# ON THE ROLE OF THE SINGLE-MESON POLE DIAGRAM IN SCATTERING OF GAMMA QUANTA BY PROTONS

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It is shown that when the sign of the $\gamma$N-scattering pole diagram connected with $\pi^0$-meson decay is correctly chosen, the contribution of the pole to the cross section for the scattering of $\gamma$ quanta by protons decreases considerably. In order to obtain information on the lifetime of the $\pi^0$ meson, the precision of the experiments must be appreciably improved.

## 1. INTRODUCTION

A few years ago Low[1] called attention to the presence of a pole diagram connected with the decay of the neutral pion, in the amplitude of elastic scattering of $\gamma$ quanta by protons. An account of this diagram, from the point of view of the double dispersion relations for $\gamma$N scattering, is equivalent to an examination of the nearest singularity in $Q^2$. Several interesting considerations in connection with the double dispersion relations for $\gamma$N scattering are contained in the paper by N. F. Nelipa and L. V. Fil'kov (preprint).* Zhizhin[2] considered a contribution of this amplitude in different states. Recently, Hyman et al[3] and in greater detail Jacob and Mathews[4] noted that the addition of the one-meson pole amplitude greatly improves the agreement between the theoretical and experimental results in the $\gamma$-quantum region from 100 to 250 Mev. This problem is considered in detail in a recently published paper by Bernadrini, Yamagata, et al.[5]

It is known that an analysis based on dispersion relations[6,7] leads to scattering cross section values greater than the experimental values in this energy region. In the present paper we wish to call attention to the sign of the pole amplitude, which is very important, since the interference terms play the principal role. From the results of Goldberger and Treiman[8] for the decay of the neutral pion, and from the dispersion relations for forward scattering, which we used previously,[7] it follows that the (relative) sign of the pole diagram differs from that used by Jacob and Mathews. Thus, the addition of the pole diagram does not improve the agreement between the theoretical and experimental results, and the discrepancy calls for a different explanation.

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## 2. SCATTERING AMPLITUDE

We denote by p and p′ the nucleon momentum vectors in the initial and final states, respectively, and by q and q′ the same quantities for the $\gamma$ quanta. Since they satisfy the conservation law

$$q + p = q' + p', \tag{1}$$

it is convenient to introduce the following four orthogonal vectors:

$$K = \tfrac{1}{2}(q + q'), \qquad Q = \tfrac{1}{2}(q' - q) = \tfrac{1}{2}(p - p'),$$
$$P' = P - K(PK)/K^2, \qquad N_\mu = i\varepsilon_{\mu\nu\sigma\rho}P'_\nu K_\sigma Q_\rho, \tag{2}$$

where $P = (p + p')/2$. From these four vectors we can construct two independent scalars:

$$Q^2, \qquad M\nu = -(PK). \tag{3}$$

The lengths of the vectors introduced in (2) are connected with $Q^2$ and $M\nu$ by the relations

$$K^2 = -Q^2, \qquad P^2 = -Q^2 - M^2,$$
$$P'^2 = P^2 - (PK)^2/K^2 = Q^{-2}[M^2\nu^2 - Q^2(Q^2 + M^2)],$$
$$N^2 = -P'^2K^2Q^2 = Q^2[M^2\nu^2 - Q^2(Q^2 + M^2)]. \tag{4}$$

The S-matrix element for $\gamma$N scattering can be represented in the form

$$\langle p'q'|S|pq\rangle = \langle p'q'|pq\rangle + \frac{i}{2\pi}\delta^{(4)}(p' + q' - p - q)\frac{MN}{(p_0p'_0q_0q'_0)^{1/2}}, \tag{5}$$

where

$$N = \bar{u}(p')e'_\mu N_{\mu\nu}e_\nu u(p) = 2\pi^2 i\left(\frac{p_0p'_0}{M^2}\right)^{1/2}\int d^4z e^{-i(Kz)}\left\langle p'\left|T\left(e'\cdot j\left(\frac{z}{2}\right)\right)\right.\right.$$
$$\times\left.\left.\left(e\cdot j\left(-\frac{z}{2}\right)\right)\right|p\right\rangle. \tag{6}$$

In the center-of-mass system (c.m.s.) the differential cross section is given by the relation

$$\frac{d\sigma}{do} = \sum_{\text{spins}} \left|\frac{M}{W} N\right|^2, \qquad (7)$$

where $W^2 = -(P+K)^2$ is the square of the total energy in the c.m.s.

