TRANSITION RADIATION IN A PLASMA WITH ACCOUNT OF TEMPERATURE

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The transition radiation that arises when a charged particle passes through the boundary of a plasma (with dielectric) described by linearized hydrodynamic equations is investigated with account of the effect of the temperature.

GINZBURG and Frank[1] (see also [2,3]) have discovered that radiation is produced when a charged particle passes through the interface between two media with different refractive indices. The formulas were derived in these papers without account of spatial dispersion of the media.

In the case of a plasma, account of the temperature gives rise to spatial dispersion. We consider below the transition radiation in a plasma with account of the temperature. The ions are assumed to be stationary, so that their effect reduces to neutralization of the equilibrium electron density.

Let a particle with velocity $\mathbf{v}$ and charge q move along the z axis and cross at the instant t = 0 the interface between a certain medium 1, having a dielectric constant $\epsilon(\omega)$, and a plasma (medium 2). The plasma fills the half-space $z < 0$.

The electromagnetic field excited in the plasma by the moving particle is described by the Maxwell equations

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}, \qquad \operatorname{div}\mathbf{E} = 4\pi\left[e\frac{\rho}{m} + q\delta(\mathbf{r}-\mathbf{v}t)\right],$$

$$\operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} + 4\pi\left[e\frac{\rho_0}{m}\mathbf{u} + q\mathbf{v}\delta(\mathbf{r}-\mathbf{v}t)\right], \qquad (1)^*$$

where $\mathbf{u}$ is the velocity of motion of the plasma, m is the electron mass, and ρ is the deviation of the electron density from the equilibrium value ρ_0. The values of $\mathbf{u}$ and ρ are determined by the following system of linearized hydrodynamic equations

$$\frac{\partial\rho}{\partial t} + \rho_0 \operatorname{div}\mathbf{u} = 0, \qquad \rho_0\frac{\partial\mathbf{u}}{\partial t} + \nabla p - \frac{e}{m}\rho_0\mathbf{E} = 0,$$

$$\nabla p = v_0^2\nabla\rho, \qquad v_0^2 = T/m, \qquad (2)$$

where T is the plasma temperature. The components of the fields in medium 1 are determined from the equations

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t}, \qquad \operatorname{div}\mathbf{D} = 4\pi q\delta(\mathbf{r}-\mathbf{v}t),$$

$$\operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{D}}{\partial t} + 4\pi q\mathbf{v}\delta(\mathbf{r}-\mathbf{v}t),$$

$$\mathbf{D}(\mathbf{r}, t) = \int_{-\infty}^{+\infty}\varepsilon(\omega)\,\mathbf{E}(\omega, \mathbf{r})\,e^{i\omega t}\,d\omega, \qquad (3)$$

and the hydrodynamic equations differ from (2) in that the term with $\mathbf{E}$ is missing. In addition to the continuity of the tangential components of $\mathbf{E}$ and $\mathbf{H}$, the conditions satisfied on the interface are equality of the pressures ($p_1 = p_2$) and equality of the normal velocity components ($u_{1z} = u_{2z}$).

Choosing as solutions waves outgoing from the origin, and using the formulated boundary conditions, we obtain for the fields in the plasma

$$H_1(r, z, t) = \int H_1 e^{i(\omega t - \varkappa_2 z)} J_1(\lambda r)\,\lambda\,d\lambda d\omega,$$

$$H_1 = \frac{q\lambda}{\pi c}\,\frac{\left\{\varepsilon_2 f_1^{-2}(\varkappa_1 \pm \omega/v) - f_2^{-2}\left[\varepsilon_1\gamma(k_2^2 - \omega^2/v^2) \pm \varepsilon_1\omega/v + \varepsilon_2\varkappa_1\right] \pm \dfrac{\gamma\varepsilon_1\omega/v}{\sqrt{k_{02}^2 - \lambda^2} \pm \omega/v}\right\}}{\varepsilon_2\varkappa_1 - \varepsilon_1\varkappa_2 + \varepsilon_1\gamma\lambda^2}, \qquad (4)$$

where $J_1(\lambda r)$ are Bessel functions of first order; the plus sign is taken for an incoming particle and the minus sign for a particle outgoing from the plasma.

