MAGNETOACOUSTIC RESONANCE IN STRONG MAGNETIC FIELDS

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In a plasma in which the magnetic energy is comparable with the electron rest energy (i.e., the electron cyclotron frequency is of the same order as the plasma frequency), the resonance frequency of magnetic sound (as a function of the concentration) passes through a maximum in a constant magnetic field. Near the peak, the magnetoacoustic resonance can be produced and observed under nonlinear conditions (for a changing concentration) and can be used for ionization of the plasma.

IN previous researches on magnetoacoustic resonance in a plasma,[1-3] it was assumed that the plasma frequency ω_0 was large in comparison with the cyclotron frequencies ω_e and ω_i. Under such conditions, the resonant frequency of magnetic sound is seen to be essentially dependent on the concentration. Under the same conditions, the observation and use of magnetoacoustic resonance becomes more difficult. The opinion[4] has already been voiced that the detailed character of the resonance can be shown to be practically unattainable because of the action of the same phenomenon on the plasma concentration. It is impossible to agree with this, in principle, since there can be no doubt of the complete possibility of observing magnetoacoustic resonance under strictly controlled linear conditions, in which the resonance fields have low amplitude and the concentration of the plasma is formed and maintained by other independent means of action. However, magnetoacoustic resonance has been observed experimentally under nonlinear conditions,[2] when the creation of the plasma and the resonance action on it were carried out by the same variable field. Moreover, it seemed that the ionization process also continues very effectively under conditions of magnetoacoustic resonance[5]; this ionization is associated by its very nature with a continuous change of the concentration.

In order to account for the possibility of resonance phenomena of the magnetoacoustic type in a plasma with a time-dependent concentration, we assume the limitation made earlier:

$$\omega_0^2 \gg \omega_e^2$$

and consider the case in which the plasma frequency is of the order of or less than the electron cyclotron frequency. This case exists either in a rarefied plasma (where the plasma frequency is low) or in very strong magnetic fields (where the cyclotron frequency is high).

A plasma in which the cyclotron frequency is high in comparison with the collision frequency is known as magnetized (in relation to the collisions). If the cyclotron frequency is higher than the plasma frequency, then the electrostatic oscillations are magnetized; it is customary to call such a plasma oscillation magnetized. We note that the ratio

$$\omega_0^2 / \omega_e^2 = 4\pi nmc^2 / H^2$$

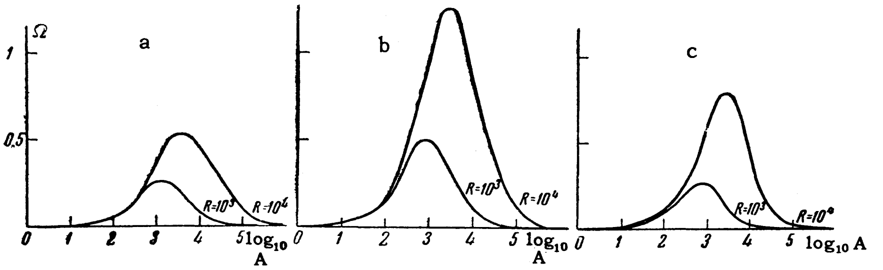
is of the order of the ratio of the electron rest energy to its magnetic energy. Thus, a plasma is magnetized in oscillation if its magnetic energy is larger than the rest energy of the electrons.

We consider first a rarefied plasma in which the number of electrons per unit length is small. Then, in accord with [3], the resonance frequency of the magnetic sound in the case of purely radial propagation is close to the lower hybrid frequency.[6] The formula developed in[3] for the lower hybrid frequency is valid only in the case when the plasma frequency is high in comparison with the cyclotron frequencies. By equating the coefficient a_{20} to zero in the general dispersion equation [Eq. (52) of [3]], we easily obtain a general expression for the lower hybrid frequency in the form

$$\omega_h^2 = \omega_i\omega_e \frac{\omega_0^2 + \omega_i\omega_e}{\omega_0^2 + \omega_e^2}. \tag{1}$$

If we assume $\omega_0^2 \gg \omega_i\omega_e$, we then get the approximate formula given in [3].

The general behavior of the lower hybrid frequency as a function of the density was also pointed out by Körper.[7] For $\omega_0^2 \ll \omega_i\omega_e$, the lower hybrid frequency approaches the ion cyclotron frequency, and for $\omega_0^2 \gg \omega_i\omega_e$, the geometric mean of the electron and ion cyclotron frequencies. However, it is



Dependence of Ω on A for B = 1840 (hydrogen), R = 10^3 and 10^4 for different values of cot θ: a – $\cot^2\theta = 0$, b – $\cot^2\theta = 10^{-3}$, c – $\cot^2\theta = 10^{-2}$ (the curves in drawing c are plotted in an ordinate scale reduced by a factor of 10).

important to note that there exists a wide intermediate region $\omega_e^2 \gg \omega_0^2 \gg \omega_i\omega_e$ in which the approximate expression for the lower hybrid frequency has the form

$$\omega_h^2 \approx \omega_0^2\omega_i/\omega_e. \qquad (2)$$

Here the lower hybrid frequency is proportional to the plasma frequency, i.e., to the square root of the concentration. For a small number of electrons per unit length, the resonant frequency of magnetic sound must behave in the same way. The heavier the ions, the greater the width of this region.

