THEORY OF ELECTRIC CONDUCTIVITY OF SEMICONDUCTORS IN A MAGNETIC FIELD. II

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We obtain an expression for the diagonal element of the transverse conductivity tensor of a semiconductor in a strong magnetic field, taking the interaction between the conduction electrons with one another into account. We assume the conduction electron dispersion law to be isotropic and quadratic. The electron-phonon scattering is considered in the Born approximation. The general expression is studied for the case where the electrons satisfy Boltzmann statistics and where the electron-electron collisions are described also in the Born approximation. As an example of an application of this expression we evaluate the height of the resonance conductivity oscillations predicted in [1].

1. In a previous paper by the authors [1] (in the following referred to as I) we obtained an expression for the diagonal element of the transverse conductivity, σ_{xx}, of semiconductors in a strong magnetic field,* caused by the inelastic scattering of the conduction electrons by the lattice vibrations. We did not take then into account the electron-electron interaction. It is the aim of the present paper to take into account the influence of this interaction upon the conductivity for the case of an isotropic, quadratic electron spectrum.

One shows easily that because of the conservation of momentum during electron-electron collisions the quantity σ_{xx} is equal to zero for a system of mutually interacting electrons. It can be different from zero only owing to interaction with some other scatterers, for instance, phonons. In that case when there is a strong magnetic field present ($\sigma_{xx}/\sigma_{xy} \ll 1$), the role of the electron-electron interaction will only make itself felt in the renormalization of the electron-phonon scattering potential as far as the first non-vanishing approximation in the scattering—to which we restrict ourselves—is concerned.

To understand qualitatively the expected character of the effects we start with a phenomenological consideration, generalizing a method proposed by Doniach [2] to the case where there is a non-vanishing magnetic field H. Let a vibration with frequency ω and wave vector $\mathbf{q}$ propagate in an isotropic crystal. When we neglect the electron-electron interaction, the correction to the electron energy when this vibration is taken into account is of the form

$$U(\mathbf{r}, t) = u + u^* = c_{\mathbf{q}} b_{\mathbf{q}} e^{-i(\omega t - \mathbf{qr})} + c_{\mathbf{q}}^* b_{\mathbf{q}}^* e^{i(\omega t - \mathbf{qr})}. \quad (1)$$

The function $c_{\mathbf{q}}$ characterizes here the electron-phonon interaction (expressions for it for different cases are given, for instance, in I) and the dimensionless quantities $b_{\mathbf{q}}$ determine the vibrational amplitude. The extra term (1) in the energy causes the free charges in the crystal to be rearranged and this leads to a violation of local neutrality and hence to the appearance of an additional electrostatic potential which, taken together with (1), renormalizes the electron-phonon interaction. Since this effect is linear in the interaction we perform a renormalization of the first term in (1). The result for the second term is obtained by taking the complex conjugate of the first term.

Let the renormalized interaction be of the form

$$\tilde{u} = \tilde{u}_{\mathbf{q}}(\omega)\, e^{i(\mathbf{qr} - \omega t)}.$$

The change in the electron density caused by it is determined by solving the equation for the density matrix ρ:

$$\partial \rho / \partial t + (i/\hbar)\,[\rho, \mathcal{H}] = 0, \quad (2)$$

where $\mathcal{H} = \mathcal{H}_0 + \tilde{u}$, $\mathcal{H}_0$ is the single-electron Hamiltonian operator while the square brackets indicate a commutator. Putting $\rho = \rho^0 + \rho'$ (ρ^0 is the equilibrium density matrix of a system of non-interacting electrons), we find

$$\rho'_{\Gamma\Delta} = -\frac{\rho^0_\Gamma - \rho^0_\Delta}{\varepsilon_\Delta - \varepsilon_\Gamma - \hbar\omega + i\nu}\, \tilde{u}_{\mathbf{q}}(\omega) \langle \Gamma | e^{i\mathbf{qr}} | \Delta \rangle,$$

where Γ and Δ stand for the totality of quantum numbers characterizing the single-electron states when there is no perturbation, $\varepsilon_\Gamma = \langle \Gamma | \mathcal{H}_0 | \Gamma \rangle$, ν is the adiabatic parameter corresponding to the

*The notation is the same as in I.

assumption of an infinitely slow switching on of the electron-phonon interaction ($\nu > 0$, $\nu \to 0$).

In the coordinate representation (σ is the spin variable) we have

$$\rho'_{\sigma\sigma'}(\mathbf{r}, \mathbf{r}') = \sum_{\Gamma\Delta} \rho'_{\Gamma\Delta} \psi^*_{\Delta}(\mathbf{r}, \sigma)\, \psi_{\Gamma}(\mathbf{r}', \sigma'),$$

from which we get for the Fourier component of the electron density

$$n_{\mathbf{q}} = \frac{1}{V_0} \sum_{\sigma} \int d^3 r e^{-i\mathbf{q}\mathbf{r}} \rho_{\sigma\sigma}(\mathbf{r}, \mathbf{r}) = -\frac{1}{\hbar V_0} K^0_{\mathbf{q}}(-\omega)\, \tilde{u}_{\mathbf{q}}, \qquad (3)$$

where V_0 is the normalizing volume and

$$K^0_{\mathbf{q}}(\omega) = \hbar \sum_{\Gamma\Delta} \frac{\rho^0_{\Delta} - \rho^0_{\Gamma}}{\varepsilon_{\Gamma} - \varepsilon_{\Delta} + \hbar\omega - i\nu} |\langle \Gamma | e^{i\mathbf{q}\mathbf{r}} | \Delta \rangle|^2. \qquad (4)$$

However, as we noticed earlier, the renormalized potential $\tilde{u}$ is the sum of the "bare" potential u and the electrostatic potential $e\varphi$, which is determined from the Poisson equation

$$\varepsilon_0 q^2 \varphi_{\mathbf{q}} = 4\pi e n_{\mathbf{q}}, \qquad (5)$$

whence

$$\tilde{u}_{\mathbf{q}} = \left[1 + \frac{4\pi e^2}{\varepsilon_0 \hbar q^2 V_0} K^0_{\mathbf{q}}(-\omega)\right]^{-1} u_{\mathbf{q}}. \qquad (6)$$

We now establish the connection between the renormalization coefficient in front of $u_{\mathbf{q}}$ in (6) and the permittivity of the crystal. To do this we note that Eq. (6) is independent of the actual form of the interaction, and depends only on the properties of its space and time periodicity. In particular, one may assume that $\mathbf{D} = -\epsilon_0 \nabla u$ is the "bare" electrical field, i.e., the electrical induction vector. In that case $\mathbf{E} = -\nabla \tilde{u}$ is the total electrical field and the coefficient connecting them,

$$\varepsilon_{\parallel}(\omega, \mathbf{q}) = \varepsilon_0 + \frac{4\pi e^2}{\hbar q^2 V_0} K^0_{\mathbf{q}}(-\omega) \qquad (7)$$

is the longitudinal component of the permittivity tensor, taking time and space dispersion into account.

