ON THE $K^+ \to \pi^+ + \pi^0 + e^+ + e^-$ *DECAY*

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The decay $K^+ \to \pi^+ + \pi^0 + e^+ + e^-$ is treated without taking final-state interactions into account. The distribution of effective masses of the pion-pion system is obtained. In the calculation, a new approximation technique is applied to calculate the integrals over phase space of the bremsstrahlung probability density when two of the four particles are very light.

In radiative kaon decay, the γ quantum can be radiated by the kaon or by pions as internal bremsstrahlung or by a block of heavy particles in the decay vertex. We call the radiation emitted by the block of heavy particles vertex radiation. As Good[1] noted, the vertex part of the amplitude plays a relatively greater role in radiative θ^+ decay than in other radiative decays, since the vertex radiation is not forbidden by the $\Delta T = \pm\frac{1}{2}$ rule, while the basic $K^+_{2\pi}$ decay is forbidden by this rule.

In this paper, internal conversion of radiative θ^+ decay is calculated. The amplitude is

$$\mathcal{M} = -i4\pi\alpha(2\pi)^4 k^{-2}(8p_0q_0r_0)^{-1/2}\langle \bar{u}_{e^-}\hat{N}u_{e^+}\rangle, \qquad (1)$$

where α is the fine-structure constant; p, q, r, and k are the four-momenta of the K^+, π^+, π^0, and the virtual γ quantum, respectively; and the four-vector N is

$$N_\nu = q_\nu\,[k^2\varphi_1 + (rk)\,\varphi_3 + 2G/[(q+k)^2 - \mu_+^2] + 2G/[(p-k)^2 - M^2]] + r_\nu\,[k^2\varphi_2 - (qk)\,\varphi_3 + 2G/[(p-k)^2 - M^2]] + i\varepsilon_{\rho\sigma\mu\nu}q_\rho r_\sigma k_\mu \varphi_{\text{mag}}. \qquad (2)$$

Here φ_1, φ_2, and φ_3 are functions of the invariants, G is the K^+ decay constant, and μ_+ and M are the masses of the π^+ and K^+, respectively.

Internal conversion differs from radiative θ^+ decay in that the vertex part of the amplitude depends not on one unknown function, but on three; however, the terms containing φ_1 and φ_2 give much smaller contributions to the amplitude than the term containing φ_3. We assume that the φ_i are functions of the single invariant $s = (r + q)^2$. In the calculations, we neglect the difference in the masses of π^0 and π^+.

After invariant integrations of the probability density by Dalitz's method[2] (see also Okun' and Shebalin[3]), the probability density will be a function only of

$$y = (r_0 + q_0)/M, \qquad x = [(r+q)^2 M^{-2}]^{1/2},$$

since the integrations over the light-particle phase space, the momentum difference of the π^0 and π^+, and the direction of the total momentum of the two pions have already been performed. The variables x and y vary within the limits

$$a \leqslant x \leqslant 1 - 2m_0,$$

$$x \leqslant y \leqslant \tfrac{1}{2}(1 + x^2) - 2m_0, \quad a = \mu/m, \quad m_0 = m/M,$$

where m is the electron mass.

The expression for the probability density is the sum of several terms:

$$W = W_{\text{BB}} + W_{3\text{B}} + W_{33} + W_{\text{mag}}. \qquad (3)$$

Here, the conversion of internal bremstrahlung of the K^+ and π^+ is given by W_{BB}, the interference of the bremstrahlung and the main term of the vertex radiation is given by $W_{3\text{B}}$, the main term of the electric vertex radiation is given by W_{33}, and the magnetic radiation is given by W_{mag}; the terms corresponding to interference of the bremstrahlung with the smaller terms in the vertex radiation are not written, since they turn out to be negligible.

