# *FLUCTUATIONS OF THE μ-MESON FLUX IN EXTENSIVE AIR SHOWERS*

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Fluctuations of the μ-meson flux in extensive air showers of a given size N ($N > 10^6$) were studied using an arrangement which simultaneously measured the total number of shower particles and the number of μ mesons in the shower. It is shown that the fluctuations can be explained by fluctuations in the height at which the shower-producing primary particle experiences its first interaction. The data obtained are used to determine the interaction mean free path for the ultra-high energy primary particles producing the extensive showers.

## INTRODUCTION

THE experimental study of the fluctuations of the μ-meson flux compared to the total flux of all charged particles in extensive air showers (EAS) is of great interest, since the character of these fluctuations is apparently determined by the fluctuations in the development of the cascade of high-energy nuclear-active particles in the atmosphere.

Recently, a number of models of EAS development were considered which predict the existence of strong fluctuations, in particular of the ratio of all charged particles to μ mesons.[1-3]

The present article presents results of a study of the fluctuations in the μ-meson flux in EAS, carried out using the array for the comprehensive study of EAS at Moscow State University.

## EXPERIMENTAL METHOD AND DESCRIPTION OF ARRAY

In order to solve the problem at hand, it is necessary simultaneously to determine the total flux of charged particles and to detect a sufficiently large number of μ mesons in individual extensive air showers. In order to determine the total number of particles, we used an array consisting of a large number of Geiger-Müller counters forming a hodoscope. The position of the counter trays and the number of counters of different areas in each tray are shown in Fig. 1a and in Table I.

The total number of particles and the position of the shower axis were determined by the usual method,[4] assuming that all showers have the same lateral-distribution function of charged particles, closely approximated by the Nishimura-Kamata function with age parameter s = 1.3.

The relative error ΔR/R in the determination of the shower-axis position (R is the distance from the shower axis to the center of the array) for showers detected using the triggering method described below amounted to 20%. The relative error in the determination of the number of particles ΔN/N amounted to ±30% for R < 60 m, and to +100%, −50% for R ~ 150 m.

The μ mesons were detected both on the surface of the earth (chambers 2, 3, 5, 6, 7, 9), and

**Table I.** Distribution of counters with different areas in different points of the array represented in Fig. 1

| Point | Unshielded counters | | Shielded counters | Point | Unshielded counters | | Shielded counters |
|---|---|---|---|---|---|---|---|
| | 330 cm² | 100 cm² | 330 cm² | | 330 cm² | 100 cm² | 330 cm² |
| *I* | 264 | 100 | — | *5* | 72 | 48 | 24×2 |
| *II* | 60 | 48 | — | *6* | 96 | 24 | 24×2 |
| *III* | 120 | 48 | — | *7* | 72 | 48 | 24×2 |
| $U_1$ (20 m w.e.) | — | — | 96 | *8* | 96 | 24 | — |
| $U_2$ (40 m w.e.) | — | — | 192 | *9* | 108 | 24 | 24×2 |
| *1* | 72 | 48 | — | *10* | 96 | 24 | — |
| *2* | 108 | 24 | 24×2 | *11* | 36 | 24 | — |
| *3* | 96 | 24 | 24×2 | *12* | 36 | 24 | — |
| *4* | 72 | 48 | — | *13* | 36 | 24 | — |
| | | | | *14* | 24 | 48 | — |
| | | | | *15* | 24 | 48 | — |

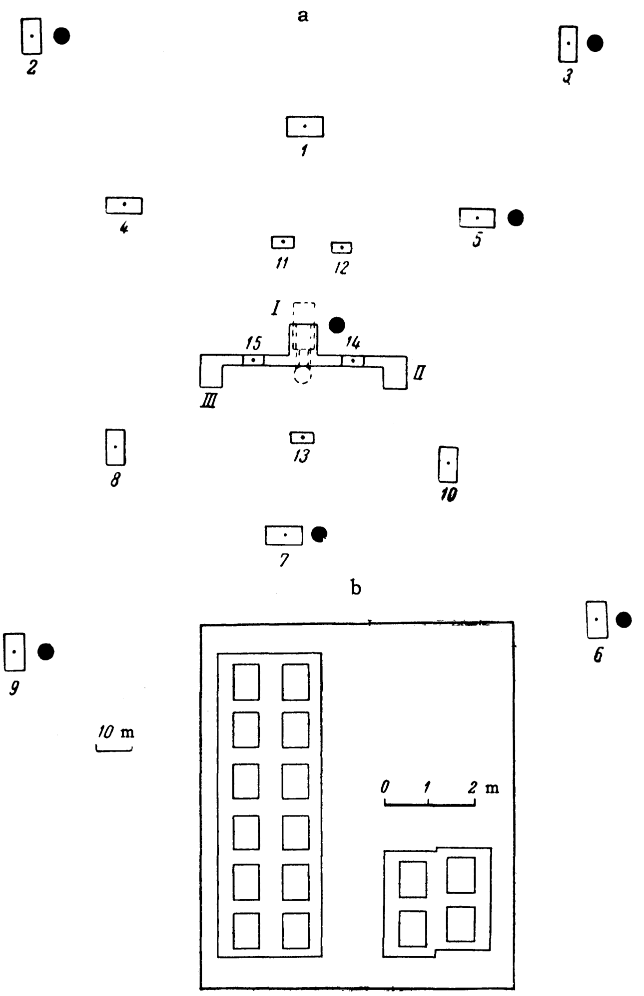



FIG. 1. a — position of the Geiger-Müller counter trays, ● — $\mu$-meson detectors; dashed line — outline of underground chambers; b — positions of detectors in the underground chamber $U_2$. The squares indicate the effective areas of the counter groups.

underground at 20 and 40 m water equivalent (w.e.) in chambers $U_1$ and $U_2$ respectively. On the surface of the earth, we used a hodoscope arrangement of Geiger counters shielded by lead and iron (see Fig. 2) for the detection of $\mu$ mesons. Underground, we used for this purpose a hodoscope of Geiger-Müller counters which was similar to the arrangement shown in Fig. 2 but without the top counter layer and the absorber above it.