The scattering amplitude N can be written as a sum of six invariant functions ($\hat{K} = \gamma_\mu K_\mu$):

$$e'_\mu N_{\mu\nu} e_\nu = \frac{(e'P')(eP')}{P'^2}[T_1 + i\hat{K}T_2] + \frac{(e'N)(eN)}{N^2}[T_3 + i\hat{K}T_4]$$
$$- \frac{(e'P')(eN) - (e'N)(eP')}{(P'^2N^2)^{1/2}} i\gamma_5 T_5$$
$$+ \frac{(e'P')(eN) + (e'N)(eP')}{(P'^2N^2)^{1/2}} \gamma_5 \hat{K} T_6. \qquad (8)$$

In some cases it is also convenient to represent the amplitude as an operator in spin space in terms of six non-covariant functions $R_i$:

$$\frac{M}{W} e'_\mu N_{\mu\nu} e_\nu = R_1(\mathbf{ee}') + R_2(\mathbf{s's}) + iR_3(\sigma[\mathbf{e'e}]) + iR_4(\sigma[\mathbf{s's}])$$
$$+ iR_5[(\sigma\mathbf{k})(\mathbf{s'e}) - (\sigma\mathbf{k}')(\mathbf{se}')]$$
$$+ iR_6[(\sigma\mathbf{k}')(\mathbf{s'e}) - (\sigma\mathbf{k})(\mathbf{se}')], \qquad (9)^*$$

where $\mathbf{s} = \mathbf{k} \times \mathbf{e}$, $\mathbf{s}' = \mathbf{k}' \times \mathbf{e}'$; $\mathbf{e}$, $\mathbf{k}$ and $\mathbf{e}'$, $\mathbf{k}'$ are the polarization of photon-momentum unit vectors before and after scattering, respectively.

## 3. MATRIX ELEMENT OF NEUTRAL-PION DECAY

The S matrix for the decay of the neutral pion has the form

$$\langle q'q|S|q_\pi\rangle = \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2\omega_k}} (2\pi)^4 \delta^{(4)}(q_\pi - q - q')$$
$$\times \langle q'q|J(0)|0\rangle, \qquad (10)$$

where q and q′ are the photon momenta; $q_\pi$ is the 4-momentum of the pion; J(x) is the current of the pion field:

$$J(x) = i\frac{\delta S}{\delta\varphi(x)} S^+ = ig_0\bar{\psi}(x)\gamma_5\tau_3\psi(x) \qquad (11)$$

[$\varphi(x)$ is the meson-field operator, $\psi(x)$ is the nucleon-field operator, and $g_0$ is the non-renormalized constant of the pion-nucleon interaction]. The Heisenberg equation for the meson field can be written in the form

$$(-\Box^2 + m_\pi^2)\varphi(x) = J(x), \qquad (12)$$

and in the notation of Goldberger and Treiman[8]

$$\mathcal{M} \equiv (2\pi)^3 \sqrt{4qq'}\langle q'q|J|0\rangle$$
$$= -i\varepsilon_{\mu\nu\sigma\lambda} e'_\mu e_\nu q_\sigma q'_\lambda F[(q+q')^2], \qquad (13)$$

where $F(q^2)$ is the form factor. The expression for the decay S matrix contains $F(-m_\pi^2)$.

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*$(\mathbf{ee}') = \mathbf{e}\cdot\mathbf{e}'$ $[\mathbf{ee}'] = \mathbf{e}\times\mathbf{e}'$.

