SOME PROCESSES INVOLVING HIGH-ENERGY NEUTRINOS

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Cross sections for processes of the type $\bar{\nu} + e^- \rightarrow \pi^- + \pi^0$, $\bar{\nu} + p \rightarrow \Lambda + \mu^+$, and $\mu^- + p \rightarrow \Lambda + \nu$ are calculated by taking into account the form factors for weak V-A coupling.

1. INTRODUCTION

A rather large number of recent papers [1-6] is devoted to processes that can occur when high energy neutrinos are absorbed. The most complete listing of such processes appears in Pontecorvo's article.[1] Lee and Yang[3] have analyzed experiments which could yield various information of interest for the theory of weak interactions. A number of authors[2,6] have studied the possibility of a practical experimental arrangement.

Since such an experiment will be possible in the comparatively near future, it is expedient to calculate the cross sections for various processes which occur when a neutrino is absorbed. Such a computation has been performed for a number of reactions, namely for elastic neutrino-electron scattering ($\bar{\nu} + e^- \rightarrow \bar{\nu} + e^-$ and $\nu + e^- \rightarrow \nu + e^-$)[7,8] and for the reactions $\bar{\nu} + p \rightarrow n + e^+$ and $\nu + n \rightarrow p + e^-$.[3-5]

In the present article cross sections are calculated for processes leading to the formation of two particles which occur when a neutrino is absorbed by an electron, a proton, or a neutron. Besides those indicated above, the following reactions are possible (their thresholds, in the laboratory system, are given in parentheses)

$$\nu + e^- \rightarrow \nu + \mu^-, \ \bar{\nu} + e^- \rightarrow \bar{\nu} + \mu^- \ (10.9 \text{ Bev}); \tag{1}$$

$$\begin{gathered}\bar{\nu} + e^- \rightarrow \pi^- + \pi^0 \ (74 \text{ Bev}), \ \bar{\nu} + e^- \rightarrow K^- + \pi^0 \ (387 \text{ Bev}),\\ \bar{\nu} + e^- \rightarrow K^- + K^0 \ (963 \text{ Bev}), \ \bar{\nu} + e^- \rightarrow \bar{K}^0 + \pi^- \ (398 \text{ Bev});\end{gathered} \tag{2}$$

$$\begin{gathered}\bar{\nu} + p \rightarrow n + \mu^+ \ (113 \text{ Mev}), \ \nu + n \rightarrow p + \mu^- \ (110 \text{ Mev}),\\ \bar{\nu} + p \rightarrow \Lambda + e^+ \ (195 \text{ Mev}), \ \bar{\nu} + p \rightarrow \Lambda + \mu^+ \ (319 \text{ Mev}),\end{gathered} \tag{3}$$

$$\begin{gathered}\bar{\nu} + p \rightarrow \Sigma^0 + e^+ \ (289 \text{ Mev}), \ \bar{\nu} + p \rightarrow \Sigma^0 + \mu^+ \ (420 \text{ Mev}),\\ \bar{\nu} + n \rightarrow \Sigma^- + e^+ \ (295 \text{ Mev}), \ \bar{\nu} + n \rightarrow \Sigma^- + \mu^+ \ (426 \text{ Mev}).\end{gathered}$$

In each of the reactions of (2) or (3), only two particles take part in the strong interactions. Their presence leads to the appearance of weak-interaction form factors, which essentially enter in all the formulas. Since strong interactions are absent in case (1), form factors of this kind do not arise.

Processes (1) and (2) will be examined further in Sec. 2, and processes (3) and further in Sec. 3. The reaction $\nu + n \rightarrow \Sigma^+ + e^-(\mu^-)$ is not included among the latter since it is forbidden in the Feynman–Gell-Mann scheme.[7] Section 4 deals with processes which are the inverse of (3) in the sense that the μ meson is absorbed by the nucleon and the neutrino arises in the final state. Processes of this kind involving the absorption of an electron have been discussed earlier.[9,10]

It will be assumed everywhere that the weak interaction includes only the V and A variants. It should not be forgotten that all processes in (2) and (3) occurring with the participation of electrons, as well as the second reaction in (1), can take place only if the electron and μ-meson neutrinos are identical (keeping in mind that the incident neutrinos are generated in $\pi_{\mu 2}$ and $K_{\mu 2}$ decay).