If we do not take electron-electron interaction into account, the probability for an electron-phonon scattering is proportional to $|c_{\mathbf{q}}|^2$. The present calculation shows that taking this interaction into account is equivalent to replacing the transition probability $|c_{\mathbf{q}}|^2$ by

$$|c_{\mathbf{q}}|^2 \left\{\left[1 + \frac{4\pi e^2}{\varepsilon_0 \hbar q^2 V_0} \operatorname{Re} K^0_{\mathbf{q}}(\omega)\right]^2 + \left[\frac{4\pi e^2}{\varepsilon_0 \hbar q^2 V_0} \operatorname{Im} K^0_{\mathbf{q}}(\omega)\right]^2\right\}^{-1} \equiv \frac{|c_{\mathbf{q}}|^2}{A^2 + B^2}. \qquad (8)$$

In the Boltzmann statistics case and when the conduction electron spectrum is quadratic and isotropic one can easily evaluate the function (4) and according to Larkin[3]

$$\operatorname{Re} K^0_{\mathbf{q}}(\omega) = 2\frac{nV_0}{qv_T}\left[W\left(\frac{\omega}{qv_T} + \frac{q}{2q_T}\right) - W\left(\frac{\omega}{qv_T} - \frac{q}{2q_T}\right)\right], \qquad (9)$$

$$\operatorname{Im} K^0_{\mathbf{q}}(\omega) = V_0 \frac{n\sqrt{\pi}\,(1 - e^{-\hbar\omega\beta})}{qv_T} \exp\left[\frac{\hbar\omega\beta}{2} - \left(\frac{\omega}{qv_T}\right)^2 - \left(\frac{q}{2q_T}\right)^2\right],$$

$$W(x) = e^{-x^2} \int_0^x e^{u^2} du \qquad (10)$$

(n is the electron concentration and $\beta = 1/kT$) when there is no magnetic field. The form of these functions and therefore also of the expressions for A^2 and B^2 in (8) depends on the relation between v_T and ω/q. When $\omega/q \gg v_T$

$$A = 1 - 4\pi e^2 n / \varepsilon_0 m\omega^2 = 1 - \omega_p^2 / \omega^2, \qquad (11)$$

B is exponentially small and expression (8) turns out to be larger than its renormalized value when ω is close to ω_p. This means physically that the electron-phonon interaction and thus also the scattering probability increases steeply when the phonon frequency approaches the frequency of the natural vibrations of the electron system, namely the plasma frequency ω_p.*

We note that the ratio ω/q is equal to the sound velocity w for acoustic phonons. But as a rule $v_T \gg w$ in the Boltzmann statistics region and it is thus practically impossible to excite plasma oscillations by sound if H = 0, since the inequality $\omega/q \gg v_T$ cannot be realized. When $\omega/q \ll v_T$

$$A = 1 + 4\pi n e^2 / \varepsilon_0 mkTq^2 = 1 + (\varkappa / q)^2$$

and the renormalized expression (6) turns out to be less than the unrenormalized expression by a factor $1 + (\kappa/q)^2$ ($1/\kappa$ is the Debye radius). This is due to the effect of the screening of the phonon potential, which decreases the electron-phonon interaction and thus also the scattering probability.

The magnetic field introduces additional complications in this picture. For instance, when H = 0 plasma effects are practically unobservable since the dispersion of the optical vibrations is small and one must select the impurity concentration specially in order that the frequency of the optical vibrations turn out to be near the plasma frequency. In a magnetic field, the plasma spectrum occupies a wide region of frequencies from

*This phenomenon is complicated by the fact that in the case of a sufficiently exact resonance the electron system may show a reciprocal influence on the phonons, changing the phonon frequency[2] (see also Appendix II).

ω_p to Ω ($\Omega = eH/mc$ is the Larmor frequency) and the equality between the plasma frequency and the frequency of the optical phonons is achieved much more simply.

Unfortunately an application of these simple methods to a rigorous analysis of the role played by the above-mentioned effects in transport phenomena is unjustified. The point is that phonons with wave vectors $q \gtrsim q_T$ are important for such phenomena, yet it does not follow directly at all that for such dimensions of the spatial inhomogeneity these effects can be described by the self-consistent field method with some kind of single-electron potential. This problem is essentially a many-particle one and one therefore needs solve it by means of many-particle quantum theory methods.

2. To construct a rigorous theory we start from Kubo's formula[4] for σ_{xx} in a form which is convenient for an expansion in powers of the electron-phonon interaction[5]

$$\sigma_{xx} = \frac{e^2\beta}{V_0} \operatorname{Re} \int_{-\infty}^{0} e^{\nu t} \left\langle e^{-\beta \mathcal{H}_1} T_C \left\{ \exp\left[\int U(z) \frac{dz}{i\hbar} \right] v^x(t)\, v^x(0) \right\} \right\rangle dt. \tag{12}$$

Here v^x is the operator of the x component of the total (summed over all electrons) velocity, T_C indicates ordering along the contour C, depicted in the figures by dot-dash lines,

$$U(z) = \exp(i\mathcal{H}_1 z/\hbar)\, U \exp(-i\mathcal{H}_1 z/\hbar), \tag{13}$$

$$\langle \ldots \rangle = \operatorname{Sp}(\ldots)/Z, \tag{14}$$

and Z is the partition function. Furthermore, $\mathcal{H} = \mathcal{H}_1 + U$, where $\mathcal{H}_1 = \mathcal{H}_e + \mathcal{H}_p$, $\mathcal{H}_e$ the Hamiltonian of a system of mutually interacting electrons in a magnetic field **H** which, when we choose the gauge **A** = (0, Hx, 0), is of the form

$$\mathcal{H}_e = \frac{\hbar^2}{2m} \sum_{i=1}^{N} \left[-\nabla_i^2 - 2ia^{-2} x_i \frac{\partial}{\partial y_i} + a^{-4} x^2 \right] + \frac{e^2}{2\varepsilon_0} \sum_{i \neq k} \frac{1}{|\mathbf{r}_i - \mathbf{r}_k|} \tag{15}$$

(N is the number of electrons, $\mathbf{r_i}$ the position coordinate of the i-th electron, $a^2 = c\hbar/eH$); $U = u + u^+$, where

$$u = \sum_{\mathbf{q}} \sum_{i} c_{\mathbf{q}} b_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}_i}, \tag{16}$$

$b_{\mathbf{q}}^+(b_{\mathbf{q}})$ the creation (annihilation) operator of a phonon in the state **q**; and $\mathcal{H}_p$ the phonon Hamiltonian.