The probability of decay of the neutral pion is

$$w = \sum_{qq'ee'} (2\pi)^3 |\langle qq'|S|q_\pi\rangle|^2/VT$$
$$= \frac{1}{(2\pi)^2}\delta^{(4)}(q_\pi - q - q')\, d^3q\, d^3q' \frac{1}{8q_{\pi 0} q_0 q'_0} q^2 q'^2$$
$$\times \sum_{ee'} [(\mathbf{es}') + (\mathbf{e's})]^2 |F|^2. \qquad (14)$$

Summing over e and e′ and integrating over the angles we obtain in the pion rest system

$$w = (m_\pi^3/64\pi)|F|^2. \qquad (15)$$

The pion lifetime $\tau$ is

$$\tau = 64\pi/m_\pi^3|F|^2. \qquad (16)$$

Using the dispersion technique, Goldberger and Treiman have shown that

$$F(0) = -\frac{ge^2}{4\pi^2 m_\pi}(1+\mu_p)\frac{I_0 + \rho I_1}{1 + (g^2/4\pi)I_1}, \qquad (17)$$

$$\rho = [2\mu_p - (\mu_p^2 - \mu_n^2)]/(1+\mu_p), \qquad (18)$$

where $\mu_p$ and $\mu_n$ are the anomalous magnetic moments of the proton and neutron, while $I_0$ and $I_1$ are positive integrals. It follows from (17) that

$$F(0)\,g < 0. \qquad (19)$$

This sign is of importance for what is to follow.

## 4. SINGLE-MESON DIAGRAM FOR THE SCATTERING OF GAMMA QUANTA BY PROTONS

The S-matrix element of the pole diagram is

$$\langle p'q'|S-1|pq\rangle = ig\frac{i}{(2\pi)^3}\bar{u}(p')\gamma_5 u(p)$$
$$\times \delta^{(4)}(p' + q' - p - q)$$
$$\times (2\pi)^4 \frac{1}{(p'-p)^2 + m_\pi^2}\langle q'|J_\pi(0)|q\rangle. \qquad (20)$$

It can be shown that

$$\langle q'|J_\pi(0)|q\rangle$$
$$= \frac{1}{(2\pi)^3}\frac{1}{(4q_0q'_0)^{1/2}}(-i)\varepsilon_{\mu\nu\sigma\lambda} e'_\mu e_\nu q'_\sigma q_\lambda F[(q'-q)^2]. \qquad (21)$$

Since the matrix element $\langle q'|J_\pi(0)|q\rangle$ is taken for the pole at $(q'-q)^2 = -m_\pi^2$, Eq. (21) contains exactly the value of F encountered in the $\pi^0$ decay.

Substituting (21) in (20) and going to the c.m.s., we obtain

$$\langle p'q'|S-1|pq\rangle = \frac{igF}{(2\pi)^6}\frac{Mq^3}{(4q_0q'_0p_0p'_0)^{1/2}}(2\pi)^4\delta^{(4)}(p' + q'$$
$$- p - q)\frac{1}{2M}[i((\sigma\mathbf{k})(\mathbf{es}') - (\sigma\mathbf{k}')(\mathbf{e's}))$$
$$- i((\sigma\mathbf{k}')(\mathbf{es}') - (\sigma\mathbf{k})(\mathbf{e's}))]. \qquad (22)$$

Comparing (22) with (9) we obtain for the contribution of the pole diagram

$$R_{1p} = R_{2p} = R_{3p} = R_{4p} = 0,$$

$$R_{5p} = -R_{6p} = \frac{gF}{8\pi W}\frac{q^3}{(p-p')^2+m_\pi^2}; \qquad (23)$$

from this we conclude that the contribution made to the amplitude by the pole diagram due to the exchange and decay of the pseudo-scalar neutral meson reduces to the combination

$$R_{5p} - R_{6p} = \frac{gFm_\pi}{8\pi W}\frac{q}{m_\pi}\frac{1}{1+m_\pi^2/2q^2-\cos\theta}. \qquad (24)$$

It is important to note that by virtue of (19)

$$R_{5p} - R_{6p} < 0, \qquad (25)$$

if it is assumed that $F(0)$ and $F(-m_\pi^2)$ do not differ greatly.

In the expression for the cross section [formula (16) in [5]] the pole term enters in the combination

$$\tfrac{1}{2}|R_5 - R_6|^2(1-\cos\theta)^3$$
$$-\operatorname{Re}(R_3 - R_4)^*(R_5 - R_6)(1-\cos\theta)^2. \qquad (26)$$

The contribution of one pole diagram has the form

$$I_0^P(\theta) = \tfrac{1}{2}|R_5 - R_6|^2(1-\cos\theta)^3$$
$$= \frac{2}{m_\pi\tau}\left(\frac{q}{W}\right)^2\frac{g^2}{4\pi}\left(\frac{1}{m_\pi}\right)^2\frac{(1-\cos\theta)^3}{(1+m_\pi^2/2q^2-\cos\theta)^2} \qquad (27)$$

which agrees with the result of Jacob and Mathews.