The total effective area of the $\mu$-meson detectors amounted to 4.75 $m^2$ on the surface of the earth, 3.2 $m^2$ at the depth of 20 m w.e., and 6.3 $m^2$ at 40 m w.e. The position of the $\mu$-meson detectors and the number of counters in them are shown in Fig. 1 and Table I.

EAS were selected by requiring a six-fold coincidence of counters with 0.132 $m^2$ area in each coincidence channel. The counters of three channels were placed into one tray, in which the distance between the counter walls amounted to 3 cm ($6 \times 55$ cm counters were used). The distance between the two trays was 2.5 m.

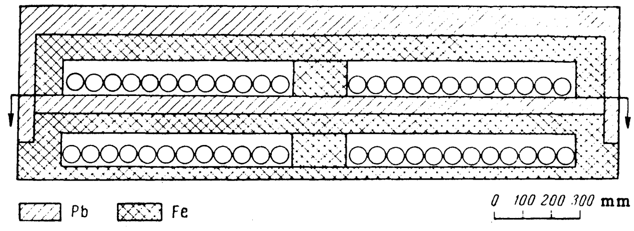



FIG. 2. $\mu$-meson detector.

The triggering array was placed in the center of chamber 1, Fig. 1a.

## REDUCTION OF DATA

From the total number of detected showers, we selected showers of large size. The selection criterion was the discharge of 100 counters out of 264 with 330 $cm^2$ area each in chamber 1, and the simultaneous discharge of at least 30 counters out of 72 at trays 1, 4, 5, 7, 8, 10. The shower selected in such a way had a total number of particles $N > 10^5$.

For the determination of the number of $\mu$ mesons (with energy $E_\mu > 4 \times 10^8$ ev) detected by the detectors at the surface of the earth, we inspected the hodoscope pictures of counter discharges in the detectors. We regarded either one of the following events as a passage of $\mu$ mesons: a) one counter is discharged in each layer of the detector, b) one counter is discharged in one layer, and two counters in the second layer. In one detector (Fig. 2), several events of the types a) or b) could be observed during the passage of several $\mu$ mesons, with the order of the discharged counters corresponding to the parallelness of the $\mu$-meson tracks.

In order to exclude the contribution of nuclear-active particles to the detected $\mu$ mesons, we analyzed only those cases which occurred at a distance of more than 50 m from the shower axis. For these showers, the probability of a discharge of two counters amounted to 12%. It follows from the experimental data[5,6] that, at distances $r > 50$ m from the shower axis, the fraction of nuclear-active particles as compared to the total number of $\mu$ mesons is small (10% at a distance of 50 m), and decreases rapidly (as $r^{-1}$) with an increasing distance from the shower axis. Using these data, we have calculated the contribution of nuclear-active particles to the detected $\mu$-meson flux for selected

**Table II.** Distribution of events with respect to the ratio of the number q of detected $\mu$ mesons to the mean expected number p of mesons

| q/p | A | | B | | C | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $0-{}^{1}/_{3}$ | 13 | 7 | 8 | 2 | 0 | 23 | 4 | 8 |
| ${}^{1}/_{3}-{}^{2}/_{3}$ | 18 | 20 | 8 | 10 | 0 | 36 | 20 | 27 |
| ${}^{2}/_{3}-1$ | 22 | 26 | 22 | 17 | 65% | 22 | 40 | 37 |
| $1-1{}^{1}/_{3}$ | 20 | 24 | 12 | 20 | 26% | 17 | 38 | 32 |
| $1{}^{1}/_{3}-1{}^{2}/_{3}$ | 15 | 14 | 1 | 5 | 6% | 18 | 15 | 13 |
| $1{}^{2}/_{3}-2$ | 6 | 6 | 4 | 2 | 1.6% | 2 | 5 | 5 |
| $2-2{}^{1}/_{3}$ | 6 | 5 | 1 | 1 | | 5 | 2 | 2 |
| $2{}^{1}/_{3}-2{}^{2}/_{3}$ | 2 | 2 | 1 | — | | — | 2 | 2 |
| $2{}^{2}/_{3}-3$ | — | 1 | | | 1.4% | — | 1 | 1 |
| $3-3{}^{1}/_{3}$ | — | 1 | | | | 1 | — | — |
| $3{}^{1}/_{3}-3{}^{2}/_{3}$ | | | | | | 3 | | |
| $3{}^{2}/_{3}-4$ | | | | | | | | |
| $4-4{}^{1}/_{3}$ | 4 | | | | | | | |
| Total number of events | 106 | 106 | 57 | 57 | 100% | 127 | 127 | 127 |
| $P(\chi^2)$ | 20 % | | 0.03% | | <0.01% | | | |

Remark: A – data from all surface detectors for $N \geq 5 \times 10^6$, B – from detector $U_1$ for $N \geq 4 \times 10^6$, C – from detector $U_2$ for $N \geq 4 \times 10^6$. Column 5 shows the distribution expected because of the spread of the real distances from the shower axis to $U_2$. Columns 1, 3, and 6 show the experimental distributions; columns 2, 4, and 7, the distributions expected according to Eq. (2); and column 8, the distribution expected because of the factor shown in column 5 and of statistical fluctuations.

showers. It was found that the contribution amounts to 5%.

The determination of the $\mu$-meson flux density with energy $E_\mu > 5 \times 10^9$ ev and $E_\mu > 10^{10}$ ev was carried out using detectors $U_1$ and $U_2$ (at 20 and 40 m w.e. respectively). It was assumed that, when one $\mu$ meson traverses the counters, we should observe the discharge of one, two, or more counters of the detector of Fig. 2 (in the lower layer). The probability of a simultaneous discharge of two or more counters due to the passage of one $\mu$ meson amounted to 8%, and was due to the production by the $\mu$ mesons of $\delta$ electrons and of secondary electron-photon showers.