To study the properties of the operator $\mathcal{H}_e$ we introduce Jacobi variables

$$\rho_k = (\mathbf{r}_1 + \mathbf{r}_2 + \ldots + \mathbf{r}_k)/k - r_{k+1},$$
$$\rho_N \equiv \mathbf{r} = (\mathbf{r}_1 + \mathbf{r}_2 + \ldots + \mathbf{r}_N)/N. \tag{17}$$

Inversely

$$\mathbf{r}_k = \mathbf{r} - \frac{k-1}{k} \rho_{k-1} + \frac{1}{k+1} \rho_k + \ldots + \frac{1}{N} \rho_{N-1},$$
$$\mathbf{r}_N = \mathbf{r} - \frac{N-1}{N} \rho_{N-1}. \tag{18}$$

One verifies easily that in the new variables

$$\mathcal{H}_e = \mathcal{H}_0(\mathbf{r}) + \mathcal{H}_2(\rho), \tag{19}$$

where $\mathcal{H}_0(\mathbf{r})$ is the operator of the orbital motion of a free particle of charge Ne and mass Nm. The Hamiltonian $\mathcal{H}_2$ commutes with $\rho_N = \mathbf{r}$. In other words, one can split off the center-of-mass motion in the system of interacting electrons in a magnetic field, and the solution of the equation

$$\mathcal{H}_e \Psi_L = E_L \Psi_L \tag{20}$$

is of the form

$$\Psi_L = \psi_\lambda(\mathbf{r})\, F_l(\rho), \tag{21}$$

where $\psi_\lambda(\mathbf{r})$ are the orbital wave functions of a particle of charge Ne and mass Nm, characterizing "external" states, and $F_l(\rho)$ stands for all wave functions describing the "internal" states of the electron system. Similarly

$$E_L = E_l + \varepsilon_\lambda. \tag{22}$$

In the new variables

$$v_x = -(i\hbar/m)\, \partial/\partial x, \tag{23}$$

$$u = \sum_{\mathbf{q}} c_{\mathbf{q}} b_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} \sum_{i} e^{i\mathbf{q}(\mathbf{r}_i - \mathbf{r})} \equiv \sum_{\mathbf{q}} c_{\mathbf{q}} b_{\mathbf{q}} e^{i\mathbf{q}\mathbf{r}} f_{\mathbf{q}}(\rho). \tag{24}$$

We assume the electron-phonon interaction to be weak and restrict ourselves, as in I, to the second-order term in the expansion of (12) in powers of U. To evaluate it we apply to (12) the standard method of disentangling exponential operator expressions under the T_C-product sign. In this way it turns out to be possible to write (12) as a sum of twelve terms, each of which corresponds to a diagram similar to the diagrams of Konstantinov and Perel',[5] but with the difference that all electron lines in it (except one) are proper ones, i.e., directed along the contour C. Each electron line going from the point z_2 to z_1 corresponds to a factor $\exp[E_L(z_1 - z_2)/i\hbar]$; the only improper electron line corresponds to an additional factor $\exp(-\beta E_L)$; each phonon line corresponds to a factor $(N_{\mathbf{q}} + 1)\exp[-i\omega_{\mathbf{q}}(z_1 - z_2)]$, if z_2 is earlier than z_1 (proper line) and to $N_{\mathbf{q}} \exp[-i\omega_{\mathbf{q}}(z_1 - z_2)]$ ($N_{\mathbf{q}}$ is the Planck function of the phonon frequency $\omega_{\mathbf{q}}$), if z_1 is earlier than z_2 (improper line); each vertex corresponds to a factor

$$c_{\mathbf{q}} \langle L | \Sigma e^{i\mathbf{q}\mathbf{r}_i} | L' \rangle = c_{\mathbf{q}} \langle \lambda | e^{i\mathbf{q}\mathbf{r}} | \lambda' \rangle \langle l | f_{\mathbf{q}} | l' \rangle; \tag{25}$$

the terminal t to a factor

$$\langle L \mid v^x \mid L' \rangle e^{i\omega_{LL'}t} = v^x_{\lambda\lambda'} e^{i\omega_{\lambda\lambda'}t} \delta_{ll'}, \tag{26}$$

and the terminal 0 to a factor

$$\langle M \mid v^x \mid M' \rangle = v^x_{\mu\mu'} \delta_{mm'}. \tag{27}$$

Here $\hbar\omega LL' = EL - EL'$. In the final expression we integrate over z_1 and z_2 between limits that follow from the form of the diagram, and we sum over $\mathbf{q}$ and over all indices of the electron system.

By virtue of the properties of the functions (22), (25), (26), and (27) the matrix elements for transitions between internal states of the system enter as independent factors and therefore all diagrams have the common factors

$$\frac{1}{Z\hbar^2} e^{-\beta E_l} |\langle l \mid f_{\mathbf{q}} \mid m \rangle|^2 |c_{\mathbf{q}}|^2 \begin{cases} N_{\mathbf{q}} \\ N_{\mathbf{q}} + 1. \end{cases} \tag{28}$$

We want to sum the diagrams a to f (see figure) and the diagrams a′ to f′ which differ from the ones illustrated in the figure by the direction of the phonon line. Using the identity

$$v^x_{\lambda\lambda'} / (\nu + i\omega_{\lambda\lambda'}) = \langle \lambda \mid x - X_\lambda \mid \lambda' \rangle \tag{29}$$