We can expect the cross section of scattering by 90° to be reduced by addition of the pole term only when the second term in (26) is negative. Since $R_4$ is large and negative, owing to the large anomalous magnetic moment of the proton, $\operatorname{Re}(R_3 - R_4)$ is a positive quantity in the region of energy under consideration. Thus, the second term in (26) is positive if $R_{5p} - R_{6p} < 0$. Consequently, assuming the analysis of Goldberger and Treiman to be correct, the pole diagram does not decrease the theoretical value of the cross section, but increases it.

If we use the results of our own analysis,[7] we find that $\operatorname{Re}(R_5 - R_6)$ is determined not only by the limit theorem, but also by the amplitudes of photoproduction of $E_2$ and $M_3$. Since in this case the "isotropic" part of the contribution of the pole amplitude is automatically taken into account, it is necessary to add to the previously-obtained amplitude not all of expression (24), but only the contribution of (24) to the higher states, i.e., the difference

$$(R_5 - R_6)_p - \frac{1}{2\pi}\int(R_5 - R_6)_p\sin\theta\, d\theta\,.$$

As a result of this procedure, which is necessary in order not to violate the unitarity of the S matrix (when $\theta = 90°$), the quantity $y_0^{-1}$ (where $y_0 = 1 + m_\pi^2/2q^2$) is replaced by

$$y_0^{-1} - \tfrac{1}{2}\ln|(y_0+1)/(y_0-1)|,$$

which leads to replacement of $2/3$ by $-0.14$ when $q^2 = m_\pi^2$ ($y_0 = 3/2$).

Thus, the contribution of the amplitude is decreased by a factor of 5, and the sign of the contribution changes. By virtue of this, a much higher accuracy is necessary before the connection between the amplitude of the neutral-pion decay and the amplitude of the scattering of $\gamma$ quanta by protons can manifest itself. It was recently shown that the lifetime of the neutral pion is $(2.0 \pm 0.4) \times 10^{-16}$ sec,[9] which also decreases the contribution of the pole diagram.

The indeterminacies in the analysis of the photoproduction cannot influence the conclusion regarding the sign of the interference term in (24), since this sign is determined by the well known theorem for low energies. The scattering amplitude at low frequencies, first obtained by Low[10] and Gell-Mann and Goldberger,[11] is reviewed in the appendix, where it is obtained as the contribution of the single-nucleon terms (see [6]).

We note, in particular, that

$$T_5^0 = \frac{e^2}{M}(1+\lambda)\frac{Q^2}{Q^4/M^2-\nu^2}. \qquad (28)$$

Let us give another, less rigorous but more illustrative proof of the correctness of the determination of the sign of the pole diagram.*

The matrix element $\langle q'|J_\pi(0)|q\rangle$ can be represented in the form

$$\langle q'|J_\pi(0)|q\rangle = i\varepsilon_{\mu\nu\sigma\lambda}e'_\mu e_\nu q_\sigma q'_\lambda\frac{1}{(2\pi)^3}F[(q-q')^2]$$
$$= -\frac{2Q^2}{(2\pi)^3}F\frac{(e'P')(eN)-(e'N)(eP')}{(P'^2N^2)^{1/2}}, \qquad (29)$$

so that

$$\langle p'q'|S-1|pq\rangle = i(2\pi)^{-2}g\delta^{(4)}(p'+q'-p-q)$$
$$\times i\bar{u}(p')\gamma_5 u(p)$$
$$\times\frac{2Q^2F}{4Q^2+m_\pi^2}\frac{(e'P')(eN)-(e'N)(eP')}{(P'^2N^2)^{1/2}}; \qquad (30)$$

hence

$$T_{5p} = \frac{gF}{\pi}\frac{Q^2}{4Q^2+m_\pi^2}. \qquad (31)$$

We now introduce the function

$$f(\nu, Q^2) = T_5(\nu, Q^2)/Q^2. \qquad (32)$$

If we regard $f(\nu, Q^2)$ as an analytic function of $Q^2$ at fixed $\nu$, we obtain from Cauchy's theorem and from (31)

