INVESTIGATION OF THRESHOLD ANOMALIES IN THE CROSS SECTIONS FOR COMPTON SCATTERING AND PHOTOPRODUCTION OF π^0 MESONS

G. K. USTINOVA

P. N. Lebedev Physics Institute, Academy of Sciences, U.S.S.R.

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Compton scattering on protons near the threshold for the production of π^0 and π^+ mesons, and photoproduction of π^0 mesons near the π^+-meson threshold, are examined phenomenologically. It is shown that if one takes into account the fact that near threshold the mesons are produced predominatly in the s-state one will obtain a peculiar energy dependence for the cross sections for elastic γp scattering and for photoproduction of neutral mesons. Analytic expressions are obtained for the total and differential cross sections for the Compton process and for π^0 photoproduction near the thresholds. Some numerical estimates of the effects to be expected are also given.

1. As is known, the unitarity of the full scattering matrix leads to a characteristic influence of one process on another in the threshold region.[1-3] In this connection Compton scattering is of particular interest; since the cross section for this process is comparatively small the effects due to the production of new particles with a large rest mass are particularly noticeable. Capps and Holladay[2] were the first to discover an anomaly in the total Compton effect cross section at the first threshold, by expanding the cross section near threshold in powers of the final state momentum. By utilizing dispersion relations, Lapidus and Chou Kuang-chao[4] obtained the characteristic threshold energy dependence for the cross section for Compton scattering and for photoproduction of neutral mesons. In the present work we investigate local threshold effects for these processes by the method of Baz',[3] generalized to include photons.

2. Let us investigate the energy dependence of the Compton effect on protons near the threshold for photoproduction of π^+ mesons ($E_t^+ = 150$ Mev). Let the z axis be taken along the direction of the momentum of the incident photon. In that case the z component of the angular momentum operator of the photon has the values ± 1, corresponding to right and left circular polarization, and the basic states of the photon + nucleon system are those in which the nucleon and photon spins are parallel or antiparallel.

For the two polarization cases the electric vector $\mathbf{E}$ is of the form[5]

$$\mathbf{E}_{n,a} = -e^{ik_0 z}\mathbf{e}_+ \chi_\pm + \mathbf{F}_{n,a} r^{-1} e^{ik_0 r};$$

$$\mathbf{F}_{n,a} = \sum_{l=1}^{\infty} \sqrt{2\pi(2l+1)} \sum_{j=l-1/2}^{l+1/2} C_{l\,1/2}(j,\, 1\pm 1/2;\; 1,\, \pm 1/2)$$
$$\times (2ik_0)^{-1}\Big\{(S^{M}_{lj}-1)\sum_{m=-1/2}^{m=1/2} C_{l\,1/2}(j,\, 1\pm 1/2;\; 1\pm 1/2 - m,\, m)$$
$$\times \mathbf{X}_{l,\,1\pm 1/2 - m}\chi_m + i(S^{E}_{lj}-1)\sum_{m=-1/2}^{m=1/2} C_{l\,1/2}(j,\, 1\pm 1/2;\; 1$$
$$\pm 1/2 - m,\, m)\left[\frac{\mathbf{r}}{r}\mathbf{X}_{l,\,1\pm 1/2 - m}\right]\chi_m\Big\}, \qquad (1)$$

where $\mathbf{e}_+$, $\chi_\pm$ are the spin functions of the photon and nucleon respectively, $\mathbf{k}_0$ is the wave vector of the relative photon—nucleon motion, $\mathbf{X}_{lm}$ is the vector spherical harmonic for $I = l$; $C_{l\frac{1}{2}}(j, M; M-m, m)$ is the Clebsch-Gordan coefficient, and $S^M_{lj} = \exp\{2i\delta^M_{lj}\}$ and $S^E_{lj} = \exp\{2i\delta^E_{lj}\}$ are the elements of the scattering matrix for magnetic and electric multipoles.