2. PROCESSES OCCURRING WHEN NEUTRINOS ARE ABSORBED BY ELECTRONS

Among the inelastic processes that occur when neutrinos interact with electrons, the reactions (1) have the lowest threshold. These reactions have been discussed earlier by, for example, Blokhintsev.[11] Their cross sections are given by the expressions (the index I refers to the reaction involving ν, II to that with $\bar{\nu}$)

$$\begin{gathered}\sigma_{\mathrm{I}} = (G^2/\pi)\mathscr{E}^2(1 - m_\mu^2/\mathscr{E}^2)^2,\\ \sigma_{\mathrm{II}} = (G^2/3\pi)\mathscr{E}^2(1 - m_\mu^2/\mathscr{E}^2)^2(1 + m_\mu^2/2\mathscr{E}^2),\end{gathered} \tag{4}$$

where $\mathscr{E}^2 \equiv (p_\nu + p_e)^2 = 2m_e W_\nu$ is the square of the total energy in the center-of-mass system and $G = 1.41 \times 10^{-49}$ erg-cm^3 is the Feynman-Gell-Mann constant. At a neutrino energy $W_\nu = 20$ Bev we have $\sigma_{\mathrm{I}} = 7 \times 10^{-41}$ cm^2 and $\sigma_{\mathrm{II}} = 3 \times 10^{-41}$ cm^2. At large energies (W_ν is expressed in Bev), σ_{I}

$= 1.6 \times 10^{-41} W_\nu \text{cm}^2$ and $\sigma_{II} = \sigma_I/3$, just as for the elastic scattering of a neutrino by an electron.

Especially interesting is the reaction

$$\bar{\nu} + e^- \to \pi^- + \pi^0. \quad (5)$$

In the Feynman–Gell-Mann scheme[7] the weak interaction responsible for this process has the same structure as the interaction between charged π mesons and the electromagnetic field. For this reason the total cross section for reaction (5)

$$\sigma = \frac{2G^2}{3\pi}\frac{k^3}{\mathscr{E}}F^2(\mathscr{E}^2) \quad (6)$$

[$k = (\mathscr{E}^2/4 - m_\pi^2)^{1/2}$ is the π-meson momentum in the center-of-mass system] contains $F(\mathscr{E}^2)$, which is the continuation of the electromagnetic form factor of the π meson into the region of timelike values of the argument. The determination of $F(\mathscr{E}^2)$ is a very interesting physical problem. Unfortunately the threshold of reaction (5) is very high.

When $W_\nu = 100$ Bev we have $\sigma = 3.3 \times 10^{-41} \times F^2(\mathscr{E}^2)\ \text{cm}^2$. It must be remembered that since $\mathscr{E}^2 > 0$, $F(\mathscr{E}^2)$ can be several times greater than unity. According to Frazer and Fulco,[12] for example, $F(\mathscr{E}^2) \sim 2$ when $W_\nu = 100$ Bev. In the laboratory system both mesons travel forwards with practically equal energy. In the center-of-mass system the angular distribution assumes the form $\sin^2\vartheta\, d\Omega$, where ϑ is the angle between the directions of incident and emerging particles; this is the result of neglecting the mass of the electron.

The same interaction as in process (5) can give rise to a completely analogous reaction, $\bar{\nu} + e^- \to K^- + K^0$. Its cross section is determined from formula (6) with the additional isotopic factor $1/2$; here $F(\mathscr{E}^2)$ is the continuation of the electric form factor of the K meson. The processes $\bar{\nu} + e^- \to K^- + \pi^0$ and $\bar{\nu} + e^- \to \bar{K}^0 + \pi^-$ are due to another interaction, responsible for one that causes lepton decays of hyperons and K mesons and has no electromagnetic analog. Here again formula (6) is correct with the added factor $1/2$, but

$$k = \{\mathscr{E}^4 - 2\mathscr{E}^2(m_K^2 + m_\pi^2) + (m_K^2 - m_\pi^2)^2\}^{1/2}/2\mathscr{E},$$

and $F(\mathscr{E}^2)$ coincides with the form factor that determines the K_{e3} decay. Whereas $F^2(0) = 1$ for $\bar{\nu} + e^- \to \pi^- + \pi^0$ and $K^- + K^0$, $F^2(0) \sim 1/30$ in the case of $\bar{\nu} + e^- \to K^- + \pi^0$ (see, for example, [13] and [14]).