*An analogous approach was used earlier[12] to obtain the Goldberger-Treiman relations.

$$f(\nu, Q^2) = \frac{gF}{\pi}\frac{1}{4Q^2+m_\pi^2} + J_Q, \quad (33)$$

where $J_Q$ is the dispersion interval, the lower limit of which is $4m_\pi^2$. In the region $Q^2 \ll 4m_\pi^2$, the integral in (33) is small and we can approximate $f(\nu, Q^2)$ by the expression

$$f(\nu, Q^2) \approx \frac{gF}{\pi}\frac{1}{4Q^2+m_\pi^2}. \quad (34)$$

On the other hand, $f(\nu, Q^2)$ is also an analytic function of $\nu$ for fixed $Q^2$. By Cauchy's theorem with account of (28) we have

$$f(\nu, Q^2) = \frac{e^2(1+\lambda)}{M}\frac{1}{Q^4/M^2-\nu^2} + J_\nu, \quad (35)$$

where $J_\nu$ is a second dispersion integral. In the region $2\nu \lesssim m_\pi$, the pole term will predominate and

$$f(\nu, Q^2) \approx \frac{e^2(1+\lambda)}{M}\frac{1}{Q^4/M^2-\nu^2}. \quad (36)$$

It is obvious that (34) does not hold near $M^2\nu^2 \approx Q^2$, and (36) does not take place when $4Q^2 = -m_\pi^2$. It is still possible, however, that expressions (34) and (36) are valid simultaneously near certain values of $\nu$ and $Q^2$. Equating these expressions for $2\nu = m_\pi$ and $Q \approx 0$, we obtain

$$F = -4\pi e^2(1+\lambda)/gM, \quad (37)$$

which is very close to the formula of Goldberger and Treiman, obtained by an entirely different method.

Actually, from (17) we obtain for $(g^2/4\pi^2)I_1 \gg 1$

$$F = -4\pi\frac{e^2(1+\lambda)}{g}\frac{I_0+\rho I_1}{I_1},$$

which coincides with (37), apart for a numerical factor.

The literature reports two different choices of the common phase for the $\gamma N$ scattering amplitude, one with a Thomson limit $+e^2/M$, the other with $-e^2/M$. The error in the published papers lies in the fact that the choice of the common factor in the one-meson amplitude does not correspond to the choice of the sign of the remaining amplitude.

A direct comparison of the amplitude used by Jacob and Mathews[4] with (9) shows that the functions $f_i$ introduced in [4] are related with $R_i$ by the equations

$$-f_1 = R_1 + R_2\cos\theta, \qquad f_2 = R_2,$$
$$f_3 = R_3 + R_4\cos\theta + (R_5+R_6)(1+\cos\theta) - (R_5-R_6)(1-\cos\theta),$$
$$f_4 = R_4, \qquad f_5 = R_4+R_5, \qquad f_6 = R_6,$$

where the difference in the common phase factor is taken into account, and from which it is clear that the sign used in [4] for the pole term differs from that proved in the present paper.

## APPENDIX

### SINGLE-NUCLEON TERMS IN THE DISPERSION RELATIONS

Recognizing that

$$T\left(e'\cdot j\left(\tfrac{z}{2}\right)\right)\left(e\cdot j\left(-\tfrac{z}{2}\right)\right) = \theta(z_0)\left[e\cdot j'\left(\tfrac{z}{2}\right), e\cdot j\left(-\tfrac{z}{2}\right)\right] + \left(e'\cdot j\left(\tfrac{z}{2}\right)\right)\left(e\cdot j\left(-\tfrac{z}{2}\right)\right), \quad (A.1)$$

we determine the retarded and advanced amplitudes:

$$N^{r,a} = \pm 2\pi^2 i\,(p_0p_0'/M^2)^{1/2} \times \int d^4z e^{\pm i(Kz)}\left\langle p'\left|\theta(\pm z_0)\left[e'\cdot j\left(\tfrac{z}{2}\right), e\cdot j\left(-\tfrac{z}{2}\right)\right]\right|p\right\rangle. \quad (A.2)$$

The vertex part of the current has the form

$$\langle p'|ej(0)|p\rangle = \frac{i\varepsilon}{(2\pi)^3}\bar u(p')\left[\hat e + i\frac{\lambda}{4M}(\hat e(\hat p'-\hat p) - (\hat p'-\hat p)\hat e)\right]u(p) = \frac{i\varepsilon}{(2)^3}u(p')\left[(1+\lambda)\hat e + \frac{i\lambda}{M}(eP)\right]u(p), \quad (A.3)$$

where $\epsilon$ is the charge of the nucleon.