In measuring the $\mu$-meson flux using the $U_1$ and $U_2$ detectors placed below the level at which the position of the shower axis was being determined, a certain uncertainty arises in the actual distance from the shower axis, owing to the unknown angle of shower arrival. This uncertainty decreases with increasing distance R from the trace of the shower axis on the surface of the earth to the vertical line passing through $U_1$ and $U_2$.

Table II (column 5) shows the probabilities of the deviation of the $\mu$-meson flux density from the average density obtained for R = H (where H is the depth of the underground chamber in meters), assuming an angular distribution as $\cos^7 \theta$ and a lateral $\mu$-meson distribution as 1/r.

For the analysis, we used showers with $R \geq H$.

Using the method for determining the number of $\mu$ mesons described above, the average lateral distributions of $\mu$ mesons with different threshold energy were obtained for showers of different size (Fig. 3).

The obtained average characteristics of the $\mu$-meson flux permitted us to determine, for each detector, the expected number of $\mu$ mesons corresponding to the detected number of shower particles, and the distance of the $\mu$-meson detector from the shower axis.

## RESULTS

As a result of the above-described data reduction, we have found for each selected shower the number q of $\mu$ mesons detected by the detectors and the number p of $\mu$ mesons expected in those detectors for a given total number of particles in the shower and for known distances of the detectors from the shower axis on the surface of the earth.

The investigated showers were divided into the following size intervals:

For the detector on the surface of the earth $\begin{cases} N = (2-5)\cdot 10^6 \\ N = (5-10)\cdot 10^6, \\ N \geqslant 10^7 \end{cases}$

For $U_1$ $\begin{cases} N = (2-4)\cdot 10^6 \\ N \geqslant 4\cdot 10^6 \end{cases}$,

For $U_2$ $\begin{cases} N = (1-2)\cdot 10^6 \\ N = (2-4)\cdot 10^6. \\ N \geqslant 4\cdot 10^6 \end{cases}$

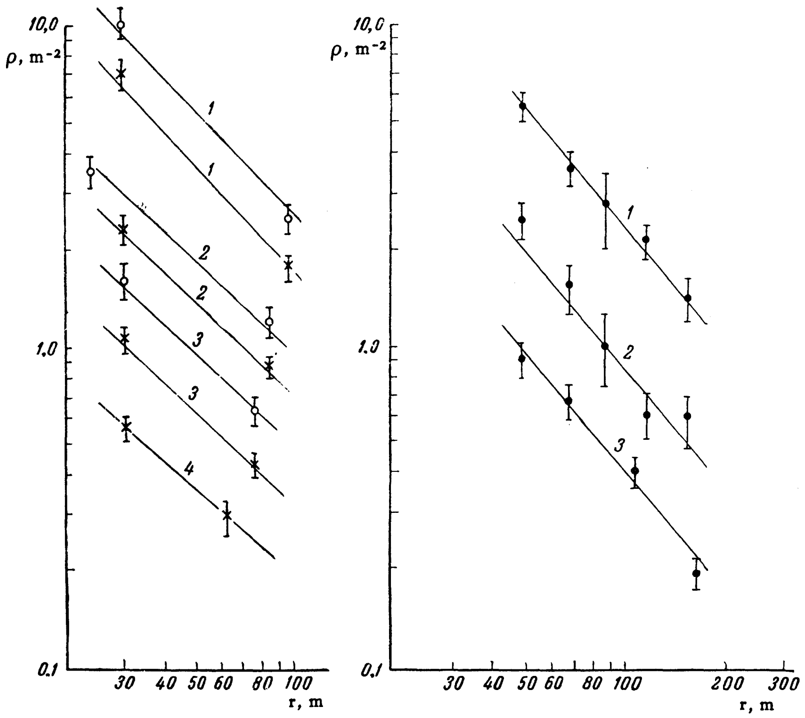


FIG. 3. Mean lateral distributions $\rho(r)$ of $\mu$-meson fluxes in showers with different numbers of particles: 1 – $\bar{N} = 2 \times 10^7$ ($N > 10^7$), 2 – $\bar{N} = 5 \times 10^6$ ($\Delta N = 4-10 \times 10^6$), 3 – $\bar{N} = 2.5 \times 10^6$ ($\Delta N = 2-4 \times 10^6$), 4 – $\bar{N} = 1 \times 10^6$ ($\Delta N = 1-2 \times 10^6$). The $\mu$-meson flux density was determined according to the detector $U_1$(O), $U_2$ (×), or the surface detector (●).