(X_λ is the coordinate of the center of the Landau oscillator in the state λ) and using the matrix multiplication rules we find that diagram a is equal to

$$\begin{aligned} &- \sum_{\lambda\lambda'} \sum_{\mu\mu'} \frac{e^{-\beta\varepsilon_{\lambda'}}}{\nu + i(\omega_{\mathbf{q}} + \omega_{\mu'\lambda'} + \omega_{ml})} \langle \mu' \mid e^{i\mathbf{qr}} \mid \lambda \rangle \langle \lambda \mid x - X_{\lambda'} \mid \lambda' \rangle \\ &\times \langle \lambda' \mid e^{-i\mathbf{qr}} \mid \mu \rangle \langle \mu \mid x - X_\mu \mid \mu' \rangle \\ &= - \sum_{\lambda'\mu'} \frac{e^{-\beta\varepsilon_{\lambda'}}}{\nu + i(\omega_{\mathbf{q}} + \omega_{\mu'\lambda'} + \omega_{ml})} \\ &\times \langle \mu' \mid e^{i\mathbf{qr}} (x - X_{\lambda'}) \mid \lambda' \rangle \langle \lambda' \mid e^{-i\mathbf{qr}} (x - X_{\mu'}) \mid \mu' \rangle \end{aligned} \tag{30}$$

(we have for the sake of simplicity not written down the factor (28) or the summation signs for summing over $\mathbf{q}$, l, and m). The expression for diagram b differs from (30) only by the general sign and by the fact that instead of the difference $x - X_{\mu'}$ the difference $x - X_{\lambda'}$ enters in it. The sum of diagrams a and b contains thus a factor

$$X_{\mu'} - X_{\lambda'} = (c\hbar / eH)\, q_y, \tag{31}$$

which is independent of the summation indices and is equal to

$$\begin{aligned} &- \mathrm{Re} \sum_{\lambda\mu} (X_\mu - X_\lambda) X_\lambda \frac{e^{-\beta\varepsilon_\lambda}}{\nu + i(\omega_{\mathbf{q}} + \omega_{\mu\lambda} + \omega_{ml})} |\langle \mu \mid e^{i\mathbf{qr}} \mid \lambda \rangle|^2 \\ &+ \mathrm{Re} \sum_{\lambda\mu} (X_\mu - X_\lambda) \frac{e^{-\beta\varepsilon_\lambda}}{\nu + i(\omega_{\mathbf{q}} + \omega_{\mu\lambda} + \omega_{ml})} \\ &\times \langle \mu \mid x e^{i\mathbf{qr}} \mid \lambda \rangle \langle \lambda \mid e^{-i\mathbf{qr}} \mid \mu \rangle. \end{aligned} \tag{32}$$

We show in the Appendix that the contribution to (12) from expressions of the form

$$\begin{aligned} &\mathrm{Re} \sum_{\mathbf{q}} \sum_{LM} e^{-\beta E_L} |c_{\mathbf{q}}|^2 |\langle l \mid f_{\mathbf{q}} \mid m \rangle|^2 \\ &\times \frac{\langle \lambda \mid e^{i\mathbf{qr}} \mid \mu \rangle \langle \mu \mid x e^{-i\mathbf{qr}} \mid \lambda \rangle}{\nu + i(\omega_{LM} \pm \omega)} \begin{cases} N_q \\ N_q + 1 \end{cases} \end{aligned} \tag{33}$$

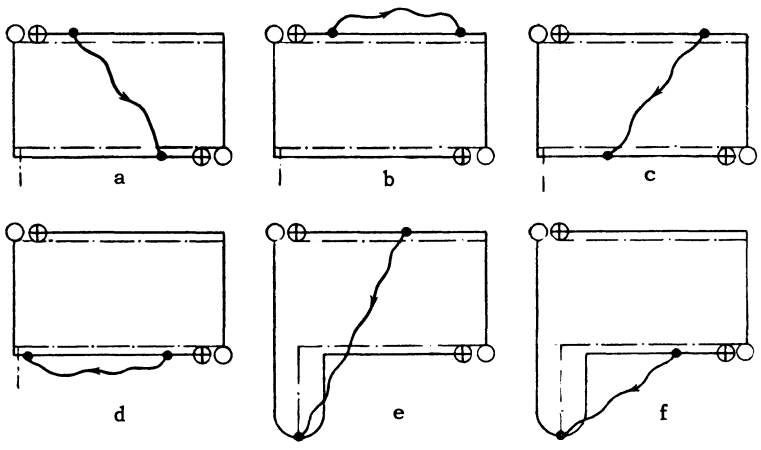
is equal to zero. The second term in (32) which after summation over $\mathbf{q}$, l, and m reduces to the form (33), can thus at once be dropped.

When diagrams c, d, e, and f are summed the factor (31) can also be split off. In the sum of these four diagrams it is also possible to perform two summations, using the rules of matrix multiplication. If we drop terms which give a zero contribution by virtue of (33), we get for this sum

$$- \mathrm{Re} \sum_{\lambda\mu} (X_\mu - X_\lambda) X_\lambda \frac{e^{-\beta\varepsilon_\lambda}}{\nu + i(\omega_{\lambda\mu} - \omega_{\mathbf{q}} + \omega_{lm})} |\langle \lambda \mid e^{i\mathbf{qr}} \mid \mu \rangle|^2. \tag{34}$$

Finally, the sum of the diagrams a to f is

$$\begin{aligned} &- \frac{2\pi}{\hbar^2} \sum_{\mathbf{q}} |c_{\mathbf{q}}|^2 (N_{\mathbf{q}} + 1) \sum_{LM} (X_\mu - X_\lambda) \\ &\times X_\lambda e^{-\beta E_L} |\langle L \mid \Sigma e^{i\mathbf{qr}_i} \mid M \rangle|^2 \delta(\omega_{LM} - \omega_{\mathbf{q}}). \end{aligned} \tag{35}$$



The sum of the diagrams a′ to f′ is the same expression, but with $-\omega_{\mathbf{q}}$ in the δ-function replaced by $\omega_{\mathbf{q}}$ and $N_{\mathbf{q}}+1$ by $N_{\mathbf{q}}$. Transposing in this expression the dummy summation indices L and M, adding this to (35) and taking into account the identity

$$e^{-\beta E_L}\delta(\omega_{LM}-\omega_{\mathbf{q}})(N_{\mathbf{q}}+1) = e^{-\beta E_M}\delta(\omega_{LM}-\omega_{\mathbf{q}})N_{\mathbf{q}} \tag{36}$$

and relation (35) we get finally

$$\sigma_{xx} = \frac{e^2\beta}{(2\pi)^2}\left(\frac{c}{eH}\right)^2\int d^3q\, q_y^2 |c_{\mathbf{q}}|^2 N_{\mathbf{q}}\Phi_{\mathbf{q}}(\omega_{\mathbf{q}}), \tag{37}$$

$$\Phi_{\mathbf{q}}(\omega) = Z^{-1}\sum_{LM}|\langle L|\sum e^{i\mathbf{q}\mathbf{r}_i}|M\rangle|^2 e^{-\beta E_L}\delta(\omega_{ML}-\omega). \tag{38}$$

(37) is exactly the same as Eq. (8) of I. The latter was derived from the formula of Kubo, Hasegawa, and Hashitsume [6] which expresses σ_{xx} in terms of the correlator of the operators of the velocity of the motion of the center of the Landau oscillator. But this formula itself was derived in [6] from an equation such as (12), using a number of additional assumptions. The present derivation starts directly from (12) and does therefore not depend on additional assumptions.