We introduce here the following notation

$$\varepsilon_2 = 1 - (\omega_0/\omega)^2, \quad \omega_0^2 = 4\pi e^2\rho_{02}/m^2, \quad k_{1,2}^2 = \omega^2\varepsilon_{1,2}/c^2,$$

$$k_{02}^2 = \omega^2\varepsilon_2/v_{02}^2,$$

$$\varkappa_{1,2}^2 = k_{1,2}^2 - \lambda^2, \quad \alpha_1^2 = (\omega/v_{01})^2 - \lambda^2, \quad \alpha_2^2 = \omega^2\varepsilon_2/v_{02}^2 - \lambda^2,$$

$$f_{1,2}^2 = \omega^2/v^2 + \lambda^2 - k_{1,2}^2, \qquad \gamma = (1-\varepsilon_2)/(\varepsilon_2\alpha_1\rho_{02}/\rho_{01} - \alpha_2).$$

*rot = curl.

Let us change to spherical coordinates by means of the relations $r = R\sin\theta$ and $z = -R\cos\theta$ (where R is the distance from the origin to the point of observation). For large values of R the integral (4) is evaluated by the method of steepest descent.[2] Carrying out this integration and calculating the energy flux in a solid angle $d\Omega = \sin\theta\, d\theta\, d\varphi$ over the entire transit time of the particle, we obtain the radiated energy for a charged particle traveling into the plasma:

$$\frac{dW}{d\Omega} = \frac{q^2v^2}{\pi^2c^3}\sin^2\theta\cos^2\theta\int_0^\infty \frac{\sqrt{\varepsilon_2}\,|A|^2\,d\omega}{|\varepsilon_1\cos\theta + \sqrt{\varepsilon_2(\varepsilon_1-\varepsilon_2\sin^2\theta)}|^2},$$

$$A = \frac{(\varepsilon_1-\varepsilon_2)(1-\varepsilon_2\beta^2-\beta\sqrt{\varepsilon_1-\varepsilon_2\sin^2\theta})}{(1-\beta^2\varepsilon_2\cos^2\theta)(1-\beta\sqrt{\varepsilon_1-\varepsilon_2\sin^2\theta})}$$

$$+\frac{v_{02}\varepsilon_1(1-\varepsilon_2)}{v\sqrt{\varepsilon_2}\,(1+\sqrt{\varepsilon_2\rho_{02}/\rho_{01}})}$$

$$\times\left[\frac{1}{1+(v/v_{02})\sqrt{\varepsilon_2(1-v_{02}^2\sin^2\theta/c^2)}} - \frac{1-\varepsilon_2\beta^2}{1-\varepsilon_2\beta^2\cos^2\theta}\right]. \quad (5)$$

This formula describes the total energy radiated from the plasma, including the Cerenkov radiation generated in medium 1.[3] When $T = 0$ this equation is equivalent to the corresponding expression (38) of the paper of Ginzburg and Frank[1] [with v replaced by $-v$ in Eq. (38)].

It follows from (5) that the temperature plays a significant role if the velocity of the charged particle exceeds the average thermal velocity of the plasma electrons. When $v/v_{02} \ll 1$, the transition radiation in the plasma is independent of the temperature. In analogy with the interaction between a particle and an unbounded plasma, we can assume that the intensity is independent of the temperature because the hydrodynamic approximation is used to solve the problem (see [4]). Apparently, a temperature dependence would obtain in the kinetic approximation.

If the particle moves in the opposite direction (from the plasma into medium 1), the formula for the radiation is obtained from (5) by substituting $-v$ for v. The divergence arising when

$$1-\frac{v}{v_{02}}\sqrt{\varepsilon_2\left(1-\frac{v_{02}^2}{c^2}\sin^2\theta\right)} = 0 \quad (6)$$

is due here to the occurrence in the plasma of a cylindrical Cerenkov wave obtained from (4) (see [2]):

$$H_2 = \frac{q}{c\sqrt{2\pi vR\sin\theta}}\int_{-\infty}^{\infty} C\exp\left\{i\left[\omega t - \frac{\omega}{v}R\left(\sqrt{1-\varepsilon_2\left(\frac{v^2}{v_{02}^2}-\beta^2\right)}\cos\theta + \sin\theta\sqrt{\frac{v^2}{v_{02}^2}\varepsilon_2-1}\right)\right]\right\}d\omega,$$

$$C = \frac{\sqrt{i\omega}\,(1-\varepsilon_2)\,(v^2\varepsilon_2/v_{02}^2-1)^{1/4}}{\left[\frac{\varepsilon_2}{\varepsilon_1}\sqrt{1-\frac{v^2}{v_{02}^2}\varepsilon_2+\beta^2\varepsilon_1}+\sqrt{1-\varepsilon_2\left(\frac{v^2}{v_{02}^2}-1\right)}\right]}\;\frac{\sqrt{i\omega}\,(1-\varepsilon_2)\,(v^2\varepsilon_2/v_{02}^2-1)^{1/4}}{\left[1+\frac{v}{v_{02}}\varepsilon_2\sqrt{\frac{\rho_{02}}{\rho_{01}}\left(1+\varepsilon_2-\frac{v_{02}^2}{v^2}\right)}\right]+\frac{v^2}{v_{02}^2}\varepsilon_2(1-\varepsilon_2)}.$$

The angle determined by relation (6) characterizes the cone of the Cerenkov waves reflected from the interface.