All these processes are no less interesting than reaction (5), but their thresholds lie fantastically high. Still higher are the thresholds for the formation of baryon pairs in the collision between $\bar{\nu}$ and e^-. The reaction $\bar{\nu} + e^- \to n + \bar{p}$, for example, begins at $W_\nu = 3450$ Bev.

3. ABSORPTION OF NEUTRINOS BY PROTONS

Processes involving the absorption of neutrinos or antineutrinos by nucleons [enumerated in (3)] are more accessible for experimental investigation. A typical process of this kind is the reaction

$$\bar{\nu} + p \to \Lambda + \mu^+, \quad (7)$$

whose matrix element, as is known, has the form

$$\begin{aligned}(G/\sqrt{2})\,(\bar{v}_\nu\gamma_\alpha(1+\gamma_5)v_\mu)\,(\bar{u}_\Lambda\{\gamma_\alpha(f_1 - g_1\gamma_5) \\ + \sigma_{\alpha\beta}(p_p - p_\Lambda)_\beta(f_2 + g_2\gamma_5) \\ + (p_p - p_\Lambda)_\alpha(f_3 + g_3\gamma_5)\}\,u_p) \quad (8)\end{aligned}$$

[the metric $(1, -1, -1, -1)$ is used, $\gamma_5 = i\gamma_0\gamma_1\gamma_2\gamma_3$, and $\sigma_{\alpha\beta} = (\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)/2$]. The form factors f_i and g_i $(i = 1, 2, 3)$ are functions of the invariant

$$Q^2 \equiv (p_p - p_\Lambda)^2 = (m_\Lambda - m_p)^2 - 2m_pW_\Lambda, \quad (9)$$

where W_Λ is the kinetic energy of the Λ hyperon in the laboratory system. The quantity Q^2 is uniquely associated with the emission angle of the Λ and can vary within the limits

$$\begin{aligned}(2\mathscr{E}^2)^{-1}\{-(\mathscr{E}^2 - m_p^2)(\mathscr{E}^2 - m_\Lambda^2 - m_\mu^2) + 2m_p^2m_\mu^2 \\ - (\mathscr{E}^2 - m_p^2)[(\mathscr{E}^2 - m_\Lambda^2 - m_\mu^2)^2 \\ - 4m_\Lambda^2m_\mu^2]^{1/2}\} \leqslant Q^2 \leqslant (2\mathscr{E}^2)^{-1}\{-(\mathscr{E}^2 - m_p^2)(\mathscr{E}^2 - m_\Lambda^2 - m_\mu^2) \\ + 2m_p^2m_\mu^2 + (\mathscr{E}^2 - m_p^2)[(\mathscr{E}^2 - m_\Lambda^2 - m_\mu^2)^2 - 4m_\Lambda^2m_\mu^2]^{1/2}\}, \quad (10)\end{aligned}$$

where again $\mathscr{E}^2 \equiv (p_\nu + p_p)^2 = m_p^2 + 2m_pW_\nu$ is the square of the total energy in the center-of-mass system. At higher energies, when $(\mathscr{E}^2 - m_\Lambda^2)/m_\mu^2 \gg 1$, (10) is simplified and assumes the form

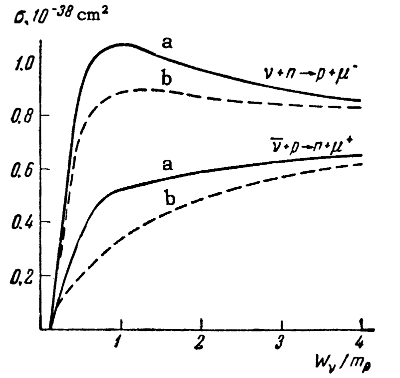
$$-\mathscr{E}^2(1 - m_p^2/\mathscr{E}^2)(1 - m_\Lambda^2/\mathscr{E}^2) \leqslant Q^2 \leqslant 0. \quad (11)$$