Moreover, it will be convenient for us to change in the expressions for $\Phi_{\mathbf{q}}(\omega)$ to a second quantization representation and to take a Gibbs average. To do this, we put

$$B_{\mathbf{q}} = \sum_i e^{i\mathbf{q}\mathbf{r}_i} = \sum_\sigma\int d^3r\psi^+(\mathbf{r},\sigma)e^{i\mathbf{q}\mathbf{r}}\psi(r,\sigma)$$

$$= \sum_{\Gamma\Delta}\langle\Gamma|e^{i\mathbf{q}\mathbf{r}}|\Delta\rangle a_\Gamma^+ a_\Delta, \tag{39}$$

where $\psi(\mathbf{r},\sigma) = \Sigma_\Delta a_\Delta\psi_\Delta(\mathbf{r},\sigma)$ is the electron wave function in the second quantization representation, $a_\Delta(a_\Delta^+)$ is the annihilation (creation) operator for an electron in the state Δ. Then we have

$$\Phi_{\mathbf{q}}(\omega) = Z^{-1}\sum_{LM}\exp[-\beta(E_L-\mu N_L)]\,\delta$$

$$\times(\omega_{ML}-\omega)|\langle M|B_{\mathbf{q}}|L\rangle|^2, \tag{40}$$

where N is the particle number operator. According to I, $\Phi_{\mathbf{q}}(\omega)$ can be expressed in terms of $K_{\mathbf{q}}(\omega)$, the Fourier component of the two-particle retarded Green's function with pairwise equal arguments, through the formula

$$\Phi_{\mathbf{q}}(\omega) = \operatorname{Im}K_{\mathbf{q}}(\omega)/\pi(1-e^{-\hbar\omega\beta}). \tag{41}$$

In [3] a method was proposed to evaluate $K_{\mathbf{q}}(\omega)$ for the case $\mathbf{H}=0$; this method was generalized in a paper by the authors [7] for the case of a non-vanishing magnetic field. According to [7]

$$K_{\mathbf{q}}(\omega) = \Pi_{\mathbf{q}}(\omega)\Big/\left[1+\frac{4\pi e^2}{\varepsilon_0\hbar q^2 V_0}\Pi_{\mathbf{q}}(\omega)\right], \tag{42}$$

where $\Pi_{\mathbf{q}}(\omega)$ corresponds to the sum of all connected Matsubara diagrams (see [3] and [7]). Taking (37) and (41) into account we get then

$$\sigma_{xx} = \frac{e^2\beta}{(2\pi)^2}\left(\frac{c}{eH}\right)^2\int d^3q q_y^2\frac{\operatorname{Im}\Pi_q(\omega)}{\pi(1-e^{-\hbar\omega\beta})}|c_{\mathbf{q}}|^2 N_{\mathbf{q}}$$

$$\times\left\{\left[1+\frac{4\pi e^2}{\varepsilon_0\hbar q^2 V_0}\operatorname{Re}\Pi_{\mathbf{q}}(\omega)\right]^2+\left[\frac{4\pi e^2}{\varepsilon_0\hbar q^2 V_0}\operatorname{Im}\Pi_{\mathbf{q}}(\omega)\right]^2\right\}^{-1}. \tag{43}$$

We see that in the general case it is not at all possible to arrive at a simple renormalization of $|c_{\mathbf{q}}|^2$ of the form discussed in Sec. 1. However, in a number of cases, we have with fair accuracy

$$\Pi_{\mathbf{q}}(\omega) = K_{\mathbf{q}}^0(\omega), \tag{44}$$

where $K_{\mathbf{q}}^0(\omega)$ can be expressed by the sum (4) and corresponds to the simplest diagram which is of the form of a loop of two electron lines.[1,3,7]

When a strong magnetic field is present ($\alpha = \hbar\Omega/2kT \gg 1$) one can easily justify the approximation (44) for the case where the Born parameter is small $e^2/\epsilon_0\hbar v_T \ll 1$ (small effective masses and large values of ϵ_0 favor the satisfying of this inequality). However, there are grounds for hoping that in reality (44) is valid when the weaker condition $e^2n^{1/3}/\epsilon_0 kT \ll 1$ (average energy of the Coulomb interaction between the electrons small compared to the thermal energy) is satisfied.

If (44) is valid,

$$\sigma_{xx} = \frac{e^2\beta}{(2\pi)^2}\left(\frac{c}{eH}\right)^2\int d^3q q_y^2|c_{\mathbf{q}}|^2 N_{\mathbf{q}}\Phi_{\mathbf{q}}^0(\omega_{\mathbf{q}})$$

$$\times\left\{\left[1+\frac{4\pi e^2}{\varepsilon_0\hbar q^2 V_0}\operatorname{Re}K_{\mathbf{q}}^0(\omega)\right]^2+\left[\frac{4\pi e^2}{\varepsilon_0\hbar q^2 V_0}\operatorname{Im}K_{\mathbf{q}}^0(\omega)\right]^2\right\}^{-1},$$

$$\Phi_{\mathbf{q}}^0(\omega) = \operatorname{Im}K_{\mathbf{q}}^0(\omega)/\pi(1-e^{-\hbar\omega\beta}). \tag{45}$$

The renormalized interaction in (45) has, indeed, the form (8). Our rigorous theory justifies thus the application of the self-consistent field method in this case.