**Table III.** Distribution of events with respect to the detected number of $\mu$ mesons q

| q | A | | B | | C | | D | | E | |
|---|---|---|---|---|---|---|---|---|---|---|
| | exp. | calc. | exp. | calc. | exp. | calc. | exp. | calc. | exp. | calc. |
| 0 | 47 | 42 | 6 | 5 | 11 | 5 | 24 | 17 | 20 | 4 |
| 1 | 42 | 51 | 9 | 8 | 19 | 9 | 27 | 28 | 28 | 12 |
| 2 | 44 | 34 | 8 | 9 | 10 | 15 | 23 | 28 | 19 | 20 |
| 3 | 12 | 18 | 8 | 8 | 12 | 15 | 17 | 19 | 22 | 25 |
| 4 | 8 | 8 | 5 | 6 | 10 | 12 | 12 | 13 | 12 | 27 |
| 5 | 2 | 2 | 8 | 5 | 4 | 10 | 5 | 7 | 10 | 24 |
| 6 | 2 | 1 | 6 | 4 | 3 | 5 | 7 | 4 | 12 | 18 |
| 7 | 1 | 1 | — | 2 | — | 4 | 1 | 2 | 3 | 12 |
| 8 | | 1 | 1 | 1 | 4 | 2 | 2 | 2 | 5 | 7 |
| 9 | | | 1 | 1 | 5 | 2 | — | 1 | 4 | 4 |
| 10 | | | | 1 | 1 | 1 | 3 | 1 | 2 | 2 |
| 11 | | | | 1 | 1 | 1 | 1 | 1 | 4 | 1 |
| 12 | | | | | | | — | | 3 | 1 |
| 13 | | | | | | | — | | 2 | |
| 14 | | | | | | | 1 | | 2 | |
| 15 | | | | | | | | | 1 | |
| 16 | | | | | 1 | | | | 1 | |
| 17 | | | | | | | | | | |
| 18 | | | | | | | | | | |
| 19 | | | | | | | | | | |
| 20 | | | | | | | | | | |
| 21 | | | | | | | | | 1 | |
| 22 | | | | | | | | | 1 | |
| 23 | | | | | | | | | | |
| 24 | | | | | 1 | | | | | |
| 25 | | | | | | | | | | |
| 26 | | | | | | | | | 1 | |
| 27 | | | | | | | | | | |
| 28 | | | | | | | | | | |
| 29 | | | | | | | | | | |
| 30 | | | | | | | | | | |
| Total number of events | 158 | 158 | 52 | 52 | 82 | 82 | 123 | 123 | 156 | 156 |
| $P(\chi^2)$ | 15% | | 50% | | 0.3% | | 20% | | 0.01% | |

Remark: A – data from all surface detectors for $N = 2-5 \times 10^6$, B – for $N = 5-10 \times 10^6$, C – data from detector $U_1$ for $N = 2-4 \times 10^6$, D – from detector $U_2$ for $N = 1-2 \times 10^6$, and E – from detector $U_2$ for $N = 2-4 \times 10^6$.

The calculation gives the distribution expected according to Eq. (1).

For groups of showers with a relatively small number of particles ($N < 4 \times 10^6$), the distributions with respect to the number of detected $\mu$ mesons q are shown in Table III. For shower groups with $N \geq 4 \times 10^6$, the distributions with respect to q/p are shown in Table II.

For the detector $U_2$, the distribution with respect to q/p for all detected showers ($N \geq 1 \times 10^6$) is shown below. This enables us to study the fluctuations of the $\mu$-meson flux in the range $q/p > 1$:

| $q/p$ | 0—1 | 1—2 | 2—3 | 3—4 | 4—5 | 5—6 | 6—7 | 7—8 | 8—9 |
|---|---|---|---|---|---|---|---|---|---|
| $I(q/p)$ | 240 | 107 | 40 | 11 | 5 | 2 | 1 | 0 | 1 |

In order to estimate the role of purely statistical fluctuations for each group of showers in Table III, we have constructed the distributions with respect to q expected because of statistical fluctuations and calculated according to the formula

$$W(q) = \sum_i w_i, \qquad w_i = p_i^q e^{-p_i} / q! \qquad (1)$$

(The summation is carried out over all showers of the given group.)

For the distributions in Table II, the statistical fluctuations of the values q/p were calculated from the formulae:

$$\sum_{q=0}^{p_i/3} W(q), \qquad (\text{for } q/p = 0 - {}^1/_3),$$

$$\sum_{q=p_i/3}^{2p_i/3} W(q) \quad (\text{for } q/p = {}^1/_3 - {}^2/_3), \text{ etc.} \qquad (2)$$

For underground detectors, the non-statistical fluctuations in the $\mu$-meson flux density may be due to the unknown true distances from the shower axis to the $\mu$-meson detectors, as mentioned above. For the distributions C (Table II), we have calculated the theoretical distribution I(q/p) taking both the spread of the true distances from the shower axis and statistical fluctuations (column 8) into account. From a comparison of columns 7 and 8 of Table II, it follows that the non-statistical fluctuations due to the deviation of the true distance of the shower axis to the underground $\mu$-meson detectors for varying angles of shower incidence are negligible.

A comparison of the experimental distributions with respect to q and q/p with the ones expected from Eqs. (1) and (2) were carried out using the $\chi^2$ test. The values of $P(\chi^2)$ are shown in Tables II and III.

It can be seen from these tables that the fluctuations observed by means of surface detectors can be fully explained by statistical fluctuations. However, the fluctuations in the $\mu$-meson flux observed by means of the underground detectors $U_1$ and $U_2$ are greater than purely statistical fluctuations if we consider sufficiently large showers.*

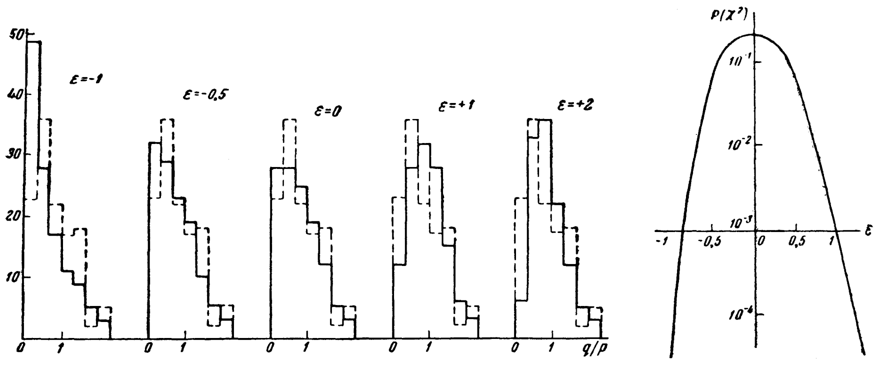
The non-statistical character of fluctuations in the latter case is confirmed by the correlation of the deviations from the mean values of the $\mu$-meson flux measured by detectors $U_1$ and $U_2$. In fact, let us consider the pair of values $x = q/p$ (for detector $U_1$) and $y = q/p$ (for detector $U_2$). If the deviations of x from unity are correlated with the deviations of y, then this means that non-statistical fluctuations in the number of $\mu$ mesons exist. However, for a small mean number of detected mesons, a large role is played by the Poisson fluctuations of q/p, which may fully mask the correlation of these values between $U_1$ and $U_2$. Therefore, in order to calculate the correlation coefficient between x and y, we have selected showers with a sufficiently large number of particles N and small distances R (ratio $N/R \geq 4 \times 10^5\ m^{-1}$). Table IV shows the distribution of the pair of values of x and y obtained. The events were grouped according to the intervals of q/p used in Table II. The