If W_{33} and W_{mag} are related to the corresponding constants $|\varphi_3|^2$ and $|\varphi_{\text{mag}}|^2$, it turns out that

$$M_{\text{mag}}/|\varphi_{\text{mag}}|^2 = W_{33}/|\varphi_3|^2 + \Delta,$$

where Δ is a small correction of the same order of magnitude as the terms we have neglected. For the probability density of the internal conversion of the K^+ and π^+ bremstrahlung, we obtain

$$W_{\text{BB}}dx\,dy = \frac{\alpha^2|G|^2 M}{6(2\pi)^3}\left(1 + \frac{m_0^2}{t-y}\right)\left(1 - \frac{2m_0^2}{t-y}\right)^{1/2} \times \left\{\frac{x^3 \ln[(1+z)/(1-z)]}{w(t-y)} - \frac{4a^2(y^2-x^2)^{1/2}(x^2-4a^2)^{1/2}}{[(1-y)^2x^2 + (x^2-4a^2)(y^2-x^2)](t-y)} - \frac{x^2(y^2-x^2)^{1/2}(x^2-4a^2)^{1/2}}{w^2(t-y)}\right\}dx\,dy + \Delta_1 dx\,dy, \qquad (4)$$

where

$$t = (1 + x^2)/2, \quad w = (1 - x^2)/2,$$
$$z = (y^2 - x^2)^{1/2}(x^2 - 4a^2)^{1/2}x(1 - y). \qquad (5)$$

and Δ_1 is a correction of the same order of magnitude as the terms dropped above.

For the interference term W_{3B} we obtain

$$W_{3B}\,dx\,dy$$
$$= \frac{\alpha^2 \operatorname{Re}(G\varphi_3^*)M}{6(2\pi)^3}\left\{-4a^2x\ln\frac{1+z}{1-z} + \frac{x^2(x^2-4a^2)^{1/2}(y^2-x^2)^{1/2}}{t-y}\right.$$
$$\left. + x(2a^2 - x^2)\ln\frac{1+z}{1-z}\right\}$$
$$\times\left(1 + \frac{m_0^2}{t-y}\right)\left(1 - \frac{2m_0^2}{t-y}\right)^{1/2} dx\,dy. \qquad (6)$$

For the pure vertex term W_{33} we have

$$W_{33}\,dx\,dy$$
$$= \frac{|\varphi_3|^2\alpha^2 M}{6(2\pi)^3}\left(1 + \frac{m_0^2}{t-y}\right)\left(1 - \frac{2m_0^2}{t-y}\right)^{1/2}\frac{1}{12}(x^2 - 4a^2)^{3/2}$$
$$\times\{-2y\sqrt{y^2-x^2} + (5x^2-1)\sqrt{y^2-x^2}$$
$$+ 2w^2\sqrt{y^2-x^2}/(t-y)\}\,dx\,dy. \qquad (7)$$

In the calculation of the integrals over y, the function $\ln[(1+z)/(1-z)]$ was expanded in a power series; even after this step, elliptic integrals appear. In order to avoid these, the following approximation was used. It is easy to show that if $\epsilon > 0$ is small enough, and if A and t are finite and F(y) is a function without poles, then the relation

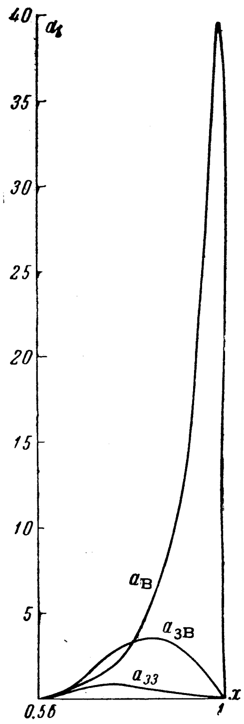
$$\int_A^{t-\varepsilon}\left[1 + \frac{\varepsilon}{2(t-y)}\right]\left(1 - \frac{\varepsilon}{t-y}\right)^{1/2}\frac{F(y)\,dy}{t-y} = \int_A^{t-\varepsilon}\frac{F(y)\,dy}{t-y}$$
$$+ F(t)\left\{\int_A^{t-\varepsilon}\left(1 - \frac{\varepsilon}{t-y}\right)^{1/2}\frac{dy}{t-y}\right.$$
$$\left. - \int_A^{t-\varepsilon}\frac{dy}{t-y} + \frac{\varepsilon}{2}\int_A^{t-\varepsilon}\left(1 - \frac{\varepsilon}{t-y}\right)^{1/2}\frac{dy}{(t-y)^2}\right\} \qquad (8)$$

is correct to any desired accuracy.

m_0^2 is a very small quantity in comparison with any of the other parameters of the system. This allows us to use formula (8), after which all the integrals over y can be carried out. The expressions thus obtained are very cumbersome; therefore, we do not give the expressions themselves, but just their graphs.