The expression for the antineutrino capture cross section at a given Q^2 is rather cumbersome and is given in the Appendix [formulas (A.1) to (A.3)]. Since the form factors are, apparently, small for large $|Q^2|$, in the limit $W_\nu \gg m_p$ the expression simplifies to

$$\begin{aligned}d\sigma = (2\pi)^{-1}G^2d(-Q^2)\,[|f_1|^2 + |g_1|^2 \\ + (-Q^2)(|f_2|^2 + |g_2|^2)]. \quad (12)\end{aligned}$$

In the case of the processes $\bar{\nu} + p \to n + \mu^+(e^+)$ and $\nu + n \to p + \mu^-(e^-)$ the form factors f_3 and g_2 go to zero as a result of isotopic invariance. The dependence of f_1 and f_2 on Q^2 in the Feynman–Gell-Mann scheme is moreover well known from experiments on the scattering of fast electrons,[15] and equals (μ_n and μ_p are the anomalous magnetic moments of the neutron and proton)

$$\begin{aligned}f_1 \approx \Phi(Q^2), \quad f_2 \approx \frac{\mu_p - \mu_n}{2m_p}\Phi(Q^2) = +\frac{3.71}{2m_p^2}\Phi(Q^2); \\ \Phi(Q^2) = (1 - Q^2a^2/12)^{-2}, \quad a = 0.8\cdot 10^{-13}\ \text{cm}. \quad (13)\end{aligned}$$



Dependence of cross sections for the reactions $\bar{\nu} + p \to n + \mu^+$ and $\nu + n \to p + \mu^-$ on the incident neutrino energy. The solid curves correspond to the possibility of case a) from formula (14), the dotted to case b).

At the present time it is impossible to say anything definite about the dependence of the form factors g_1 and g_3 on Q^2. It is only known that $g_1(0) = -1.2$. In [3-5] it was assumed that, like f_1 and f_2, $g_1 = -1.2\,\Phi(Q^2)$, and cross sections for the processes $\bar{\nu} + p \to n + e^+$ and $\nu + n \to p + e^-$ at various (anti) neutrino energies were calculated in this approximation. In both processes the "pseudoscalar" form factor g_3 yields a negligibly small contribution. In the reactions $\bar{\nu} + p \to n + \mu^+$ and $\nu + n \to p + \mu^-$, however, the contribution from g_3 is substantial. Goldberger and Treiman,[16] using dispersion techniques, found that $m_\mu g_3(-m_\mu^2) \approx 8\,g_1(0)$ when $Q^2 = -m_\mu^2$ and that $g_3(Q^2) \sim 1/(-Q^2 + m_\pi^2)$ at small Q^2. Such a dependence of g_3 on Q^2 cannot, strictly speaking, be extended to $-Q^2 \gg m_\pi^2$, but it can be adopted as an estimate. An alternate possibility is the weaker dependence $g_3 \sim \Phi(Q^2)$ for small Q^2.

The figure shows the total cross sections for the reactions $\bar{\nu} + p \to n + \mu^+$ and $\nu + n \to p + \mu^-$ as a function of incident (anti) neutrino energy W_ν. Relations (13) are adopted for the vector form factors. [Although f_1 falls off, apparently, more slowly than $\Phi(Q^2)$ when $-Q^2$ is of the order of $8 \times 10^{26} - 2 \times 10^{27}$ cm^{-2}, the correction necessitated by this fact would not be very great at these energies.] Two possibilities are considered for g_1 and g_3:

$$\text{a) } m_\mu g_3(Q^2) \approx 8 g_1(Q^2), \quad g_1(Q^2) \approx -1.2\Phi(Q^2);$$

$$\text{b) } m_\mu g_3(Q^2) \approx 8 g_1(0)\,(m_\mu^2 + m_\pi^2)/(-Q^2 + m_\pi^2),$$
$$g_1(Q^2) \approx -1{,}2\Phi(Q^2). \tag{14}$$