Near threshold the mesons are produced in the s state. The total angular momentum in the final state is $I = \frac{1}{2}$ since the spin of the meson is zero. The parity of the final state is $(-)$. The total angular momentum of the initial state is composed of the angular momentum of the photon l and the spin of the nucleon $\frac{1}{2}$, so that we must have the equality $l \pm \frac{1}{2} = I = \frac{1}{2}(-)$. It therefore follows that the photoproduction is due to electric dipole radiation in the state in which the photon and nucleon spin are antiparallel. Then, following Baz',[3,6] one may assume that

$$S^{\mathrm{E}}_{1\,1/2} = |S^{0\,\mathrm{E}}_{1\,1/2}| \exp\{2i\delta^{\mathrm{E}}_{1/2}\}\,(1 - a_1 kR),$$
$$S^{\mathrm{E}}_{lj} = |S^{0\,\mathrm{E}}_{lj}| \exp\{2i\delta^{\mathrm{E}}_{lj}\},\quad (l \neq 1,\ j \neq \tfrac{1}{2}),$$
$$S^{\mathrm{M}}_{lj} = |S^{0\mathrm{M}}_{lj}| \exp\{2i\delta^{\mathrm{M}}_{lj}\}. \tag{2}$$

Analogously, if D_l stands for the matrix element for the photoproduction of π^0 mesons, then near the threshold for the production of π^+ mesons the conservation of angular momentum and parity leads to the conclusion that

$$D_0 = |D_0^{(0)}| \exp\{i\delta_0\}\,(1 - a_2 kR),$$
$$D_l = |D_l^{(0)}| \exp\{i\delta_l\}\quad (l \neq 0). \tag{3}$$

In Eqs. (2) and (3) $S^0_{lj} = |S^0_{lj}| \exp\{2i\delta_{lj}\}$ and $D_l^{(0)} = |D_l^{(0)}| \exp\{i\delta_l\}$ are the values of S_{lj} and D_l at the threshold' E_t^+ ($\mathbf{k} = 0$, where $\mathbf{k}$ is the momentum vector of the relative π^+-meson-nucleon motion); δ_{lj} and δ_l are real phase shifts respectively for the scattering of photons of various multipolarities and scattering of π^0 mesons in l state (at $k = 0$). The coefficients a_1 and a_2 are determined, in accordance with Baz' and Okun',[6] by

$$|S^{0\,\mathrm{E}}_{1\,1/2}|\,a_1 = m_0'^2/2,\qquad |D_0^{(0)}|\,a_2 = m_0' m_0''/2, \tag{4}$$

where m_0' appears in the matrix element for direct photoelectric production of π^+ mesons in s states: $M_0 = (kR)^{1/2} m_0'$, and m_0'' is analogous to m_0' but in the matrix element for "internucleon charge exchange" of π^0 mesons in s states.

From a knowledge of the energy dependence of S_{lj} one can calculate the energy dependence of the cross section for Compton scattering near the threshold for π^+ production:

$$\sigma_c(\theta, E) = \sigma_c(\theta,\ E_t^+) - \mathrm{Re}\, a_1 kR \mathbf{F}_a^* \mathbf{G}^{\mathrm{E}}_{a1}, \tag{5}$$

$$\mathbf{G}^{\mathrm{E}}_{a1} = \frac{\sqrt{6\pi}}{2k_0} S^{0\,\mathrm{E}}_{1\,1/2} \left\{ \frac{2}{3}\left[\frac{\mathbf{r}}{r}\mathbf{X}_{1,1}\right]\chi_{-1/2} - \frac{\sqrt{2}}{3}\left[\frac{\mathbf{r}}{r}\mathbf{X}_{1,0}\right]\chi_{1/2}\right\}. \tag{6}$$