The figure shows the quantities

$$a_B = W_{BB}\frac{6(2\pi)^3}{|G|^2\alpha^2 M}, \qquad a_{3B} = W_{3B}\frac{6(2\pi)^3}{\alpha^2 M \operatorname{Re}(G\varphi_3^*)}\left|\frac{G_0}{G}\right|,$$
$$a_{33} = W_{33}\frac{6(2\pi)^3}{\alpha^2 M|\varphi_3|^2}\left|\frac{G_0}{G}\right|^2.$$



In the figure a_{3B} is multiplied by 20 and a_{33} by 400, which corresponds to a ratio of the square moduli of the θ^0 and θ^+ decay constants of $|G_0/G|^2 = 400$.

The average value of the internal bremstrahlung conversion coefficient is found to be $\bar{\rho}_{BB} = 1/73$.

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[1] I. D. Good, Phys. Rev. **113**, 352 (1959).
[2] R. H. Dalitz, Phys. Rev. **99**, 915 (1955).
[3] L. B. Okun' and E. P. Shebalin, JETP **37**, 1775 (1959), Soviet Phys. JETP **10**, 1252 (1960).

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ERRATA

Vol	No	Author	page	col	line	Reads	Should read
13	2	Gofman and Nemets	333	r	Figure	Ordinates of angular distributions for Si, Al, and C should be doubled.	
13	2	Wang et al.	473	r	2nd Eq.	$\sigma_\mu = \frac{e^2f^2}{4\pi^3}\omega^2 (\ln\frac{2\omega}{m_\mu} - 0.798)$	$\sigma_\mu = \frac{e^2f^2}{9\pi^3}\omega^2 \left(\ln\frac{2\omega}{m_\mu} - \frac{55}{48}\right)$
			473	r	3rd Eq.	$(e^2f^2/4\pi^3)\,\omega^2 \geqslant \ldots$	$(e^2f^2/9\pi^3)\,\omega^2 \geqslant \ldots$
			473	r	17	242 Bev	292 Bev
14	1	Ivanter	178	r	9	1/73	1.58×10^{-6}
14	1	Laperashvili and Matinyan	196	r	4	statistical	static
14	2	Ustinova	418	r	Eq. (10) 4th line	$[-\frac{1}{4}(3\cos^2\theta - 1)\ldots$	$-\left[\frac{1}{4}(3\cos^2\theta - 1)\ldots\right.$
14	3	Charakhchyan et al.	533	Table II, col. 6 line 1		1.9	0.9
14	3	Malakhov	550	The statement in the first two phrases following Eq. (5) are in error. Equation (5) is meaningful only when s is not too large compared with the threshold for inelastic processes. The last phrase of the abstract is therefore also in error.			
14	3	Kozhushner and Shabalin	677 ff		The right half of Eq. (7) should be multiplied by 2. Consequently, the expressions for the cross sections of processes (1) and (2) should be doubled.		
14	4	Nezlin	725	r	Fig. 6 is upside down, and the description "upward" in its caption should be "downward."		
14	4	Geĭlikman and Kresin	817	r	Eq. (1.5)	$\ldots\left[b^2\sum_{s=1}^{\infty}K_2(bs)\right]^2$	$\ldots\left[b^2\sum_{s=1}^{\infty}(-1)^{s+1}K_2(bs)\right]^2$
			817	r	Eq. (1.6)	$\Phi(T) = \ldots$	$\Phi(T) \approx \ldots$
			818	1	Fig. 6, ordinate axis	$\frac{\varkappa_s(T)}{\varkappa_n(T_c)}$	$\frac{\varkappa_s(T)}{\varkappa_n(T)}$
14	4	Ritus	918	r	4 from bottom	two or three	2.3
14	5	Yurasov and Sirotenko	971	l	Eq. (3)	$1 < d/2 < 2$	$1 < d/r < 2$
14	5	Shapiro	1154	l	Table	2306	23.6