group velocity is equal to V. The frequency $\bar{\psi}_t$, the wave number $\bar{\psi}_z$, and the coordinate $\bar{\xi}$ of this point, from which we shall henceforth measure ξ, are determined from (5) and from

$$\bar{\psi}_z + \beta(\sqrt{\varepsilon}/c)\bar{\psi}_t(1 - \omega_0\chi^2/\omega_M) = 0. \qquad (7)$$

The vicinity of $\bar{\xi}$ reflects a wave whose group velocity is greater than the velocity of the magnetization-field wave. We shall henceforth call this the reflected wave. As $\beta \to 1$, the reflected wave appears only when the maximum value of the magnetization field tends to infinity. When $\beta \geq 1$, there can be no reflected wave, and the field at any point is described by formulas (4) — (6), which in this case have for $\omega_{inc} > \omega_B(-\infty)$ only one solution, describing a wave incident with respect to the magnetization-field wave.

To determine the connection between the amplitudes and phases of the incident and reflected waves, we assume, as is usually done,[8,9] that the coefficient of refraction varies linearly in the region where geometric optics is invalid. In the case considered below, it is sufficient for this purpose to put

$$\omega_B = \bar{\omega}_B(1 + 2\alpha\xi). \qquad (8)$$

When $\alpha V \ll \bar{\psi}_t$, a solution of (2) can be sought in the form

$$h_x(t, z) = [h_0(\xi) + h_1(\xi) + \ldots]\exp[i(\bar{\psi}_t t + \bar{\psi}_z z)],$$
$$\mathbf{m}(t, z) = [\mathbf{m}_0(\xi) + \mathbf{m}_1(\xi) + \ldots]\exp[i(\bar{\psi}_t t + \bar{\psi}_z z)],$$
$$e_y(t, z) = [e_0(\xi) + e_1(\xi) + \ldots]\exp[i(\bar{\psi}_t t + \bar{\psi}_z z)]. \qquad (9)$$

Here $|h_0| \gg |h_1| \gg \ldots$; $|\mathbf{m}_0| \gg |\mathbf{m}_1| \gg \ldots$; $|e_0| \gg |e_1| \gg \ldots$ are slowly varying functions, so that $|\partial\mathbf{m}_0/\partial t| \ll |\psi_t\mathbf{m}_0|$ and so on, but their second derivatives cannot be regarded as small.

We restrict ourselves to a case when the magnetization-field wave velocity is so high that $0 < (1-\beta) \ll 1$. We then obtain in the zeroth approximation the following equation for the amplitude of the magnetic field

$$d^2h_0/d\eta^2 - (2\alpha\bar{\psi}^2/\beta^2)^{-2}\eta h_0 = 0, \quad \eta = 2\alpha(\psi_z^2\xi/\beta^2 - i\bar{\psi}_z), \qquad (10)$$

the solution of which has the form[8,9]

$$h_0(\xi) = A(2\alpha\bar{\psi}_z^2/\beta^2)^{-1/3}\Phi[(2\alpha\bar{\psi}_z^2/\beta^2)^{-2/3}\eta]. \qquad (11)$$

Here Φ is the Airy function. From the properties of this function[8,9] it is seen that when $\xi > 0$ the amplitude of the refracted wave is a damped function, which for large ξ is proportional to*

$$\exp\left\{-\frac{\sqrt{\varepsilon}}{c}\int_0^{\xi}\left[(1-\beta^2)\omega_M\omega_0 - \psi_{t0}^2\right]^{1/2}\frac{d\xi}{1-\beta^2}\right\}.$$

From a comparison of (11) with the solution (4) — (6), in the approximation of geometric optics, it is seen that when ξ is negative and large in absolute value, the field is a sum of the incident and reflected waves $h_{0;1}$ and $h_{0;2}$ respectively:

*This can be seen also from the fact that the point ξ_1 has no singularities in a coordinate system moving with velocity V.