Thus, for a particle with velocity greater than the mean thermal velocity of the plasma electrons, we obtain for the transition-radiation energy

$$\frac{dW_{\text{tr}}}{d\Omega} = \frac{q^2v^2}{\pi^2c^3}\sin^2\theta\cos^2\theta\int_0^\infty \frac{\sqrt{\varepsilon_2}\,|A|^2\,d\omega}{|\varepsilon_1\cos\theta+\sqrt{\varepsilon_2}\sqrt{\varepsilon_1-\varepsilon_2\sin^2\theta}\,|^2},$$

$$A = \frac{(\varepsilon_1-\varepsilon_2)(1-\varepsilon_2\beta^2+\beta\sqrt{\varepsilon_1-\varepsilon_2\sin^2\theta})}{(1-\beta^2\varepsilon_2\cos^2\theta)(1+\beta\sqrt{\varepsilon_1-\varepsilon_2\sin^2\theta})}$$

$$+\frac{v_{02}\varepsilon_1(1-\varepsilon_2)(1-\beta^2\varepsilon_2)}{v\sqrt{\varepsilon_2}\,(1+\sqrt{\varepsilon_2\rho_{02}/\rho_{01}})(1-\beta^2\varepsilon_2\cos^2\theta)}. \quad (8)$$

For the Cerenkov radiation we obtain the energy flux through an annular area $r, r+dr$ over the entire transit time of the particle:

$$\frac{dW_{\text{Cer}}}{dr} = \frac{q^2}{v^2}\int_0^\infty \frac{(1-\varepsilon_2)^2\,\{(v^2\varepsilon_2/v_{02}^2-1)\,[1-\varepsilon_2(v^2/v_{02}^2-v^2/c^2)]\}^{1/2}\cdot\omega\,d\omega}{\varepsilon_2\,|S|^2},$$

$$S = \left[\frac{\varepsilon_2}{\varepsilon_1}\sqrt{1-\frac{v^2}{v_{02}^2}\varepsilon_2+\beta^2\varepsilon_1}+\sqrt{1-\varepsilon_2\left(\frac{v^2}{v_{02}^2}-1\right)}\right]$$

$$\times\left[1+\frac{v}{v_{02}}\varepsilon_2\sqrt{\frac{\rho_{02}}{\rho_{01}}\left(1+\varepsilon_2-\frac{v_{02}^2}{v^2}\right)}\right]+\frac{v^2}{v_{02}^2}\varepsilon_2(1-\varepsilon_2). \quad (9)$$

Here, as in formula (7), the integration extends over the frequencies

$$\omega_0\left(1-\frac{v_{02}^2}{v}\right)^{-1/2} \leqslant \omega \leqslant \omega_0\frac{\left[\left(1-\frac{v_{02}^2}{c^2}\sin^2\theta\right)\right.}{\left.\left(1-\frac{v_{02}^2}{v^2}-\frac{v_{02}^2}{c^2}\sin^2\theta\right)\right]^{1/2}}.$$

Let us mention also the transition radiation produced when a particle crosses the interface between a plasma and a dielectric, on which interface the normal component of the plasma velocity vanishes, $u_{2Z} = 0$. The boundary conditions for the fields **E** and **H** remain the same as before. All the results are obtained for this case from formulas (5) — (9) by putting $\rho_{01} = \infty$. In order to obtain the corresponding expressions for the vacuum-plasma case it is sufficient to put $\epsilon_1 = 1$ and $\rho_{01} = \infty$ in (5) — (9). It is assumed here, naturally, that the interface is a solid wall.

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[1] V. L. Ginzburg and I. M. Frank, JETP **16**, 15 (1945).

[2] G. M. Garibyan, JETP **33**, 1403 (1957), Soviet Phys. JETP **6**, 1079 (1958).

[3] V. E. Pafomov, JETP **36**, 1853 (1959), Soviet Phys. JETP **9**, 1321 (1960).

[4] A. I. Akhiezer and A. G. Sitenko, JETP **23**, 161 (1952).

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