**Table IV**

| $U_1$ \ $U_2$ | $0-\frac{1}{3}$ | $\frac{1}{3}-\frac{2}{3}$ | $\frac{2}{3}-1$ | $1-\frac{4}{3}$ | $\frac{4}{3}-\frac{5}{3}$ | $\frac{5}{3}-2$ | $2-\frac{7}{3}$ | $\frac{7}{3}-\frac{8}{3}$ | $\frac{8}{3}-3$ | >3 |
|---|---|---|---|---|---|---|---|---|---|---|
| >3 | | | | | | 1 | | | 1 | 1 |
| $\frac{8}{3}-3$ | | | | | | | | | | 1 |
| $\frac{7}{3}-\frac{8}{3}$ | | 1 | | | 1 | | | | | |
| $2-\frac{7}{3}$ | 1 | | | 1 | | 1 | | | | 2 |
| $\frac{5}{3}-2$ | 1 | | 1 | 1 | 5 | 1 | | 1 | | |
| $\frac{4}{3}-\frac{5}{3}$ | 1 | 6 | 1 | 1 | 3 | 3 | | 1 | | 1 |
| $1-\frac{4}{3}$ | 2 | 5 | 6 | 2 | | 1 | 1 | | | |
| $\frac{2}{3}-1$ | 5 | 5 | 5 | 5 | 3 | | | | | |
| $\frac{1}{3}-\frac{2}{3}$ | 2 | 3 | 5 | 2 | 1 | | | | | |
| $0-\frac{1}{3}$ | 8 | 4 | 1 | 2 | 2 | | 1 | | | |

*The data of Tables II and III show only that, for a limited area of $\mu$-meson detectors, the study of fluctuations is possible if N is sufficiently large and R sufficiently small, i.e., the mean number of $\mu$ mesons incident on the detector area is large. In a contrary case, purely statistical fluctuations may mask the effect.

A comparison of the value of $P(\chi^2)$ in columns d and e of Table III shows that a change in the mean number of mesons incident upon the area of detector $U_2$ from 2.5 to 4.2 leads to a clear-cut manifestation of non-Poisson fluctuations.




a b

FIG. 4. a – theoretical distributions with respect to q/p calculated using Eq. (10) (solid histograms) and experimental distribution with respect to q/p from Table II (dotted histograms). The quantity q/p ranges from 0 to $2\frac{1}{3}$, $\Delta q/p = \frac{1}{3}$. b – probability $P(\chi^2)$ of different values of $\varepsilon$ calculated from the comparison of experimental and theoretical distributions.

correlation coefficient R calculated from the data of Table IV according to the formula

$$r = \left(\sum xy - n\bar{x}\bar{y}\right)\left(\sum x^2 - n\bar{x}^2\right)^{-1/2}\left(\sum y^2 - n\bar{y}^2\right)^{-1/2}$$

where n is the number of (x, y) pairs taken into consideration amounts to $0.60 \pm 0.15$.

The probability that the resultant value of r (or rather of its lower limit, $r = 0.45$) occurs when there is no correlation between the quantities x and y amounts to 0.1% (the total number of pairs of the measured values being $n = 109$).[7] Consequently, the value $r = 0.60 \pm 0.15$ means that a correlation exists between the quantities x and y due to non-statistical fluctuations in the $\mu$-meson flux.

The set of the distributions with respect to q/p gives the total picture of fluctuations in the $\mu$-meson flux with energy $E_\mu > 10^{10}$ ev (detector $U_2$). In the range $q/p > 1$, we can represent the dependence $I(q/p)$ as a power law $I(q/p) \sim (q/p)^{-m}$ where $m > 3.0$.

Table II shows the variation of $I(q/p)$ in the range $q/p < 1$. In Fig. 4, the data of column 6 of Table II are presented together with the theoretically expected fluctuations discussed below.

## DISCUSSION OF RESULTS

Let us consider some methodological problems arising in connection with the experiment.

1. The $\mu$-meson flux is observed over a relatively small area, since a small fraction of the total number of $\mu$ mesons in the showers is detected ($\sim 0.1\%$). Moreover, for the case of the detectors $U_1$ and $U_2$, the detection of $\mu$ mesons occurs at one point, and therefore the fluctuations of the detected number of $\mu$ mesons may refer both to fluctuations of the total flux of $\mu$ mesons and to the fluctuations in the lateral distribution function of the $\mu$ mesons.

In the case of surface detectors, the detection takes place at several points at different distances from the axis, which enables us in principle to distinguish the fluctuations in the total $\mu$-meson flux from the fluctuations in their lateral distribution function. However, the area of the detectors is relatively small, and chance variations in the number of detected $\mu$ mesons play a large role.

Therefore, in order to compare the experiment with theory, it is necessary to take into account the theoretical prediction both with respect to the fluctuations of the total $\mu$-meson flux and with respect to the fluctuations of the $\mu$-meson lateral distribution function.

2. A number of errors of random nature (random errors in the determination of the shower size, the spread of distances of the $\mu$-meson detectors from the shower axis due to angular distribution of the shower axis, especially for detectors $U_1$ and $U_2$, etc.) lead to an increase in the observed fluctuations of the $\mu$-meson flux. Therefore, the observed fluctuations represent an upper limit for possible fluctuations of the $\mu$-meson flux.