Cases a) and b) are shown in the figure by the solid and dotted lines respectively. When W_ν exceeds 500 Mev, the curve for case b) practically coincides with the cross section for the reaction that leads to the formation of e instead of μ. A similar agreement occurs for the curve of case a) only beyond 4 Bev. The difference is principally due to the component proportional to $g_3^2 m_\mu^2$, which in case a) is large for all Q^2, and in case b) is large only for small Q^2. The interference terms are insignificant since $g_3 m_\mu$ is multiplied by m_μ, which is small in comparison with the other energy quantities in (A.3). Analogously, the differential cross sections of electron and μ-meson reactions in case a) differ essentially in the term $g_3^2 m_\mu^2(-Q^2) \times (-Q^2 + m_\mu^2)/2$ in (A.3). Therefore, if indeed $m_\mu g_3 \gg g_1$, then by investigating experimentally the quantity

$$\frac{8\pi}{G^2 m_\mu^2}\,\frac{(\mathscr{E}^2 - m_p^2)^2}{(-Q^2)(-Q^2 + m_\mu^2)}\left[\frac{d\sigma_\mu}{d(-Q^2)} - \frac{d\sigma_e}{d(-Q^2)}\right], \tag{15}$$

we can determine with good accuracy the dependence of g_3 on Q^2. Here, of course, expression (15) must not depend on $\mathscr{E}^2$.

It is, no doubt, impossible to regard the hypotheses (14) as proved. There is no basis for assuming, for example, that f_1 and g_1 have the same order of magnitude for all Q^2. It can be noted that the cross sections for the processes $\bar{\nu} + p \to n + e^+$ and $\nu + n \to p + e^-$ differ only in the sign of the term containing $f_1 g_1$. If for some Q^2 one of these form factors is much smaller than the other, the cross sections in this region must coincide. Otherwise they will differ considerably.

Reactions (3), which involve the formation of strange particles, are also very interesting to study. When evaluating the order of magnitude of the cross sections it is possible to neglect in (A.3) or (12) the form factors f_2, g_2, f_3, and g_3 and consider that $f_1 = -g_1 = \text{const}$ when $-Q^2 \lesssim m_K^2$ and that $f_1 = g_1 = 0$ when $-Q^2 > m_K^2$.[10,14] In this approximation the cross sections for the processes

$$\bar{\nu} + p \to \Lambda + \mu^+(e^+), \quad \bar{\nu} + p \to \Sigma^0 + \mu^+(e^+),$$
$$\bar{\nu} + n \to \Sigma^- + \mu^+(e^+)$$

tend in the limit of $W_\nu \to \infty$ to the value $4 \times 10^{-39}\,|f_1|^2\text{cm}^2$. If we assume that $|f_1|^2 \sim 1/20$, which is similar to what apparently occurs in the decay $\Lambda \to p + e^- + \bar{\nu}$, $\sigma \sim 2 \times 10^{-40}\text{cm}^2$. When $W_\nu = 1$ Bev the cross sections for the production of Λ and Σ hyperons reach 0.7 and 0.6 of the limiting value.

4. THE PROCESS $\mu^- + p \to \Lambda + \nu$

Considerable information about the weak interactions of strange particles can be obtained by studying the process

$$\mu^- + p \rightarrow \Lambda + \nu, \tag{16}$$

as well as the completely analogous reactions $\mu^- + p \rightarrow \Sigma^0 + \nu$ and $\mu^- + n \rightarrow \Sigma^- + \nu$. A similar process involving the participation of an electron has been previously examined by one of the authors.[10] Great experimental difficulties are associated with the observation of this process, principally because of the fact that Λ hyperons are hard to detect against the large background.* As was observed several years ago by L. B. Okun', however, process (16) is more favorable in this respect. The fact is that the threshold for the photoproduction of π^- in the collision between a μ^- meson and nucleon, 166 Mev, exceeds the 82 Mev threshold for reaction (16). The thresholds for all other processes capable of leading to the formation of π^- mesons in a μ^--p interaction lie still higher. Thus in the energy region between the thresholds the Λ hyperon can be observed through the decay into a π^- meson ($\Lambda \rightarrow p + \pi^-$). This is impossible in the case of electrons, since the thresholds lie in the reverse order due to the smallness of m_e.

The expression for the matrix element of reaction (16) is analogous to that for (8), and the limits for variations of Q^2 [determined from Eq. (9)] are given by formula (10) if we make the exchanges $m_p \longleftrightarrow m_\Lambda$ and $\mathscr{E} \rightarrow \mathscr{E}_1$, where $\mathscr{E}_1^2 \equiv (p_\mu + p_p)^2 = (m_p + m_\mu)^2 + 2m_pW_\mu$ and W_μ is the kinetic energy of the μ meson in the laboratory system.