For a given magnetic field (ω_e = const) in a dense plasma, the resonant frequency of magnetic sound decreases, and, in a rarefied plasma, increases with the concentration. It must pass through a maximum in the intermediate region. Near the maximum, the concentration dependence of the resonant frequency should be weak and, with account of the finite width of the resonance, the resonance can be observed in a wide range of concentrations in the case of a sufficiently broad maximum.

In order to represent the concentration dependence of the resonant frequency in general form for a constant magnetic field, it is convenient to introduce dimensionless variables in a somewhat different way from what was done in [3]. Preserving the definition of the dimensionless frequency

$$\Omega = \omega^2/\omega_i\omega_e, \qquad (3)$$

the square of the Alfvén index of refraction

$$A = \omega_0^2/\omega_i\omega_e \qquad (4)$$

and the ratio of the cyclotron frequencies

$$B = \omega_e/\omega_i, \qquad (5)$$

we introduce new dimensionless parameters:

$$R = k_1^2c^2 / \omega_i\omega_e = k_1^2\tilde{r}_i\tilde{r}_e, \quad \mathrm{tg}\,\theta = k_1/k_3. \qquad (6)^*$$

Here ω is the resonant, ω_0 the plasma, ω_e and ω_i the electron and ion cyclotron angular frequencies, k_1 and k_3 are the radial and longitudinal wave numbers; $\tilde{r}_e$ and $\tilde{r}_i$ are the cyclotron radii for the velocity of light, $\tilde{r} = c/\omega$. The angle θ lies between 0 and $\pi/2$.

*tg = tan.

Expanding the dispersion equation in powers of Ω, we reduce it to the form

$$\begin{aligned}
&\Omega^5 - b_4\Omega^4 + b_3\Omega^3 - b_2\Omega^2 + b_1\Omega - b_0 = 0;\\
&b_4 = 3A + B + 2R\,(1 + \mathrm{ctg}^2\,\theta),\\
&b_3 = A^2 + 3AB + B^2 - [2A + B + R\,(1 + \mathrm{ctg}^2\,\theta)]^2,\\
&b_2 = (A + B)\,[A + R\,(1 + \mathrm{ctg}^2\,\theta)]^2 - AB\,(A + R),\\
&b_1 = AR\,[A + R + BR\,\mathrm{ctg}^2\,\theta\,(1 + \mathrm{ctg}^2\,\theta)],\\
&b_0 = AR^2\,\mathrm{ctg}^2\,\theta\,(1 + \mathrm{ctg}^2\,\theta).
\end{aligned} \qquad (7)^*$$

If we neglect all the coefficients except b_2 and b_1, then we obtain the approximate formula (for $\cot^2\theta \ll 1$)

$$\Omega = \left(1 + \frac{BR}{A+R}\mathrm{ctg}^2\,\theta\right)\Big/\left(\frac{A}{R} + 1 + \frac{B}{A}\right), \qquad (8)$$

which is identical with the "long cylinder approximation."[3] This approximation gives the root for Ω closest to unity if the parameters A and R are large and $AR(A + R)/B \gg 1$.

In dimensionless form, the concentration dependence of the resonant frequency for a constant magnetic field is represented as the dependence of Ω on A for constant R. Under ordinary experimental conditions, the long-cylinder approximation is shown to be sufficiently accurate. In this approximation, for $\theta = \pi/2$ ($k_3 = 0$, i.e., purely radial oscillations), the maximum lies at $A = \sqrt{BR}$ and the maximum value of the dimensionless frequency is

$$\Omega_m = \sqrt{BR}\,/\,(2B + \sqrt{BR}). \qquad (9)$$

For $k_3 \neq 0$ the expressions for the location and height of the maximum are obtained very crudely and we shall not write them down. As is seen from the graphs, the position of the maximum is shifted slightly, but the height of it increases sharply: resonance becomes possible at frequencies above the hybrid.[3]

The phenomenon under consideration of the maximum of the resonance frequency of the magnetic sound as a function of concentration for a constant magnetic field is of great value for interpretation of experiments on the magnetoacoustic resonance under nonlinear conditions.[2] Far from the maximum, resonance is possible

*ctg = cot.

only for strictly fixed concentration of the plasma. In the region close to the maximum, the magnetoacoustic resonance can occur and be observed even for variable concentration. In particular, the ionization with the help of magnetic sound[5] should effectively take place precisely in the region of concentration close to the condition of maximum. Experiments whose detailed description will be published later confirm the existence of a maximum and the remaining qualitative conclusions of the theory.

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