3. A decrease in the observed fluctuations of the $\mu$-meson flux may be caused by a systematic selection of showers with a given type of the electron lateral distribution function, if the latter depends on the $\mu$-meson flux. Experimentally, the showers are selected by the triggering arrangement and by means of an additional requirement of a given electron density at two points 60 m apart. As calculations carried out by one of the authors (V. I. Solov'eva) have shown, such a selection leads to a considerable decrease in the selection efficiency only for showers with a lateral distribution function corresponding to electron-photon showers with age $s < 1.0$. At present, there is a lack of sufficient experimental data on the age distribution of showers. According to preliminary results obtained by the Tokyo group,* there are no sharp deviations from the average lateral distribution function for $s = 1.3$, at least at distances

*Fukui, Hasegawa, Matano, Miura, Oda, Ogita, Suga, Tanahashi, and Tanaka (private communication).

greater than 30 m from the shower axis. In such a case, a selection of showers according to the electron density should not influence the fluctuations in the $\mu$-meson flux.

Taking the methodological problems mentioned above into consideration, we shall compare the results obtained with the theoretical predictions.

1. Comparison of the results with the model of EAS developing the atmosphere without fluctuations is simplest when the fluctuations of the $\mu$-meson flux in the showers with a given number of particles are only due to the fluctuations in the altitude of the first interaction of the primary particles.[3] In such a case, the shower development in the atmosphere is described by a cascade curve with a maximum, whose depth depends logarithmically ($x_m = B \ln E_0 + \text{const}$) on the energy of the primary particle. The absorption of the shower particles beyond the maximum is exponential with a mean free path $\Lambda = 200$ g/cm$^2$. The $\mu$-meson flux $n_\mu$ with $E_\mu \geq 10^{10}$ ev is not absorbed beyond the shower maximum, while the total number of $\mu$ mesons is proportional to the primary energy. In such a model, it is assumed[3] that the lateral distribution functions of all shower particles and of $\mu$ mesons do not fluctuate, and our experimental results therefore permit an unambiguous interpretation.

The number of particles in showers whose axes are inclined to the vertical by an angle $\theta$ at the observation level $x_0$ depends on the depth of the first interaction, as shown by

$$N = c_1 N_m E_0^{B/\Lambda} \exp\,[-(x_0 - x)\sec\theta/\Lambda], \tag{3}$$

where $N_m = c_2 E_0$.

The number of primary particles with energy $E_0$ interacting at the depth $x$ equals

$$I(E_0, x)\, dE_0\, dx = c_3 E_0^{-\gamma-1} \exp\,[-x\sec\theta/\lambda]\sec\theta\, dE_0\, dx/\lambda, \tag{4}$$

where $\lambda$ is the interaction mean free path of primary particles, and $\gamma$ is the exponent of their energy spectrum.

Substituting $x$ for $N$ according to Eq. (3), we obtain

$$I(E_0, N)\, dE_0\, dN = c_4 E_0^{(\Lambda+B)/\lambda-\gamma-1} N^{-(1+\Lambda/\lambda)} \exp(-x_0\sec\theta/\lambda)\, dE_0\, dN. \tag{5}$$

Integrating over $E_0$ from $E_{0m} = N/c_2$ to $E_{0\,max}^{1+B/\Lambda} = N\exp(x_0\sec\theta/\Lambda)/c_1c_2$, we obtain the angular distribution and the altitude dependence of showers with a given number of particles, which should satisfy the experimental data (see, e.g., [5,8]).

Obviously, the result depends essentially on the quantity $\epsilon = (\Lambda+B)/\lambda - \gamma - 1$. If $(\Lambda+B)/\lambda < \gamma$, then the angular distribution has the form

$$\exp\,[-\gamma x_0 \sec\theta/(\Lambda + B)],$$

If $(\Lambda+B)/\lambda > \gamma$, then $\exp(-x_0\sec\theta/\lambda)$. The experimentally observed variation is of the form

$$\exp(-x_0\sec\theta/(100-120)).$$

It is found that the distribution of showers with respect to the number of $\mu$ mesons $n_\mu$ is very sensitive to $\epsilon$, and, from a comparison of experimental data with the model under consideration, we can determine this quantity and, knowing $\Lambda$ and $\gamma$, also find the value of $\lambda$.

The experimental data were obtained with an array which detected EAS with all angles of incidence $\theta$, and we therefore integrate Eq. (5) over $\theta$ assuming an isotropic distribution of the primary particles. Two regions of integration are important, $E_0 < E_{01}$ and $E_0 > E_{01}$, where

$$E_{01}^{1+B/\Lambda} = c_1^{-1}c_2^{-1}N\exp(x_0/\Lambda)$$

is the maximum possible energy for a vertical shower with a given number of particles.

For $E_0 < E_{01}$ we have

$$I(E_0, N)\, dE_0\, dN = c_4 E_0^{\epsilon} N^{-(1+\Lambda/\lambda)} \times \left[\lambda \exp\left(-\frac{x_0}{\lambda}\right)\Big/ x_0 - \int\limits_{x_0/\lambda}^{\infty} \frac{e^{-t}}{t}\,dt\right] dE_0 dN. \tag{6}$$

At sea level, $x_0/\lambda$ is large ($> 10$), and therefore

$$\lambda/x_0 \exp(x_0/\lambda) - \int\limits_{x_0/\lambda}^{\infty} \frac{e^{-t}}{t}\,dt \sim c_5 \exp(-x_0/\lambda), \tag{7}$$

where $c_5$ is independent of $x_0/\lambda$.