We obtained in I an expression for $\operatorname{Im}K_{\mathbf{q}}^0(\omega)$ for the case $H \neq 0$ in the form of a series which was particularly convenient to use in the quantum limiting case when it converges very rapidly*

$$\operatorname{Im}K_{\mathbf{q}}^0(\omega) = \pi(1-e^{-\hbar\omega\beta})\Phi_{\mathbf{q}}^0(\omega) = \frac{n\sqrt{\pi}V_0}{|q_z|v_T}(1-e^{-\hbar\omega\beta})$$

$$\times\exp\left(\frac{\hbar\omega\beta}{2}-\frac{q_\perp^2a^2\operatorname{cth}\alpha}{2}\right)$$

$$\times\sum_{N=-\infty}^{\infty}I_N\left(\frac{q_\perp^2a^2}{2\operatorname{sh}\alpha}\right)\exp\left[-\frac{(N\Omega-\omega)^2}{q_z^2v_T^2}-\frac{q_z^2}{4q_T^2}\right]. \tag{46}$$

*cth = coth, sh = sinh.

Using similar methods we find

$$\mathrm{Re}\, K_{\mathbf{q}}^{0}(\omega) = \frac{2nV_0}{|q_z|v_T} \exp\left(-\frac{q_\perp^2 a^2}{2}\,\mathrm{cth}\,\alpha\right) \sum_{N=-\infty}^{\infty} e^{\alpha N} I_N\left(\frac{q_\perp^2 a^2}{2\,\mathrm{sh}\,\alpha}\right) \times \left[W\left(\frac{\hbar q_z^2/2m + \omega + N\Omega}{|q_z|v_T}\right) + W\left(\frac{\hbar q_z^2/2m - \omega + N\Omega}{|q_z|v_T}\right)\right]. \quad (47)$$

3. Using Eq. (45) we can in principle consider the effects caused by the Coulomb interaction between carriers which were mentioned in Sec. 1. Here we study one such effect. In I we predicted theoretically peculiar resonance oscillations of the transverse conductivity in a magnetic field which are caused by the scattering of the electrons by phonons with a non-vanishing limiting frequency ω_0 (optical phonons). The oscillation maxima are determined by the condition $\omega_0 = N_0\Omega$ (N_0 is an integer) and, for instance, when $\alpha \gg 1$ we have near $\omega_0 = \Omega$ ($N_0 = 1$)*

$$\sigma_{xx} = \sigma_{xx}^{\mathrm{cl}} \frac{3\alpha}{2\sqrt{\pi\hbar\omega_0\beta}} \ln\frac{1}{\alpha|\delta|} = \sigma_{xx}^{\mathrm{cl}} \frac{3(\hbar\omega_0\beta)^{1/2}}{4\sqrt{\pi}} \ln\frac{1}{\alpha|\delta|}, \quad (48)$$

where $\sigma_{xx}^{\mathrm{cl}}$ is the classical value of the low-temperature transverse conductivity evaluated in I and $\delta = (\omega_0 - \Omega)/\omega_0$. Expression (48), as also Eqs. (48) and (50) in I which are suitable for arbitrary N, give an infinite height of the oscillating peak for $\omega_0 = N_0\Omega$.

Mathematically, this infinity is caused by the fact that when ω_0 is exactly equal to $N_0\Omega$ the integral over q_z of the N_0-th term of the series (46) diverges at the lower limit. We did not consider in I the mechanisms limiting this infinity. If, however, we take these into account, the logarithm in (48) has a finite value at $\omega_0 = N_0\Omega$ and is equal to

$$\ln(1/\alpha\delta_c), \quad (49)$$

where $\delta_c \ll 1$. The second term in the denominator of (45) is proportional to the square of (46) and thus tends to infinity as $1/q_z^2$ when $q_z \to 0$. The Coulomb interaction between the scatterers is thus able to limit the height of the resonance peak. This mechanism is, of course, not the only one, but the cut-off determined by it can in a number of cases turn out to be the most effective one; in the present paper we shall evaluate it.

We restrict ourselves to one of the experimentally most interesting cases: $N_0 = 1$ (first oscillation), $\alpha = \hbar\Omega/2kT \gg 1$ (quantum limit). We shall here not be interested in the shape of the resonance line and simply evaluate σ_{xx} for $\Omega = \omega_0$. It is clear that small q_z ($q_z \ll q_T$) and $q_\perp \lesssim 1/a$ play a role in the resonance region. However, in the important region of q

$$\varkappa^2/q^2 \approx \varkappa^2 q_\perp^2 \sim \varkappa^2 a^2 \ll 1, \quad (50)$$

and when $q_\perp \sim 1/a$ the first term in the denominator in (45) can be assumed to be equal to unity. The interval $q_\perp \ll 1/a$ cannot play a role as long as the first term does not tend to zero in it. In actual fact, it takes on a zero value, generally speaking, but one can show that all contributions from the region of small $q_\perp$ are negligibly small, all the same.

Moreover, in the series (46) only the term with N = 1 will play a role and all other terms are exponentially small. Finally, for the interaction with the optical phonons

$$|c_{\mathbf{q}}|^2 = A/q^2 \approx A/q_\perp^2,$$

where the expression for A was given in I. The problem is thus reduced to an evaluation of the following expression

$$\sigma_{xx} = \frac{ne^2\beta A}{2\pi^{3/2}v_T} N_{\mathbf{q}} e^{\hbar\omega_0\beta/2} \int_0^\infty \frac{dq_z}{q_z} \int_0^\infty q_\perp dq_\perp I_1\left(\frac{q_\perp^2 a^2}{2\,\mathrm{sh}\,\alpha}\right) \times \exp\left(-\frac{q_\perp^2 a^2}{2} - \frac{q_z^2}{4q_T^2}\right)\left\{1 + 4\pi\left(\frac{\varkappa}{q_\perp}\right)^4\left(\frac{q_T}{q_z}\right)^2 \times \mathrm{sh}^2\frac{\hbar\omega_0\beta}{2} I_1^2\left(\frac{q_\perp^2 a^2}{2\,\mathrm{sh}\,\alpha}\right)\exp\left(-\frac{q_z^2}{2q_T^2} - q_\perp^2 a^2\right)\right\}^{-1}. \quad (51)$$