The pole term has in the region of positive frequencies the form

$$A^0 = -\frac{(2\pi)^6}{4}\sum_n \delta^{(4)}(k-p+p_n)\langle p'|e\cdot j(0)|p_n\rangle \times \langle p_n|e'j(0)|p\rangle = \frac{\varepsilon^2}{4}\int d^3p_n\,\delta^{(4)}(k-p+p_n)$$
$$\times\bar u(p')\left[(1+\lambda)\hat e + \frac{i\lambda}{M}ep'\right]u(P-K)\bar u(P-K) \times\left[(1+\lambda)\hat e' + \frac{i\lambda}{M}e'p\right]u(p). \quad (A.4)$$

Using the relations

$$\sum u(P-K)\bar u(P-K) = \frac{-i(\hat P-\hat K)+M}{2p_{n0}}, \quad (A.5)$$

$$(2p_{n0})^{-1}d^3p_n = d^4p_n\theta(p_{n0})\,\delta(p_n^2+M^2), \quad (A.6)$$

we obtain

$$A^0 = \frac{\varepsilon^2}{4}\delta(p_n^2+M^2)\bar u(p')\left[(1+\lambda)\hat e + \frac{i\lambda}{M}(ep')\right] \times[-i(\hat P-\hat K)+M]\left[(1+\lambda)\hat e' + \frac{i\lambda}{M}(e'p)\right]u(p). \quad (A.7)$$

We can express $A^0$ in terms of the fundamental invariants:

$$A^0 = \frac{(e'P')(eP')}{P'^2}A_1^0 + \frac{(e'N)(eN)}{N^2}A_2^0 + \frac{(e'P')(eN)-(e'N)(eP')}{(P'^2N^2)^{1/2}}A_3^0 + \frac{(e'P')(eN)+(e'N)(eP')}{(P'^2N^2)^{1/2}}A_4^0. \quad (A.8)$$

Comparing (A.7) and (A.8), we obtain

$$A_1^0P'^2 = \frac{\varepsilon^2}{4}\delta(p_n^2+M^2)\bar u(p')\left[(1+\lambda)\hat P' + \frac{i\lambda}{M}(P'p')\right] \times[-i(\hat P-\hat K)+M]\left[(1+\lambda)\hat P' + \frac{i\lambda}{M}(P'p)\right]u(p). \quad (A.9)$$

It is easy to verify that

$$(P'p') = (P', P - Q) = (P'p) = P'^2, \quad (A.10)$$

$$\bar{u}(p')[\hat{P}'(-i(\hat{P}-\hat{K})+M)\hat{P}']u(p)$$
$$= \bar{u}(p')\left\{-i\hat{K}P'^2 + 2P'^2M + 2i\frac{(PK)}{K^2}\hat{K}P'^2\right\}u(p), \quad (A.11)$$

$$\bar{u}(p')\hat{P}'[-i(\hat{P}-\hat{K})+M]u(p) = \bar{u}(p')\left[M\left(iM - \frac{(PK)}{K^2}\hat{K}\right)\right.$$
$$\left. - i(P^2+(PK)) + i\left((\hat{P}\hat{K}) + \frac{(PK)}{K^2}\hat{K}\hat{P}\right)\right]u(p), \quad (A.12)$$

$$\bar{u}(p')[(-i(\hat{P}-\hat{K})+M)\hat{P}']u(p)$$
$$= \bar{u}(p')\left[M\left(iM - \frac{(PK)}{K^2}\hat{K}\right) - i(P^2+(PK))\right.$$
$$\left. + i\left(\hat{K}\hat{P} + \frac{(PK)}{K^2}\hat{P}\hat{K}\right)\right]u(p), \quad (A.13)$$

$$\bar{u}(p')[-i(\hat{P}-\hat{K})+M]u(p) = \bar{u}(p')[i\hat{K}+2M]u(p). \quad (A.14)$$