For $E_0 > E_{01}$ we have

$$I(E_0, N)\, dE_0\, dN = c_4 E_0^{\epsilon} N^{-(1+\Lambda/\lambda)} \times \int\limits_0^{(\cos\theta)_1} \exp(-x_0\sec\theta/\lambda)\, d\cos\theta\, dE_0\, dN,$$

where

$$(\cos\theta)_1 = [1 + (\Lambda + B)\, x_0^{-1} \ln(E_0/E_{01})]^{-1}.$$

We therefore obtain the expression

$$I(E_0, N)\, dE_0\, dN = c_4 c_5 E_0^{\epsilon} N^{-(1+\Lambda/\lambda)} \times \exp\{-x_0\lambda^{-1}[1 + (\Lambda + B)\, x_0^{-1}\ln(E_0/E_{01})]\}\, dE_0\, dN = c_6 E_0^{-\gamma-1} N^{-(1+\Lambda/\lambda)}\, dE_0\, dN. \tag{8}$$

Substituting in Eqs. (8) and (6) $n_\mu$ instead of $E_0$, we obtain the distribution of events according to the number of $\mu$ mesons of the form

$$I(n_\mu)\,dn_\mu = \begin{cases} n_\mu^{\varepsilon}\,dn_\mu & n_\mu < n_{\mu\,min}^{\Lambda/(\Lambda+B)} \exp\,[x_0/(\Lambda+B)] \\ n_\mu^{-\gamma-1}\,dn_\mu & n_\mu > n_{\mu\,min}^{\Lambda/(\Lambda+B)} \exp\,[x_0/(\Lambda+B)]. \end{cases} \quad (9)$$

From the distribution (9), we can obtain the theoretically expected distributions shown in Table II. For this, it is necessary for each shower with a number of particles N and falling at a distance R from the $\mu$-meson detectors, to consider its spectrum $I(p', p)\,dp'$ of the $\mu$-meson numbers $p'$ detected by the detectors, which has the form (9) and which, in addition, satisfies the condition

$$\int_{p'_{min}}^{\infty} I(p', p)\,dp' = 1, \qquad \int_{p'_{min}}^{\infty} p' I(p', p)\,dp' = p,$$

where p is the average number of $\mu$ mesons obtained for showers with a given N and R. We then obtain the theoretically expected distribution $I_T(q/p)$ analogously to formula (2):

$$\sum_i \sum_{q=0}^{p_i/3} \int_{p'_{min}}^{\infty} p'^q e^{-p'} I(p', p)\,dp'/q! \qquad \text{for} \quad q/p = 0 - {}^1/_3,$$

$$\sum_i \sum_{q=p_i/3}^{2p_i/3} \int_{p'_{min}}^{\infty} p'^q e^{-p'} I(p', p)\,dp'/q! \qquad \text{for} \quad q/p = {}^1/_3 - {}^2/_3. \quad (10)$$

The distributions $I_T(q/p)$ for the detector $U_2$ calculated according to Eq. (10) are shown in Fig. 4. The separate distributions (solid histograms a — e) are calculated for different values of $\epsilon$.

The comparison with the experimentally observed distributions $I(q/p)$ (the dotted histograms in Fig. 4) for the detector $U_2$ taken from Table II were carried out using the $\chi^2$ test. Figure 4b shows the probabilities of agreement $P(\chi^2)$ between the experimentally and the theoretically expected distributions for different $\epsilon$. The values of $\epsilon$ from $-0.5$ to $+0.5$ occur with a probability greater than 10%, which, for $\Lambda = 200$ g/cm$^2$, $B = 30$ g/cm$^2$, and $\gamma = 2$ corresponds to values of $\lambda$ from 92 to 66 g/cm$^2$. As has been mentioned above, methodological errors of the experiment lead to an increase in the observed fluctuations, so that, from the comparison of the experimental and theoretical distributions, it follows only that $\epsilon > -0.5$ or $\lambda < 92$ g/cm$^2$.

2. In recent years, several models of EAS development were proposed in which an essential role is played by the fluctuations of the nuclear-interaction characteristics[1] and the fluctuations of the height of the nuclear interaction which determines the number of particles in the shower.[2] We shall consider the model of EAS development proposed by Cranshaw and Hillas.[2] This model assumes that electron-photon showers in EAS have a small range, so that the number of particles in the shower at observation level is determined only by the last interactions of nuclear-active particles in the shower core. The altitude of the last interaction may vary, which leads to variations in the age of observed showers and in the lateral distribution function of shower particles. The number of high-energy $\mu$ mesons is proportional to the energy lost by the primary particle up to the observation level.

Selecting the showers in our experiment in the manner described above, we choose showers with a lateral distribution function corresponding to $s > 1.0$. (We assume that, according to the model under consideration, there are fluctuations in the lateral distribution function of the shower.) The detection of a given number of particles in a shower then shows that $E_{n.a.} > E_c$, and that the depth of the last interaction $x_0 < x_c$. (The values of $E_c$ and $x_c$ correspond to the case where the shower age parameter $s = 1$.) Because of the falling spectrum of nuclear-active particles, the main contribution to the number of detected showers will be due to particles with energy $E_{n.a.} \sim E_c$ and $x_0 \sim x_c$, and we can therefore assume that the given number of particles in the shower determines $E_{n.a.}$ and $x_0$. The $\mu$-meson flux, however, may fluctuate, since the given value of $E_{n.a.}$ at the level $x_0$ can occur for different energies of the primary particle, which undergoes a different number of interactions along its path.

If $\beta$ is the energy fraction conserved in each interaction by the primary particle, then $E_0 = E_{n.a.}\beta^{-i}$, where i is the number of interactions. The probability that i interactions occur is

$$W_i = \left(\frac{x_0}{\lambda}\sec\theta\right)^i \exp\left(-\frac{x_0}{\lambda}\sec\theta\right) \Big/ i!$$

and the number of primary particles which are responsible for the appearance of a particle with energy $E_{n.a.}$ at the level $x_0$ is given by the equation

$$I(E_{n.a.})\,dE_{n.a.} = c'\,\frac{dE_{n.a.}}{E_{n.a.}^{\gamma+1}}\,\beta^{i\gamma} \exp\left(-\frac{x_0}{\lambda}\sec\theta\right) \frac{1}{i!}\left(\frac{x_0}{\lambda}\sec\theta\right)^i, \quad (11)$$

where $c'$ is the absolute coefficient in the energy spectrum of primary particles.