We take further into account that apart from exponentially small terms

$$I_1(q_\perp^2 a^2/2\,\mathrm{sh}\,\alpha) = q_\perp^2 a^2/4\,\mathrm{sh}\,\alpha,$$

and we change to new integration variables $x = q_\perp^2 a^2/2$, $y = q_z^2/4q_T^2$:

$$\sigma_{xx} = \frac{ne^2}{4\sqrt{\pi}\,\Omega^2 t_0} e^{-\hbar\omega_0\beta}\left(\frac{\hbar\omega_0}{kT}\right)^{3/2} \int_0^\infty x e^{-x}\,dx \times \int_0^\infty \frac{e^{-y}dy}{y + (\pi/16)(\varkappa a)^4 e^{-2(x+y)}}, \quad (52)$$

where $t_0 = (2\pi\hbar^2/A)(\hbar\omega_0/2m)^{1/2}$. When $(\varkappa a)^2 \ll 1$ the integral over y is equal to

$$\ln\frac{16}{\pi C_1(\varkappa a)^4} + 2x, \quad (53)$$

where $C_1 = e^C$, C = 0.577 is Euler's constant. Performing the integration over x we get finally

$$\sigma_{xx} = \sigma_{xx}^{\mathrm{cl}} \frac{3}{4\sqrt{\pi}} \sqrt{\hbar\omega_0\beta}\, \ln\frac{4e^2}{\sqrt{\pi C_1}(\varkappa a)^4}, \quad (54)$$

where according to (49) of I

$$\sigma_{xx}^{\mathrm{cl}} = (2ne^2/3m\Omega^2 t_0)\,\hbar\omega_0\beta e^{-\hbar\omega_0\beta}. \quad (55)$$

Comparing (48), (49), and (54) we find

*We note that in the corresponding Eq. (50) of I the coefficient $1/\pi^{1/2}$ was omitted by mistake.

$$\delta_c = \sqrt{\pi C_1}\,(\varkappa R)^2 / 4e^2, \tag{56}$$

where $R = mv_T c/eH$.

To elucidate when this mechanism of cut-off plays the main role we determine the cut-off, taking the dispersion of the optical phonons into account. Let the optical phonon frequency be

$$\omega_{\mathbf{q}} = \omega_0 (1 - a_0^2 q^2), \tag{57}$$

where a_0 is of the order of the crystal lattice constant. Calculations completely similar to the ones we just performed give the following result

$$\delta_c = e (a_0 / a)^2. \tag{58}$$

This quantity is as a rule considerably less than (56).

We must finally give an estimate of the cut-off caused by the broadening of the energy levels due to collisions. Very rough estimates in a paper by Adams and Holstein[8] give

$$\delta_c = 1 / \Omega\tau, \tag{59}$$

where τ is a characteristic life time of the electron in the given state. Attempts to estimate this quantity more accurately run up against serious difficulties. From a comparison of (59) and (56) it is, however, clear that the electron-electron interaction may in a number of cases be the main mechanism restricting the height of the oscillations.

APPENDIX I

We show that an expression of the type

$$Z^{-1}\mathrm{Re}\sum_{\mathbf{q}} |c_{\mathbf{q}}|^2 N_{\mathbf{q}} \times \sum_{LM} (X_\lambda - X_\mu)\, e^{-\beta E_L} \frac{\langle L | x\Sigma e^{i\mathbf{q}\mathbf{r}_i} | M\rangle \langle M | \Sigma e^{-i\mathbf{q}\mathbf{r}_i} | L\rangle}{\nu + i(\omega_{LM} + \omega_{\mathbf{q}})} \tag{A.1}$$

when summed with the same expressions, where $N_{\mathbf{q}}$ and $\omega_{\mathbf{q}}$ are replaced by $N_{\mathbf{q}}+1$ and $-\omega_{\mathbf{q}}$ gives zero. We put

$$\mathrm{Re}\frac{\langle L | x\Sigma e^{i\mathbf{q}\mathbf{r}_i} | M\rangle \langle M | \Sigma e^{-i\mathbf{q}\mathbf{r}_i} | L\rangle}{\nu + i(\omega_{LM} + \omega_{\mathbf{q}})} = \pi\delta(\omega_{LM} + \omega_{\mathbf{q}}) \times \mathrm{Re}\{\langle L | x\Sigma e^{i\mathbf{q}\mathbf{r}_i} | M\rangle \langle M | e^{-i\mathbf{q}\mathbf{r}_i} | L\rangle\} + \mathrm{P}\frac{\mathrm{Im}\{\langle L | x\Sigma e^{i\mathbf{q}\mathbf{r}_i} | M\rangle \langle M | e^{-i\mathbf{q}\mathbf{r}_i} | L\rangle\}}{\omega_{LM} + \omega_{\mathbf{q}}} \tag{A.2}$$

(P indicates the principal value). The contribution in (A.1) from the first term in (A.2) to the sum with the same expression, but with $N_{\mathbf{q}}$ and $\omega_{\mathbf{q}}$ replaced by $N_{\mathbf{q}}+1$ and $-\omega_{\mathbf{q}}$ is zero. To verify that, it is sufficient to transpose in the expression with $N_{\mathbf{q}}+1$ the summation indices L and M, to replace $\mathbf{q}$ by $-\mathbf{q}$ and to take the identity (36) into account. We use further the relation

$$\mathrm{Im}\{\langle\lambda | xe^{i\mathbf{q}\mathbf{r}} | \mu\rangle \langle\mu | e^{-i\mathbf{q}\mathbf{r}} | \lambda\rangle\} = -\tfrac{1}{2}\frac{\partial}{\partial q_x} |\langle\lambda | e^{i\mathbf{q}\mathbf{r}} | \mu\rangle|^2. \tag{A.3}$$

However, by definition

$$\pi\sum_{\lambda\mu}\delta(\omega_{\lambda\mu} + \omega) |\langle\lambda | e^{i\mathbf{q}\mathbf{r}} | \mu\rangle|^2 e^{-\beta\varepsilon_\lambda} = Z^{-1}\Phi_{\mathbf{q}}^0(\omega), \tag{A.4}$$

where Z is the appropriate partition function. Using the representation (45) one obtains easily the rule for differentiating this function:

$$\frac{\partial}{\partial q_x}\Phi_{\mathbf{q}}^0(\omega) = -q_x a^2 \,\mathrm{cth}\,\alpha\,\Phi_{\mathbf{q}}^0(\omega) + \frac{q_x a^2}{2\,\mathrm{sh}\,\alpha}[\Phi_{\mathbf{q}}^0(\omega + \Omega) + \Phi_{\mathbf{q}}^0(\omega - \Omega)]. \tag{A.5}$$