*The refracted wave is described, apart from a constant factor, by formulas (12) and (14), where the index used is 1.

$$h_{0;1,2}=\mp h_{1,2}\left\{\frac{[\psi_{z;1,2}+\beta(\sqrt{\varepsilon}/c)\psi_{t;1,2}(1+\omega_0\chi^2/\omega_B)]_{-\infty}}{[\psi_{z;1,2}+\beta(\sqrt{\varepsilon}/c)\psi_{t;1,2}(1+\omega_0\chi^2/\omega_B)]_{\xi}}\right\}^{1/2}$$
$$\times\exp\left\{\frac{1}{2}\frac{\beta\sqrt{\varepsilon}}{c}\int\limits_{-\infty}^{\xi}\psi_{t;1,2}\frac{\omega_M(\omega_B^2+\psi_{t;1,2}^2)}{(\psi_{t;1,2}^2-\omega_0\omega_B)^2}\right.$$
$$\times\left[\psi_{z;1,2}+\frac{\beta\sqrt{\varepsilon}}{c}\psi_{t;1,2}\left(1+\frac{\omega_0\chi^2}{\omega_B}\right)\right]^{-1}\frac{d\omega_B}{d\xi}d\xi$$
$$\left.+i\left(\psi_{1,2}\mp\frac{\pi}{4}\right)\right\};$$
$$e_{0;1,2}=(c\psi_{z;1,2}/\varepsilon\psi_{t;1,2})h_{0;1,2},\qquad \psi_{1,2}=-\int\limits_0^{\xi}\psi_{z;1,2}d\xi+\psi_{t0}t. \tag{12}$$

The ratio of the amplitude of the incident wave (h_1) to the amplitude of the reflected wave (h_2) far ahead of the front of the magnetization-field wave is approximately equal to the ratio of the wave frequencies in this region:

$$h_2/h_1\approx\psi_{t;2}(-\infty)/\psi_{t;1}(-\infty). \tag{13}$$

The dependence of the frequency and of the wave numbers on ξ is determined from Eq. (5), which in the case $\omega_0 \ll \psi_t$ can be approximately written in a more convenient form:

$$\psi_{t;1,2}=\frac{1}{1-\beta^2}\left\{\psi_{t0}\mp\beta\left[\psi_{t0}^2-(1-\beta^2)\omega_B^2\right.\right.$$
$$\left.\left.\times\left(1-\frac{\omega_0}{\omega_B}\left(1-\frac{\omega_B^2}{\chi_{t;1,2}^2}\right)\right)\right]^{1/2}\right\},$$
$$\psi_{z;1,2}=\frac{\sqrt{\varepsilon}}{c(1-\beta^2)}\left\{-\beta\psi_{t0}\pm\left[\psi_{t0}^2-(1-\beta^2)\omega_B^2\right.\right.$$
$$\left.\left.\times\left(1-\frac{\omega_0}{\omega_B}\left(1-\frac{\omega_B^2}{\psi_{t;1,2}}\right)\right)\right]^{1/2}\right\}. \tag{14}$$

The square bracket in (14) vanishes at the point of reflection $\bar{\xi}$, and consequently the frequencies and wave numbers of the incident and reflected waves are equal to each other. On going further into the region of weaker magnetization field, the frequency of the incident wave decreases, while that of the reflected wave increases. Consequently, an increase in frequency takes place upon reflection, and, as can be readily verified, this increase obeys the usual Doppler formula. According to (13), the amplitude of the reflected wave increases, as does also the total energy of the reflected wave packet, W_2. Indeed, the energy of the wave packet is proportional to the energy flux density averaged over the period and to the duration of the wave packet, which varies upon reflection as the reciprocal of the frequency. It then follows from (12) – (14) that the ratio of W_2 (total energy of the reflected wave packet) to W_1 (total energy of the incident wave packet) is

$$\frac{W_2}{W_1}=\left(\frac{h_2}{h_1}\right)^2\frac{\psi_{t;1}^2(-\infty)\,|\,\psi_{z;2}(-\infty)\,|}{\psi_{t;2}^2(-\infty)\,|\,\psi_{z;1}(-\infty)\,|} \tag{15}$$

and is greater than unity.