Carrying out a summation over all values of i, we obtain the angular distribution and altitude dependence of showers with fixed N and age

$$I(E_{n.a.})\,dE_{n.a.} = c'\,\frac{dE_{n.a.}}{E_{n.a.}^{\gamma+1}} \exp\left[-\frac{x_0}{\lambda}\sec\theta\,(1-\beta^\gamma)\right]. \quad (12)$$

In order to explain the observed angular distribution of showers with a number of particles in the range $10^5 - 10^7$, Cranshaw and Hillas [2] assumed $\lambda = 75$ g/cm$^2$ and $\beta = 0.5$.

Assuming these values of $\lambda$ and $\beta$, we can obtain the distribution with respect to i from Eq. (11). Since the experimental array detects showers with all angles of incidence $\theta$, we integrate (11) over $\theta$ assuming an isotropic angular distribution of the primary particles. The results of the integrations are given below. W is the probability that i collisions of the primary particles occur at any angle $\theta$. The corresponding values of the $\mu$-meson flux $n_\mu$ are also given:

| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $W$ | 0.12 | 0.16 | 0.21 | 0.17 | 0.13 | 0.075 | 0.046 | 0.018 | 0.0074 | 0.003 | $9.3 \cdot 10^{-4}$ | $3.2 \cdot 10^{-4}$ |
| $n_\mu/n_{\mu min}$ | 1 | 3 | 7 | 15 | 31 | 63 | 127 | 255 | 511 | 1023 | 2047 | 4095 |

In fact, it follows from our assumptions that the $\mu$-meson flux $n_\mu$ is proportional to the energy lost by the primary particle, i.e.,

$$\begin{aligned} n_\mu &\sim (1-\beta) E_0 (1+\beta+\ldots+\beta^i) \\ &= (1-\beta) E_{\text{n.a.}} (\beta^{-i}+\ldots+1) \\ &= (1-\beta) E_{\text{n.a.}} (\beta^{-i}-1) \end{aligned} \tag{13}$$

and $n_\mu/n_{\mu \min} = \beta^{-i} - 1$.

From this relation between $n_\mu$ and i, we obtain the distribution with respect to $n_\mu$. The mean value of the $\mu$-meson flux is found to be $\bar{n}_\mu/n_{\mu \min} = 36$, and the corresponding number of interactions $\approx 5$.

Let us consider the number of showers in which $n_\mu < \bar{n}_\mu$ and $n_\mu > \bar{n}_\mu$ (using the nomenclature of the previous section, $q < p$ and $q > p$). From the distribution with respect to $n_\mu/n_{\mu \min}$ given above, it follows that $I(q < p)/I(q > p) = 5.6$. The experimental distribution from Table II, column 6 gives $I(q < p)/I(q > p) = 1.75 \pm 0.2$. Consequently, the model under consideration predicts a large number of showers with a relatively small $\mu$-meson flux, which contradicts the experimental data.

We will show that the fluctuations in the lateral distribution of $\mu$ mesons cannot, in the model under consideration, lead to the observed difference between the theoretical and experimental distributions $I(q/p)$. In order to explain this difference, it is necessary to assume that the lateral distribution of the $\mu$-meson flux for $i < 5$ is steeper than for $i > 5$. A small number of $\mu$ mesons for $i < 5$ will then be compensated by the large concentration of $\mu$ mesons and, vice-versa, a large number of $\mu$ mesons for $i > 5$ will be compensated by the small concentration of the $\mu$ mesons since the $\mu$-meson density will undergo small fluctuations with respect to the mean value.

However, the variation in the lateral distribution of $\mu$ mesons would mean a change in the mean altitude of $\mu$-meson production. Since the main contribution to the number of $\mu$ mesons is due to the interactions of the primary particle occuring before the last interaction, which determines the number of particles in the shower [see Eq. (13)], and the altitude distribution of these events is independent of the number i, the mean altitude of $\mu$-meson production should not depend on the number of interactions i. This means that the mean lateral distribution of the $\mu$-meson flux should be independent of i, and therefore the fluctuations in the lateral distribution of $\mu$-meson flux cannot lead to the difference between the theoretical and experimental distributions $I(q/p)$.

## CONCLUSIONS

1. Experimentally observed small fluctuations in the $\mu$-meson flux in showers with a given number of particles contradict the model of shower development proposed by Cranshaw and Hillas. [2]

2. The fact that the observed fluctuations in the $\mu$-meson flux are not greater than the theoretically predicted fluctuations due only to the altitude fluctuations in EAS indicates a small role of fluctuations in the development of EAS with a large number of particles ($N > 10^6$).

Calculations carried out by Fukui et al [3] and in the present article show that, if EAS develop without fluctuations, the distribution with respect to the $\mu$-meson number $n_\mu$ in a shower with a given number of particles is very sensitive to the quantity $\epsilon = (\Lambda + B)/\lambda - \gamma - 1$. The values of $\Lambda$ and $\gamma$ are well-known, and therefore the magnitude $\lambda$ of the interaction mean free path of primary particles follows from the exact shape of the distribution with respect to $n_\mu$. Thus, the study of the exact form of the distribution of $\mu$ mesons in EAS with a known number of particles enables us to determine the interaction mean free path of primary ultra-high energy particles producing the EAS. Furthermore, in order to obtain the exact distribution of the $\mu$-meson flux, it is necessary to increase the accuracy of the experimental method. This involves the use of large-area $\mu$-meson detectors located at several points, the distance between which should be sufficiently large in order to determine the role of fluctuations in the $\mu$-meson lateral distribution function; and also the exact determination of the distance between the $\mu$-meson detectors and the shower axis, etc.

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