If one chooses the unit vector $\mathbf{n}$ in the direction of $\mathbf{F}_\alpha$ and the unit vector $\mathbf{m}$ in the direction of $\mathbf{G}^{\mathrm{E}}_{\alpha 1}$ then

$$\mathbf{F}_a = e^{i\alpha(\theta)} |F_a| \mathbf{n} = e^{i\alpha(\theta)} \sqrt{\sigma_c^a(\theta,\ E_t^+)}\,\mathbf{n},$$

$$\mathbf{G}^{\mathrm{E}}_{a1} = \exp(2i\delta^{\mathrm{E}}_{1/2})\,|G^{\mathrm{E}}_{a1}|\,\mathbf{m} = (1/2k_0)\,|S^{0\,\mathrm{E}}_{1/2}| \exp(2i\delta^{\mathrm{E}}_{1/2})\,\mathbf{m}, \tag{7}$$

and if we denote the angle between $\mathbf{n}$ and $\mathbf{m}$ by $\beta(\theta)$, we will get

$$\sigma_c(\theta, E) = \sigma_c(\theta, E_t^+) - (k_0/4\pi)\sqrt{\sigma_c^a(\theta, E_t^+)}$$
$$\times \sigma^{+\prime}_{\mathrm{ph}}(|k|) \cos\beta(\theta) \begin{cases} \sin(2\delta^{\mathrm{E}}_{1/2} - \alpha(\theta)), & E > E_t^+ \\ \cos(2\delta^{\mathrm{E}}_{1/2} - \alpha(\theta)), & E < E_t^+ \end{cases}, \tag{8}$$

where $\sigma^{+\prime}_{\mathrm{ph}}(|k|) = 2\pi\,|S^{0\mathrm{E}}_{1/2}|\,a_1\,|k|\,R/k_0^2$ coincides with the cross section for direct photoelectric production of positive mesons above threshold E_t^+, $\sigma_C(\theta, E_t^+)$ is the Compton cross section at the threshold E_t^+, $\alpha(\theta)$ is the phase of the amplitude at $E = E_t^+$, and $\sigma_C^a(\theta, E_t^+)$ is the cross section for Compton scattering of photons fully polarized antiparallel to the proton spin.

For $\theta = 0°$ it turns out that

$$\sigma_c(0, E) = \sigma_c(0, E_t^+) - (k_0/4\pi)\sqrt{\sigma_c^a(0, E_t^+)}$$
$$\times \sigma^{+\prime}_{\mathrm{ph}}(|k|) \begin{cases} \sin(2\delta^{\mathrm{E}}_{1/2} - \alpha(0)), & E > E_t^+ \\ \cos(2\delta^{\mathrm{E}}_{1/2} - \alpha(0)), & E < E_t^+, \end{cases} \tag{9}$$

i.e., $\beta(0) = 0$. For other angles it is not possible to obtain a similar exact formula because of the infinite number of interference terms. If, on the other hand, we limit ourselves to the dipole approximation and take into account that the phase shifts are small ($\delta_{lj} \sim e^2 k_0/M \sim 0.001$) so that $\sin\delta_{lj} = \delta_{lj}$ and $\cos\delta_{lj} = 1$, then we find for the elastic scattering cross section through an arbitrary angle

$$\sigma_c(\theta, E) = \sigma_c(\theta,\ E_t^+) - \sigma^{+\prime}_{\mathrm{ph}}(|k|)/4\pi$$
$$\times \begin{cases} \frac{1}{2}[|S^{0\,\mathrm{E}}_{1/2}| + \frac{1}{4}(3\cos^2\theta - 1)\,|S^{0\,\mathrm{E}}_{3/2}| + \frac{1}{2}\cos\theta\,[2\,|S^{0\mathrm{M}}_{1/2}| + |S^{0\,\mathrm{M}}_{3/2}|] \\ \qquad - \frac{3}{4}(1 + \cos\theta)^2], \quad E > E_t^+; \\ [-\frac{1}{4}(3\cos^2\theta - 1)\,|S^{0\,\mathrm{E}}_{3/2}|\,(\delta^{\mathrm{E}}_{1/2} - \delta^{\mathrm{E}}_{3/2}) + \frac{1}{2}\cos\theta\,[2\,|S^{0\,\mathrm{M}}_{1/2}|(\delta^{\mathrm{E}}_{1/2} - \delta^{\mathrm{M}}_{1/2}) \\ \qquad + |S^{0\,\mathrm{M}}_{3/2}|\,(\delta^{\mathrm{E}}_{1/2} - \delta^{\mathrm{M}}_{3/2})] - \frac{3}{4}(1 + \cos\theta)^2\,\delta^{\mathrm{E}}_{1/2}], \quad E < E_t^+. \end{cases} \tag{10}$$