REFLECTION OF PLANE ELECTROMAGNETIC WAVES IN UNBOUNDED PLASMA FROM A LONGITUDINAL MAGNETIC-FIELD WAVE, FROM A UNIFORMLY MOVING INHOMOGENEOUS PLASMA, AND FROM AN IONIZATION WAVE IN A STATIONARY PLASMA

For all the three cases mentioned in the heading, Maxwell's equations can be readily written in the form

$$\frac{\partial^2\mathbf{e}}{\partial z^2}-\frac{\varepsilon}{c^2}\frac{\partial^2\mathbf{e}}{\partial t^2}-\frac{4\pi}{c^2}\frac{\partial\mathbf{j}}{\partial t}=0,\quad \mathrm{rot}\,\mathbf{e}=-\frac{1}{c}\frac{\partial\mathbf{h}}{\partial t}. \tag{16}*$$

We confine ourselves to a very simple plasma model, without pretending to describe fully the processes occurring in the plasma. Namely, we assume that in the case of reflection from a magnetic field that varies in space and in time, and from a moving plasma, the current density $\mathbf{j}$ is

$$\mathbf{j}=-eN\mathbf{v}. \tag{17}$$

In this equation N is the electron density, which is constant in the former case and depends on $\xi = Vt - z$ in the latter (V is the velocity of motion of the plasma); $\mathbf{v}$ is the forced solution of the equation of motion of the electrons:

$$d\mathbf{v}/dt=-(e/m)\,\mathbf{e}-(e/mc)\,[\mathbf{vH}]. \tag{18}‡$$

In the case of reflection from an ionization wave† it is necessary to take into account the alternating current produced by the uniform motion of the electrons, at a velocity determined by the initial conditions at the instant of ionization. Assuming the initial velocity upon ionization to be equal to zero and replacing the summation by integration we find that in an ionization wave

$$\mathbf{j}=-eN\mathbf{v}+e\int\limits_{-\infty}^{t}\mathbf{v}(t)\frac{\partial N}{\partial t}dt. \tag{19}$$

Investigating the solution of (16) – (19) by the same method as the solution of (2), we find that far in front of the reflecting point the field is equal to the sum of the fields of the incident and reflected waves ($\mathbf{e}_{0;1}$ and $\mathbf{e}_{0;2}$ respectively).

*rot = curl.

†[vH] = v × H.

‡An ionization wave, namely a moving boundary of a region with increased electron concentration, can be obtained by exposing a stationary gas to ionizing radiation of intensity that varies in space and in time. Obviously, no limitations are imposed on the velocity of the ionization wave.

In reflection from a moving plasma, we confine ourselves to the case of nonrelativistic plasma velocity,* $V \ll c$, so that the effect of the magnetic field of the wave on the electrons can be disregarded, and we can assume $d\mathbf{v}/dt = \partial\mathbf{v}/\partial t$. In reflection from a magnetic-field wave, we consider only a right-hand polarized wave, and also assume that $\omega_H \ll \omega_{pl} < \psi_t$ and $\beta^2 \ll 1$. Then the dependence of the frequencies and wave numbers of the incident and reflected waves on ξ will have in all three cases the following explicit form:

$$\psi_{t;1,2} = \frac{1}{1-\beta^2}\ \{\psi_{t0} \mp \beta\ [\psi_{t0}^2 - (1-\beta^2)\ \omega_{pl}^2 - \psi_{t0}\omega_H]^{1/2}\},$$
$$\psi_{z;1,2} = \frac{\sqrt{\varepsilon}}{c\,(1-\beta^2)}\ \{-\beta\psi_{t0} \pm [\psi_{t0}^2 - (1-\beta^2)\ \omega_{pl}^2 - \psi_{t0}\omega_H]^{1/2}\}. \tag{20}$$

From a comparison of (20) with (14) we see that in all three cases the change in frequency and wave number is qualitatively similar to that in reflection from a magnetization-field wave in a ferrite.

The amplitudes of the incident wave and the wave reflected from a magnetic-field wave vary as

$$e_{0;1,2} = e_{1,2}\left\{\left[\psi_{z;1,2} + \frac{\sqrt{\varepsilon}}{c}\beta(\psi_{t;1,2} + \omega_{pl}^2\omega_H)/(\psi_t - \omega_H)^2\right]_{-\infty} \times \left[\psi_{z;1,2} + \frac{\sqrt{\varepsilon}}{c}\beta\,(\psi_{t;1,2} + \omega_{pl}^2\omega_H/(\psi_t - \omega_H)^2)\right]_{\xi}^{-1}\right\}^{1/2}$$
$$\times \exp\left\{\beta\frac{\sqrt{\varepsilon}}{c}\frac{\omega_{pl}^2}{4}\int_{-\infty}^{\xi}\frac{d\omega_H}{d\xi}\left[(\psi_{t;1,2} - \omega_H)^2 \times\left[\psi_{z;1,2} + \frac{\sqrt{\varepsilon}}{c}\beta\left(\psi_{t;1,2} + \frac{\omega_{pl}^2\omega_H}{(\psi_t - \omega_H)^2}\right)\right]\right]^{-1} d\xi\right\}. \tag{21}$$