Substituting (A.5) and (A.4) into (A.1) and taking into account that a non-vanishing contribution can only come from the second term in (A.2) we get finally the following expression for (A.1):

$$\frac{a^4}{2Z}\,\mathrm{P}\int\frac{d\omega}{\omega}\sum_{\mathbf{q}} q_x q_y |c_{\mathbf{q}}|^2 N_{\mathbf{q}} [\mathrm{cth}\,\alpha\,\Phi_{\mathbf{q}}(\omega - \omega_{\mathbf{q}}) - \Phi_{\mathbf{q}}(\omega - \omega_{\mathbf{q}} + \Omega)/2\,\mathrm{sh}\,\alpha - \Phi_{\mathbf{q}}(\omega - \omega_{\mathbf{q}} - \Omega)/2\,\mathrm{sh}\,\alpha]. \tag{A.6}$$

Since $\Phi_{\mathbf{q}}$, $|c_{\mathbf{q}}|^2$, and $\omega_{\mathbf{q}}$ are essentially even functions of q_x this expression vanishes which concludes our proof.

APPENDIX II

When analyzing the causes producing the cut-off of the oscillations we must bear in mind still the two following factors which occur because of the interaction between the phonons and the electrons: damping of the phonons and renormalization of their frequency, thanks to which there occurs for them an additional, rather strong dispersion. We show that both these factors are unimportant for the present problem.

We are interested in the case of a low electron concentration and a weak electron-phonon interaction. One can show that under those conditions when taking these effects into account it is sufficient to introduce renormalized phonon lines to each of which must be assigned a causal Green's function which is equal to[9,10]

$$iD(\mathbf{q}, z_1 - z_2) = \begin{cases} \int_{-\infty}^{\infty} \rho_{\mathbf{q}}(\omega)\, e^{-i\omega(z_1-z_2)}\, d\omega, & \text{if the line is a proper one,} \\ \int_{-\infty}^{\infty} \rho_{\mathbf{q}}(\omega)\, e^{-\hbar\omega\beta - i\omega(z_1-z_2)}\, d\omega, & \text{if the line is an improper one;} \end{cases} \tag{A.7}$$

$$\rho_{\mathbf{q}}(\omega) = \sum_{LM} e^{-\beta E_L} |\langle L | b_{\mathbf{q}} + b^{+}_{-\mathbf{q}} | M \rangle|^2 \delta(\omega - \omega_{ML}). \tag{A.8}$$

As a result we get instead of (37)

$$\sigma_{xx} = (e^2\beta / 8\pi^2)(c/eH)^2 \int_{-\infty}^{\infty} d\omega \int d^3q\, q_y^2 |c_{\mathbf{q}}|^2\, \Phi_{\mathbf{q}}(\omega)\, \rho_{\mathbf{q}}(-\omega). \tag{A.9}$$

This expression corresponds to a sum of Matsubara diagrams with electron lines renormalized by taking phonons into account and with electron-phonon vertices.

In accordance with [9]

$$\rho_{\mathbf{q}}(\omega) = -\pi (1 - e^{-\hbar\omega\beta})^{-1} \operatorname{Im} \{D^T(\mathbf{q}, \omega_N)|_{\omega_N \to i\omega + \nu}\}, \tag{A.10}$$

where $D^T(\mathbf{q}, \omega_N)$ is the temperature dependent Green's function, and $\omega_N = 2\pi NkT/\hbar$. It satisfies the equation (cf. [10])

$$D^T(\mathbf{q}, \omega_N) = D_0^T(\mathbf{q}, \omega_N) + \hbar^{-2} D_0^T(\mathbf{q}, \omega_N)\, K_{\mathbf{q}}(-i\omega_N)\, D^T(\mathbf{q}, \omega_N).$$

According to Eq. (7b) of I, $|c_{\mathbf{q}}|^2 = 2\pi\hbar\omega_0 e^2/V_0 q^2 \epsilon_c$, $\epsilon_c^{-1} = \epsilon_0^{-1}(\infty) - \epsilon_0^{-1}(0)$. We have then

$$\rho_{\mathbf{q}}(\omega) = -\frac{2\omega_{\mathbf{q}}}{\pi(1 - e^{-\hbar\omega\beta})} \operatorname{Im} \left\{ \omega_{\mathbf{q}}^2 \left(1 - \frac{\varepsilon_0}{\varepsilon_c}\right) + \omega_{\mathbf{q}}^2 \frac{\varepsilon_0}{\varepsilon_c} \frac{1}{1 + \frac{4\pi e^2}{\varepsilon_0 \hbar V_0 q^2} \Pi_{\mathbf{q}}(\omega)} - \omega^2 \right\}^{-1}. \tag{A.11}$$

The poles of the function $D^T(\mathbf{q}, i\omega)$ give the renormalized phonon frequencies and the damping of the phonons due to their interaction with the electrons. In those $q_\perp$ and q_z regions where $\operatorname{Im} \Pi_{\mathbf{q}}(\omega)$ is large and leads to a limiting of the oscillations, the second term within the curly brackets in (A.11) is unimportant and in that region we obtain only an inappreciable shift of the resonance frequency* (when the coupling parameter is small, $\epsilon_0/\epsilon_c \ll 1$) Far from $\omega = N\Omega$ we obtain rather interesting singularities of the renormalized phonon spectrum, but they are unimportant for the present effect.

In conclusion we turn our attention to one interesting singularity of expression (A.9). For free phonons

$$\rho_{\mathbf{q}}(\omega) = N_{\mathbf{q}} \delta(\omega + \omega_{\mathbf{q}}) + (N_{\mathbf{q}} + 1)\, \delta(\omega - \omega_{\mathbf{q}}).$$

If we substitute this expression into (A.9) the integration over $d\omega$ disappears and we get Eq. (37). This is because when the electrons interact with free phonons there corresponds to a given value of the momentum transfer $\hbar\mathbf{q}$ a completely determined value of energy transfer $\hbar\omega_{\mathbf{q}}$. When the phonon damping is taken into account there occurs an uncertainty in the phonon energy and momentum. As a result the energy and momentum transfer are no longer connected with one another during electron-phonon collisions and σ_{xx} has the form of an integral over all possible values of the energy and momentum transfer (cf. [7]).

*We note that according to Born and Huang[11] the electron-electron interaction operator in an ionic crystal contains $\varepsilon_0(\infty)$, the permittivity on the plateau of the dispersion curve.

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