The energy dependence of the total cross section for the Compton process is given by

$$\sigma_c(E) = \sigma_c(E_t^+) - \sigma^{+\prime}_{\mathrm{ph}}(|k|) \begin{cases} \frac{1}{2}[|S^{0\mathrm{E}}_{1/2}| - 1], & E > E_t^+ \\ \delta^{\mathrm{E}}_{1/2}, & E < E_t^+ \end{cases}. \tag{11}$$

All the analysis above refers to the case of unpolarized particles. It is relevant, however, that only the photon-nucleon combination with antiparallel spins contributes to the singularity. It is therefore possible to generalize the formulas to the case of arbitrary polarization. Thus in the case of Eq. (8) we get

$$\sigma_c(\theta, E, \xi^0, \zeta^0) = \sigma_c(\theta, E_t^+, \xi^0, \zeta^0) - (k_0/4\pi)\,(1 - \xi_3^0\zeta_z^0)$$
$$\times \sqrt{\sigma_c^a(\theta, E_t^+)}\,\sigma^{+\prime}_{\mathrm{ph}}(|k|) \cos\beta(\theta) \begin{cases} \sin(2\delta^{\mathrm{E}}_{1/2} - \alpha(\theta)), & E > E_t^+ \\ \cos(2\delta^{\mathrm{E}}_{1/2} - \alpha(\theta)), & E < E_t^+, \end{cases} \tag{12}$$

where $\boldsymbol{\zeta}^0$ (ζ_x^0, ζ_y^0, ζ_z^0) is the polarization vector of the incident proton, $\boldsymbol{\xi}^0$ (ξ_1^0, ξ_2^0, ξ_3^0) is the Stokes parameter of the incident electromagnetic radiation; ξ_3^0 is the degree of circular polarization of the incident photon. The same factor has to be introduced in all other formulas as well. For totally polarized particles with antiparallel spins ($\xi_3^0\zeta_z^0 = -1$) the singularity in the cross section is twice as large as in the unpolarized case, and for parallel spins ($\xi_3^0\zeta_z^0 = 1$) the singularity disappears.

3. The energy dependence of the Compton scattering cross section near the threshold for photoproduction of neutral mesons ($E_t^0 = 145$ Mev) can be formally obtained from the previous equations if it is remembered that S_{lj} satisfies at the threshold E_t^0 the condition $|S_{lj}^0| = 1$.[3] Therefore everywhere in Eqs. (8) — (12) all threshold quantities should be evaluated at 145 Mev (i.e., E_t^+ should be replaced by E_t^0) and $|S_{lj}^0|$ should be set equal to unity. For example, instead of Eq. (10) we get for the γp-scattering cross section near the threshold for π^0 production the following expression

$$\sigma_c(\theta, E) = \sigma_c(\theta, E_t^0) - (1/4\pi)\,\sigma_{ph}^0(|k'|) \times \begin{cases} 0, & E > E_t^0 \\ -[\frac{1}{4}(3\cos^2\theta - 1)(\delta_{1/2}^E - \delta_{3/2}^{E}) + \frac{1}{2}\cos\theta\,[2(\delta_{1/2}^E - \delta_{1/2}^M) + (\delta_{1/2}^E - \delta_{3/2}^M)] - \frac{3}{4}(1+\cos\theta)^2\delta_{1/2}^E], & E < E_t^0 \end{cases} \tag{13}$$

where all δ_{lj} stand for the real phase shifts of Compton scattering at the threshold E_t^0 ($k' = 0$), and $\sigma_{ph}^0(|k'|) = 2\pi a |k'| R/k_0^2$ coincides with the cross section for the photoproduction of π^0 mesons above the threshold E_t^0: $\mathbf{k}'$ is the π^0-meson-nucleon relative-motion vector.

