ELECTROMAGNETIC INTERACTION OF A NEUTRAL VECTOR MESON

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The interaction between a neutral vector meson and a photon is considered. The two particles can transform into each other. In this connection the problem of diagonalization of the Green's function for the vector meson and photon is examined.

THE literature has been recently full of discussions on the existence of a hypothetical neutral vector meson ρ^0, endowed with strong interactions.[1-4] The interaction of the ρ^0 meson with the conserved current of strongly interacting particles j_s

$$\sqrt{4\pi}\, gB_\mu j_{s\mu}$$

(B_μ is the ρ^0 meson field, g is the coupling constant) resembles strongly the interaction of a photon with the current of charged particles j_e

$$\sqrt{4\pi}\, eA_\mu j_{e\mu}$$

(A_μ is the photon field, e is the electric charge). Since the spin and space and charge parities of the ρ^0 meson and photon are the same, the virtual transition $\rho^0 \rightarrow \gamma$ is possible since there exist strongly interacting charged particles, which contribute to both currents j_s and j_e.* From gauge invariance considerations it follows that the amplitude for such a transition will be of the form

$$P^{\rho\gamma}_{\mu\nu} = (\delta_{\mu\nu}k^2 - k_\mu k_\nu)\, P(k^2) \qquad (P(k^2) \sim ge),$$

where k is the momentum of the meson, μ, ν are polarization indices of the meson and photon. Since both j_s and j_e are conserved it follows that the second term in $P^{\rho\gamma}_{\mu\nu}$ never contributes and may be discarded. In what follows we shall take

$$P^{\rho\gamma}_{\mu\nu} = k^2\delta_{\mu\nu}P(k^2).$$

The simplest diagram for $P(k^2)$ is shown in Fig. 1. Using the conventional way of calculating, one obtains from such a diagram a quadratically divergent expression, which violates gauge invariance. One must therefore discard the quadratically divergent part of the integral, just as is done in calculating vacuum polarization in electrodynamics. (Let us note that the loop under consideration is exactly the same as the loop involved in photon vacuum polarization.) The remaining part of the integral contains a logarithmic divergence. In electrodynamics this divergence is absorbed into charge renormalization. In the present case we must introduce for the ρ-γ transition a special constant.

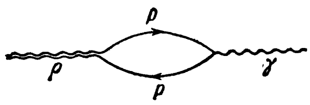

FIG. 1

The existence of the ρ-γ transition leads[1] to the appearance of a pole at $k^2 = m^2$ (where m is the mass of the meson) in the amplitude for p-e (n-e) scattering, as a result of the diagrams shown in Fig. 2. The corresponding matrix element is of the form

$$M \sim eg\frac{1}{k^2 - m^2}k^2P(k^2)\frac{1}{k^2} = \frac{egP(k^2)}{k^2 - m^2}.$$

Such a pole appears also in e-e scattering as a consequence of more complicated diagrams.

As a result of the ρ-γ transition it is possible to emit in electron collisions the ρ^0 meson with an amplitude, proportional to the corresponding vertex part (Fig. 3), which is given by

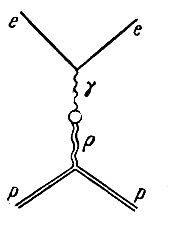

FIG. 2

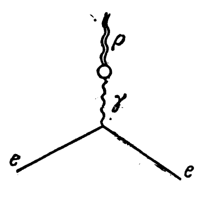

FIG. 3

*Within the framework of Sakata's model we may assume that

$$j_s = \bar{p}\gamma_\alpha p + \bar{n}\gamma_\alpha n + \bar{\Lambda}\gamma_\alpha\Lambda,$$
$$j_e = -\bar{p}\gamma_\alpha p + \bar{e}\gamma_\alpha e + \bar{\mu}\gamma_\alpha\mu$$

(in this connection see reference 2). The proton enters into both these currents.

$$ek^{-2}P(k^2)k^2|_{k^2=m^2} = eP(m^2).$$

We note that the amplitude for the ρ-γ transition chosen by us does not contribute to the photon vertex part at $k^2 = 0$, and therefore has no effect on the emission of real photons. If that were not the case Ward's identity in electrodynamics would be upset and the equality of the electromagnetic charges of the electron and the proton would no longer hold.[5]

We see that the presence of the ρ-γ transition leads in effect to the appearance of an interaction between the ρ^0 meson and the electric current, with a coupling constant equal to $eP(m^2) \sim e^2g$. We shall now carry out a more detailed discussion of this problem. We introduce the following D functions: for the ρ meson $D^{\rho\rho}$ (Fig. 4a), for the photon $D^{\gamma\gamma}$ (Fig. 4b), and also a new function $D^{\rho\gamma}$ for the diagram pictured in Fig. 4c. Let us note that the cross-hatched loops indicate the totality of diagrams (connected and disconnected).