The ratio of the amplitudes of these waves at infinity $e_{0;1}(-\infty)/e_{0;2}(-\infty) = e_1/e_2$ is equal in this case to the ratio of the corresponding exponential factors in (21) with an upper integration limit equal to zero. This ratio is less than unity, i.e., the amplitude of the wave increases upon reflection, and, as can be readily shown,† in a way as to increase also the energy of the wave packet.

In reflection from a moving plasma (a) and from an ionization wave in a stationary plasma (b), the amplitudes are determined respectively by the equations

$$e_{0;1,2} = e_{1,2}\{[\psi_{t0}^2 - (1-\beta^2) \times \omega_{pl}^2(-\infty)]/[\psi_{t0}^2 - (1-\beta^2)\ \omega_{pl}^2(\xi)]\}^{1/4}\varphi(\xi);$$
$$e_2/e_1 = \psi_{t;2}(-\infty)/\psi_{t;1}(-\infty),\quad \varphi = \psi_{t;1,2}(\xi)/\psi_{t;1,2}(-\infty);$$
$$e_2/e_1 = 1,\qquad \varphi = 1. \tag{22}$$

It follows from (12), (13), (21), and (22) that reflections from a magnetization-field wave in a two-dimensional waveguide filled with a ferrite, from a magnetic-field wave in a homogeneous moving plasma, and from a moving inhomogeneous plasma all are qualitatively alike; in addition to increasing the frequency, the reflection increases the amplitude of the reflected wave and the energy of the reflected wave packet.

The increase in energy of the reflected wave packet can be attributed in the first two cases to the fact that the magnetic moment per unit volume of the ferrite, deflected by the high-frequency field from the direction of the magnetizing field (or the magnetic moment of the electron produced by the high-frequency field) is situated in an increasing magnetic field. This increases energy of interaction between these moments and the magnetic field,* and therefore more energy is reradiated than was originally expended by the high-frequency field.

In reflection from an ionization wave, the frequency increases in exactly the same way as in reflection from a moving plasma. The amplitude of the reflected wave, however, is equal to the amplitude of the incident wave, and consequently the energy of the wave packet is decreased by reflection. Part of the energy of the incident wave packet goes into the kinetic energy of uniform motion of the electrons.

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*At relativistic plasma velocities, a solution in the geometrical-optics approximation was obtained by Ostrovskii.[10]

†In the calculation of the ratio of the energies of the reflected and incident wave packet it is necessary to replace the ratio of the squares of the magnetic-field amplitudes in (15) by the ratio of the squares of the electric-field amplitudes.

*From the equation for the adiabatic invariant[11] it is seen that for an electron this increase is equal to the increase in kinetic energy of its rotational motion.

[1] M. A. Lampert, Phys. Rev. **102**, 299 (1956).
[2] Ya. B. Faĭnberg and V. S. Tkalich, ZhTF **29**, 491 (1959), Soviet Phys.-Tech. Physics **4**, 438 (1959).
[3] Zagorodnov, Faĭnberg, and Egorov, JETP **38**, 7 (1960), Soviet Phys. JETP **11**, 4 (1960).
[4] H. Suhl and L. Walker, Waveguide Propagation of Electromagnetic Waves in Gyrotropic Media (Russ. Transl.) IIL, 1955.
[5] A. V. Gaponov and G. I. Freĭdman, Izv. Vuzov —Radiofizika **3**, 80 (1960).
[6] G. I. Freĭdman, ibid. **3**, 277 (1960).
[7] L. A. Ostrovskii, ibid. **2**, 833 (1959).
[8] L. D. Landau and E. M. Lifshitz, Electrodynamics of Continuous Media, Pergamon, 1961.
[9] Al'pert, Ginzburg, and Feĭnberg, Rasprostranenie radiovoln (Propagation of Radio Waves), GITTL, 1953.
[10] L. A. Ostrovskii, op. cit. [5] **4**, 293 (1961).
[11] L. D. Landau and E. M. Lifshitz, Classical Theory of Fields, Addison-Wesley, 1951.

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