4. The differential cross section for photoproduction of π^0 mesons near the threshold E_t^+ may be written with the help of Eq. (3) as follows

$$\sigma_{ph}^0(\theta, E) = \frac{k'}{k_0}\left|\sum_{l=0}^{\infty}\frac{2l+1}{2\sqrt{k_0 k'}} D_l P_l(\cos\theta)\right|^2 = \sigma_{ph}^0(\theta, E_t^+) - \frac{k_0}{2\pi}\sqrt{\sigma_{ph}^0(\theta, E_t^+)}\,\sigma_{ph}^{+''}(|k|)\begin{cases}\sin(\delta_0 - \gamma(\theta)), & E > E_t^+\\ \cos(\delta_0 - \gamma(\theta)), & E < E_t^+\end{cases} \tag{14}$$

or, if we introduce the known phase shifts δ_1 and δ_3 for s-wave scattering in states with isotopic spin ½ and 3/2 respectively,

$$\sigma_{ph}^0(\theta, E) = \sigma_{ph}^0(\theta, E_t^+) - (2k_0/6\pi)\sqrt{\sigma_{ph}^0(\theta, E_t^+)}\,\sigma_{ph}^{+''}(|k|) \times \begin{cases}[-(M/m - 1)\sin(\delta_1 - \gamma(\theta)) + (M/m+1)\sin(\delta_3 - \gamma(\theta))], & E > E_t^+\\ [-(M/m - 1)\cos(\delta_1 - \gamma(\theta)) + (M/m+1)\cos(\delta_3 - \gamma(\theta))], & E < E_t^+.\end{cases} \tag{15}$$

Here M is the nucleon and m the meson mass; $\gamma(\theta)$ is the phase of the amplitude

$$f_{ph}^0(\theta, E_t^+) = e^{i\gamma(\theta)}\sqrt{\sigma_{ph}^0(\theta, E_t^+)}$$

at the threshold E_t^+, and we have used the abbreviation

$$\sigma_{ph}^{+''}(|k|) = 2\pi |D_0^{(0)}|\, a_2 |k| R/k_0^2. \tag{16}$$

For the total cross section for the production of neutral mesons near the threshold for positive mesons we get

$$\sigma_{ph}^0(E) = \sigma_{ph}^0(E_t^+) - \begin{cases}|D_0^{(0)}|\,\sigma_{ph}^{+''}(|k|), & E > E_t^+\\ 0, & E < E_t^+.\end{cases} \tag{17}$$

5. The formulas (8) — (13) and (15) are valid as long as $kR \ll 1$, $k'R \ll 1$, i.e., $k, k' \ll 1/R$, where $R = \hbar/mc = 1.4 \times 10^{-13}$ cm is the range of the meson-nucleon interaction. The width of the threshold singularities is of the same order. It is reasonable to make the estimates at k and k' $= 0.124 \times 10^{13}\ \text{cm}^{-1}$, which corresponds to a distance away from the threshold of $E - E_t = k^2/2\mu = 2.5$ Mev (this should make it possible to resolve the singularities in the Compton effect cross section near the thresholds for the production of π^0 and π^+ mesons [$\mu = mM/(m+M)$].

Let us investigate the behavior of the Compton scattering cross section near the threshold E_t^+, as given by Eq. (10). If we take the phase shifts δ_{lj} from Gell-Mann, Goldberger, and Thirring[5] we get

$$\sigma_c(\theta, E) = \sigma_c(\theta, E_t^+) + (1/4\pi)\,\sigma_{ph}^+(|k|) \times \begin{cases}0.00021 + 0.00001\cos\theta, & E > E_t^+,\\ 0.00017 - 0.00041\cos\theta + 0.00057\cos^2\theta, & E < E_t^+\end{cases} \tag{18}$$

i.e., at all angles the cross section increases on both sides of the threshold (the singularity is in the form of a cusp, cf. [4]).