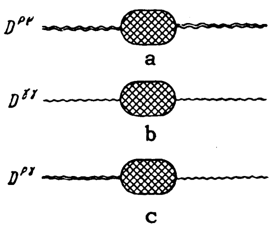

FIG. 4

Let us consider now electron-electron (e-e), electron-proton (e-p), electron-neutron (e-n), and proton-proton (p-p) scattering. The corresponding pole amplitudes are given by

$$M_{ne} = geD^{\rho\gamma}, \qquad M_{nn} = g^2D^{\rho\rho},$$
$$M_{pe} = geD^{\rho\gamma} + e^2D^{\gamma\gamma}, \qquad M_{pp} = g^2D^{\rho\rho} + 2geD^{\rho\gamma} + e^2D^{\gamma\gamma},$$
$$M_{ee} = e^2D^{\gamma\gamma} \tag{1}$$

For simplicity we shall ignore the diagonal vacuum polarizations $P^{\rho\rho}$ and $P^{\gamma\gamma}$ and take into account only the amplitude $P^{\rho\gamma}$, which describes the totality of connected diagrams for the ρ-γ transition. By summing chains of diagrams we obtain

$$D^{\gamma\gamma} = 1/k^2 + P^2(k^2) / (k^2 - m^2 - k^2P^2(k^2)),$$
$$D^{\rho\rho} = 1 / (k^2 - m^2 - k^2P^2(k^2)),$$
$$D^{\rho\gamma} = P(k^2) / (k^2 - m^2 - k^2P^2(k^2)). \tag{2}$$

We now introduce diagonalized propagation functions for the photon and ρ meson:

$$D_{diag}^{\gamma\gamma} = 1/k^2, \quad D_{diag}^{\rho\rho} = 1/(k^2 - m^2 - k^2P^2(k^2)).$$

It then follows from Eqs. (1) and (2) that

$$M_{ee} = e^2D_{diag}^{\gamma\gamma} + (eP)^2D_{diag}^{\rho\rho},$$
$$M_{nn} = g^2D_{diag}^{\rho\rho}, \quad M_{ne} = gePD_{diag}^{\rho\rho},$$
$$M_{pe} = (g + eP)(eP)D_{diag}^{\rho\rho} + e^2D_{diag}^{\gamma\gamma}. \tag{3}$$

The formulas (3) may be interpreted as follows: 1) the photon mass vanishes, as before; 2) the electric charge remains unchanged; 3) the pole of $D_{diag}^{\rho\rho}$ determines the renormalized mass M^2 of the ρ meson, where

$$M^2 - m^2 - M^2P^2(M^2) = 0 \text{ or } M^2 = m^2/(1 - P(M^2));$$

4) the renormalized charge for the interaction of the ρ meson with j_s is given by

$$g_{ren} = gZ^{1/2},$$

where

$$Z = [1 - P^2(M^2) - M^2\partial P^2/\partial k^2]^{-1}|_{k^2=M^2};$$

5) as a result of diagonalization an interaction of the ρ meson with the current j_e has been produced with the vertex part $P(k^2)$ and renormalized charge given by

$$h_{ren} = eP(M^2)Z^{1/2}.$$

Since $P(k^2)$ contains a logarithmic divergence h_{ren} cannot in fact be calculated but must be introduced as a new constant.

The diagonalization here performed can be illustrated by the following example. Consider the Lagrangian of the ρ and γ fields, containing the ρ-γ transition and the interaction with external currents J_s and J_e:

$$L = \tfrac{1}{2}B_\mu(k^2 - m^2)B_\mu$$
$$+ \tfrac{1}{2}A_\mu k^2A_\mu - A_\mu B_\mu \lambda k^2 - eJ_{e\mu}A_\mu - gJ_{s\mu}B_\mu$$

(A_μ and B_μ are respectively the photon and meson fields). Let us now introduce the renormalized (diagonal) fields*

$$A' = A - \lambda B, \quad B' = \sqrt{1 - \lambda^2}B;$$
$$A = A' - \lambda B'/\sqrt{1 - \lambda^2}, \quad B = B'/\sqrt{1 - \lambda^2}.$$

Then the Lagrangian may be written in the form

$$A'_\mu k^2A'_\mu + B'_\mu(k^2 - M^2)B'_\mu - g_{ren}B'_\mu J_{s\mu} - eA'_\mu J_{e\mu} - hJ_{e\mu}B'_\mu,$$

where

$$M^2 = m^2/(1 - \lambda^2), \quad g_{ren} = g/\sqrt{1 - \lambda^2},$$
$$h = \lambda e/\sqrt{1 - \lambda^2}.$$

It is seen that as a result of diagonalization the mass of the ρ meson and the strong interaction coupling constant g underwent renormalization,

*An analogous diagonalization procedure is contained in[6].

and the interaction of the ρ meson with the current J_e with the constant h appeared. The expressions obtained here for M^2, g_{ren} and h coincide with those obtained earlier in the case when $P(k^2) = \lambda = \text{const}$.

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Note added in proof (July 14, 1961). The ρ-γ transition is discussed to some extent in the paper by Huff [R. W. Huff, Phys. Rev. **112**, 1021 (1958)] with which we have become familiar after our work was sent to press.

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91