The expression for the cross section of process (16) (if the beam of μ mesons is unpolarized) is given by formulas (A.4) — (A.3) of the Appendix. In the case of pure V-A coupling it will assume the form

$$d\sigma_0 = \frac{G^2}{2\pi} \frac{\mathscr{E}_1^2 - m_\Lambda^2}{[(\mathscr{E}_1^2 - m_p^2 - m_\mu^2)^2 - 4m_p^2 m_\mu^2]^{1/2}} \frac{f_1^2 d(-Q^2)}{|\mathbf{v}_\mu|}, \tag{17}$$

where v_μ is the velocity of the μ meson in the laboratory system. If f_1 is considered to be a constant, the total cross section is

$$\sigma_0 = \frac{G^2}{2\pi} \frac{f_1^2}{|\mathbf{v}_\mu|} \mathscr{E}_1^2 (1 - m_\Lambda^2 / \mathscr{E}_1^2)^2. \tag{18}$$

Again assuming $f_1^2 \sim 1/20$, we find that for the cross section at the photoproduction threshold ($W_\mu = 166$ Mev) $\sigma_0 = 0.7 \times 10^{-41}$ cm^2.

*B. Pontecorvo has indicated the interesting possibility of discovering Λ particles produced in the reaction $e^- + p \rightarrow \Lambda + \nu$. All π^0 mesons formed in the interaction of the beam of electrons with the target decay in the target. The π^0 mesons can appear in the space behind the target only as a result of the decay of a strange particle. Registering a π^0 meson is thus equivalent to observing a Λ.

Real μ mesons generated in $\pi_{\mu 2}$ or $K_{\mu 2}$ decay are always polarized. In this case the cross section for process (16) is given by the more complicated expression of (A.3) — (A.8), which for pure V-A coupling has the form

$$d\sigma = d\sigma_0 (1 - \zeta \mathbf{v}_\mu),$$

where ζ is the polarization vector of the μ meson, definable as the average value of the spin in its rest system. According to (19), in pure V-A coupling azimuthal asymmetry of Λ hyperon emission does not arise, and the polarization of the μ meson leads only to the suppression or strengthening of the process according to whether the meson spin is directed forwards or backwards. We also point out that in the case of V-A coupling the Λ hyperon is completely polarized in a direction opposite to its own momentum in the center-of-mass system, independently of the polarization of the μ meson. If the meson spin is not parallel to its momentum and $f_1 \neq -g_1$, or if the contribution of other form factors is substantial, then, as follows from (A.4) and (A.7), azimuthal asymmetry of the type $p_\Delta \zeta_\perp$ arises in Λ-particle emission. When time parity is not conserved, an asymmetry of the type $p_\Delta [p_\mu \zeta]$ would also be added.

If ζ is expressed in terms of the μ and π meson momenta in the laboratory system, $d\sigma/d\sigma_0$ is according to (A.5) equal to

$$\varkappa = 2m_\mu^2 (E_\pi - E_\mu) / (m_\pi^2 - m_\mu^2) E_\mu, \tag{20}$$

where E_π and E_μ are the total energies of the mesons in the laboratory system. Since

$$[E_\pi (m_\pi^2 + m_\mu^2) - p_\pi (m_\pi^2 - m_\mu^2)] / 2m_\pi^2 \leqslant E_\mu \leqslant [E_\pi (m_\pi^2 + m_\mu^2) + p_\pi (m_\pi^2 - m_\mu^2)] / 2m_\pi^2 \tag{21}$$

($p_\pi = \sqrt{E_\pi^2 - m_\pi^2}$), in (20) χ varies within the limits

$$2m_\mu^2 / [(E_\pi - p_\pi)^2 + m_\pi^2] \geqslant \varkappa \geqslant 2m_\mu^2 / [(E_\pi + p_\pi)^2 + m_\pi^2]. \tag{22}$$

When $E_\pi = 300$ Mev, for example, and when $74 \text{ Mev} \leq W_\mu = E_\mu - m_\mu \leq 187$ Mev, these limits are equal to 1.63 and 0.07, and when $E_\pi \gg m_\pi$, $2 \geq \chi \geq 0$. When studying reaction (16) the upper limit of χ decreases, since the left inequality in (21) must be replaced by $E_\mu \geq (m_\Lambda^2 - m_p^2 - m_\mu^2)/2m_p = 188$ Mev.