This is also the form of the singularity in the total Compton effect cross section according to Eq. (11). The size of the anomaly is determined by the cross section for meson photoproduction. It can be obtained, according to the calculations by Feld, by making use of perturbation theory (which is quite reliable near threshold) with a small correction for the finiteness of the nucleon mass and the possibility of meson charge exchange scattering in the field of the nucleon:[7]

$$\sigma_{ph}(|k|) = 4\pi|\zeta|^2\sigma_0 = 4\pi|\zeta|^2\, 25k/k_0\ \ \mu\text{b};\quad \zeta = \zeta_r + i\zeta_i,$$
$$\zeta_r = 0.105,\quad \zeta_i = -0.127k'R \text{ — for } \pi^0 \text{ mesons}$$
$$\zeta_r = 1,\quad \zeta_i = 0.057\,kR \text{ — for } \pi^+ \text{ mesons.} \tag{19}$$

Taking the threshold values of the Compton cross section from Capps,[8] we find that 2.5 Mev away from the threshold the relative increase in the cross section at individual angles amounts to 10 — 30% above threshold and 30 — 50% below threshold.

Since the cross section for s-wave production of neutral mesons is approximately 100 times smaller than the corresponding cross section for charged mesons [see Eq. (19)], it follows that the singularity in the Compton process cross section at the threshold E_t^0 will amount to a fraction of a percent. The total cross section for π^0 photoproduction decreases by 1 — 2% at 25 Mev above the threshold E_t^+.

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103

ERRATA

Vol	No	Author	page	col	line	Reads	Should read
13	2	Gofman and Nemets	333	r	Figure	Ordinates of angular distributions for Si, Al, and C should be doubled.	
13	2	Wang et al.	473	r	2nd Eq.	$\sigma_\mu = \frac{e^2f^2}{4\pi^3}\omega^2(\ln\frac{2\omega}{m_\mu} - 0.798)$	$\sigma_\mu = \frac{e^2f^2}{9\pi^3}\omega^2\left(\ln\frac{2\omega}{m_\mu} - \frac{55}{48}\right)$
			473	r	3rd Eq.	$(e^2f^2/4\pi^3)\omega^2 \geqslant \ldots$	$(e^2f^2/9\pi^3)\omega^2 \geqslant \ldots$
			473	r	17	242 Bev	292 Bev
14	1	Ivanter	178	r	9	1/73	1.58×10^{-6}
14	1	Laperashvili and Matinyan	196	r	4	statistical	static
14	2	Ustinova	418	r	Eq. (10) 4th line	$[-\frac{1}{4}(3\cos^2\theta - 1)\ldots$	$-[\frac{1}{4}(3\cos^2\theta - 1)\ldots$
14	3	Charakhchyan et al.	533	Table II, col. 6 line 1		1.9	0.9
14	3	Malakhov	550	The statement in the first two phrases following Eq. (5) are in error. Equation (5) is meaningful only when s is not too large compared with the threshold for inelastic processes. The last phrase of the abstract is therefore also in error.			
14	3	Kozhushner and Shabalin	677 ff		The right half of Eq. (7) should be multiplied by 2. Consequently, the expressions for the cross sections of processes (1) and (2) should be doubled.		
14	4	Nezlin	725	r	Fig. 6 is upside down, and the description "upward" in its caption should be "downward."		
14	4	Geĭlikman and Kresin	817	r	Eq. (1.5)	$\ldots\left[b^2\sum_{s=1}^{\infty}K_2(bs)\right]^2$	$\ldots\left[b^2\sum_{s=1}^{\infty}(-1)^{s+1}K_2(bs)\right]^2$
			817	r	Eq. (1.6)	$\Phi(T) = \ldots$	$\Phi(T) \approx \ldots$
			818	l	Fig. 6, ordinate axis	$\frac{\varkappa_s(T)}{\varkappa_n(T_c)}$	$\frac{\varkappa_s(T)}{\varkappa_n(T)}$
14	4	Ritus	918	r	4 from bottom	two or three	2.3
14	5	Yurasov and Sirotenko	971	l	Eq. (3)	1 < d/2 < 2	1 < d/r < 2
14	5	Shapiro	1154	l	Table	2306	23.6