Using (A.10) – (A.14) and noting that at the pole we have

$$(P-K)^2 = P^2 + K^2 - 2(PK)$$
$$= 2K^2 - 2(PK) - M^2 = -M^2$$

or that

$$K^2 = (PK), \quad (A.15)$$

we obtain

$$A_1^0 = \frac{1}{4}\varepsilon^2\delta(2K^2 - 2(PK))\bar{u}(p')(2M + i\hat{K})u(p)$$
$$= \frac{\varepsilon^2}{8M}\delta\left(\nu - \frac{Q^2}{M}\right)\bar{u}(p')(2M + i\hat{K})u(p). \quad (A.16)$$

Analogously,

$$A_2^0 = -\frac{\varepsilon^2}{8M}\delta\left(\nu - \frac{Q^2}{M}\right)\bar{u}(p')\,i\hat{K}\,u(p)\,(1+\lambda)^2, \quad (A.17)$$

$$(A_3^0 + A_4^0)(P'^2N^2)^{1/2}$$
$$= \frac{\varepsilon^2(1+\lambda)}{8M}\delta\left(\nu - \frac{Q^2}{M}\right)\bar{u}(p')\left[\frac{P'^2}{M}\hat{N}(-iM + \hat{K})\right]u(p), \quad (A.18)$$

$$(A_4^0 - A_3^0)(P'^2N^2)^{1/2}$$
$$= \frac{\varepsilon^2(1+\lambda)}{8M}\delta\left(\nu - \frac{Q^2}{M}\right)\bar{u}(p')\left[\frac{P'^2}{M}(\hat{K} - iM)\hat{N}\right]u(p). \quad (A.19)$$

From (A.18) and (A.19) we find

$$A_4^0(P'^2N^2)^{1/2} = \frac{\varepsilon^2(1+\lambda)}{8M}\delta\left(\nu - \frac{Q^2}{M}\right)(-iP'^2)\bar{u}(p')\hat{N}u(p),$$
$$A_3^0(P'^2N^2)^{1/2} = \frac{\varepsilon^2(1+\lambda)}{8M}\delta\left(\nu - \frac{Q^2}{M}\right)\frac{P'^2}{M}\bar{u}(p')\hat{N}\hat{K}u(p). \quad (A.20)$$

It can be shown that

$$\bar{u}(p')\hat{N}\hat{K}u(p) = (P'^2N^2)^{1/2}\,i\bar{u}(p')\gamma_5 u(p),$$
$$i\bar{u}(p')\hat{N}u(p) = K^2\bar{u}(p')\gamma_5\hat{K}u(p).$$

If we now take into account the fact that $(P'^2N^2)^{1/2} = P'^2Q^2$ by virtue of (4), we obtain from (A.20)

$$A_3^0 = -\frac{\varepsilon^2(1+\lambda)}{8M}\delta\left(\nu - \frac{Q^2}{M}\right)i\bar{u}(p')\gamma_5 u(p),$$
$$A_4^0 = \frac{\varepsilon^2(1+\lambda)}{8M}\delta(\nu - Q^2/M)\bar{u}(p')\gamma_5\hat{K}u(p). \quad (A.21)$$

Finally from (A.16), (A.17), and (A.21) we obtain

$$T_1^0 = \frac{\varepsilon^2}{2\pi M}\frac{Q^2}{Q^4/M^2 - \nu^2}, \qquad T_2^0 = \frac{\varepsilon^2}{4\pi M}\frac{\nu}{Q^4/M^2 - \nu^2},$$
$$T_3^0 = 0, \qquad T_4^0 = -\frac{\varepsilon^2(1+\lambda)^2}{4\pi M}\frac{\nu}{Q^4/M^2 - \nu^2},$$
$$T_5^0 = MT_6^0 = \frac{\varepsilon^2(1+\lambda)}{4\pi M}\frac{Q^2}{Q^4/M^2 - \nu^2} \quad \left(\frac{\varepsilon^2}{4\pi} = \frac{1}{137}\right), \quad (A.22)$$

which coincides with the previously-obtained results and has the correct signs.

In all the calculations of the single-nucleon terms it is assumed that parity is conserved in the electromagnetic interactions. The results obtained remain valid also in the presence of CP invariance.

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