The authors thank L. B. Okun' and B. Pontecorvo for discussing the possibility of observing the reactions considered in Sec. 4.

APPENDIX

A. The differential cross section for the reaction $\bar{\nu} + p \rightarrow \Lambda + \mu^+$ can be written as

$$d\sigma = \frac{G^2}{4\pi}\frac{d(-Q^2)}{(\mathcal{E}^2 - m_p^2)^2}A, \qquad \text{(A.1)}$$

where A is a function of the invariants $\mathcal{E}^2 = (p_p + p_\nu)^2$, $Q^2 = (p_p - p_\Lambda)^2$, and $\Delta^2 = (p_p - p_\mu)^2$, of which only two are independent, since

$$\mathcal{E}^2 + Q^2 + \Delta^2 = m_p^2 + m_\Lambda^2 + m_\mu^2. \qquad \text{(A.2)}$$

The quantity $A(\mathcal{E}^2, Q^2, \Delta^2)$ is expressed in terms of the form factors $f_i(Q^2)$ and $g_i(Q^2)$ introduced into (8):

$$\begin{aligned}
A = {} & |f_1 - g_1|^2(-\Delta^2 + m_\Lambda^2)(-\Delta^2 + m_p^2 + m_\mu^2) \\
& + |f_1 + g_1|^2(\mathcal{E}^2 - m_p^2)(\mathcal{E}^2 - m_\Lambda^2 - m_\mu^2) \\
& + [|f_2|^2 + |g_2|^2]\{Q^2[(\mathcal{E}^2 - m_p^2)(\Delta^2 - m_\Lambda^2) \\
& + (\mathcal{E}^2 - m_\Lambda^2)(\Delta^2 - m_p^2)] - m_\mu^2[(m_\Lambda^2 - m_p^2)(\mathcal{E}^2 - \Delta^2) \\
& + \tfrac{1}{2}(-Q^2 + m_\mu^2)(m_p^2 + m_\Lambda^2 + Q^2)]\} \\
& + 2\,\mathrm{Re}\, f_2 g_2^*(m_\Lambda^2 - m_p^2)[Q^2(\mathcal{E}^2 - \Delta^2) + m_\mu^2(m_\Lambda^2 - m_p^2)] \\
& + \tfrac{1}{2}[|f_3|^2 + |g_3|^2]m_\mu^2(-Q^2 + m_\mu^2)(-Q^2 + m_\Lambda^2 + m_p^2) \\
& + m_p m_\Lambda(-Q^2 + m_\mu^2)\{2[|g_1|^2 - |f_1|^2] \\
& + (2Q^2 + m_\mu^2)[|g_2|^2 - |f_2|^2] - m_\mu^2[|g_3|^2 - |f_3|^2]\} \\
& + [Q^2(\mathcal{E}^2 - \Delta^2) + m_\mu^2(m_\Lambda^2 - m_p^2)] \\
& \times 2\,\mathrm{Re}[f_1 g_2^*(m_\Lambda - m_p) - g_1 f_2^*(m_\Lambda + m_p) \\
& + (f_2 f_3^* + g_2 g_3^*)\tfrac{1}{2}m_\mu^2] \\
& + 2\,\mathrm{Re}(f_1 f_2^* + g_1 g_2^*)m_p[Q^2(Q^2 - m_p^2 + m_\Lambda^2) \\
& + m_\mu^2(\mathcal{E}^2 - m_\Lambda^2 - m_\mu^2)] \\
& + 2\,\mathrm{Re}(f_1 f_2^* - g_1 g_2^*)m_\Lambda[Q^2(Q^2 - m_\Lambda^2 + m_p^2) \\
& + m_\mu^2(\Delta^2 - m_p^2 - m_\mu^2)] + 2\,\mathrm{Re}(f_1 f_3^* + g_1 g_3^*)m_\mu^2 m_p \\
& \times(-\Delta^2 + m_\Lambda^2) + 2\,\mathrm{Re}(f_1 f_3^* - g_1 g_3^*)m_\mu^2 m_\Lambda(\mathcal{E}^2 - m_p^2).
\end{aligned} \qquad \text{(A.3)}$$

After renaming the indices, formulas (A.1) — (A.3) are of course correct for all the processes of antineutrino absorption in (3), and it is possible to neglect the contribution from f_3 and g_3 in reactions involving formation of e because of the smallness of m_e. In the case of the process $\bar{\nu} + p \to n + \mu^+(e^+)$ the form factors f_3 and g_2 reduce to zero. The cross section for the analogous process involving the absorption of a neutrino $[\nu + n \to p + \mu^-(e^-)]$ is also described by formulas (A.1) — (A.3) if we reverse the sign of g_1 and g_3.

B. For the reaction $\mu^- + p \to \Lambda + \nu$ with a polarized beam of μ^- mesons, the cross section in the laboratory system equals

$$\begin{aligned}
d\sigma = {} & \frac{G^2}{8\pi\,|\mathbf{v}_\mu|}\frac{d(-Q^2)\,(d\varphi/2\pi)}{(\mathcal{E}_1^2 - m_p^2 - m_\mu^2)[(\mathcal{E}_1^2 - m_p^2 - m_\mu^2)^2 - 4m_p^2 m_\mu^2]^{1/2}} \\
& \times\{A_1 - 2m_p B\,\mathbf{p}_\mu\boldsymbol{\zeta} - 2m_\mu C\,\mathbf{p}_\Lambda\boldsymbol{\zeta}_\perp + 4m_p m_\mu D\,\mathbf{p}_\Lambda[\mathbf{p}_\mu\boldsymbol{\zeta}]\},
\end{aligned} \qquad \text{(A.4)}$$

where φ is the azimuthal angle of the vector $\mathbf{p}_\Lambda$, ζ is the depolarization vector of the μ meson, and $\zeta_\perp$ is its component orthogonal to the momentum $(\zeta_\perp = \zeta - \mathbf{p}_\mu(\zeta\mathbf{p}_\mu)/\mathbf{p}_\mu^2)$.

If μ mesons are produced in the decay of π^- mesons, they will be completely polarized along their own momentum in the π rest system. In the laboratory system

$$\zeta = -\frac{2m_\mu \mathbf{p}_\pi}{m_\pi^2 - m_\mu^2} + \frac{W_\mu \mathbf{p}_\mu}{(m_\pi^2 - m_\mu^2)\,\mathbf{p}_\mu^2}[(m_\pi + m_\mu)^2 + 2m_\mu W_\pi], \qquad \text{(A.5)}$$

where W_π and W_μ are the kinetic energies of the mesons.

The expression for A_1 coincides with (A.3) if we substitute

$$\Delta^2 \to \mathcal{E}_1^2 \equiv (p_p + p_\mu)^2 = (m_p + m_\mu)^2 + 2m_p W_\mu,$$
$$\mathcal{E}^2 \to \Delta_1^2 = m_p^2 + m_\Lambda^2 + m_\mu^2 - \mathcal{E}_1^2 - Q^2.$$

The quantities B, C, and D are given by the formulas

$$B = B_1 - \frac{(\mathcal{E}_1^2 - m_p^2 - m_\mu^2)(\mathcal{E}_1^2 + Q^2 - m_p^2) - 2m_\mu^2(m_\Lambda^2 + m_p^2 - Q^2)}{(\mathcal{E}_1^2 - m_p^2 - m_\mu^2)^2 - 4m_p^2 m_\mu^2}C, \qquad \text{(A.6)}$$

$$\begin{aligned}
B_1 = {} & |f_1 - g_1|^2(\mathcal{E}_1^2 - m_\Lambda^2) + 2\,\mathrm{Re}\, f_2 g_2^*(m_\Lambda^2 - m_p^2)(\mathcal{E}_1^2 - m_\Lambda^2) \\
& + [|f_2|^2 + |g_2|^2][(-\Delta_1 + m_p^2)(-Q^2 - m_\Lambda^2 + m_p^2) \\
& + \tfrac{1}{2}m_\mu^2(m_\Lambda^2 + m_p^2 - Q^2)] - \tfrac{1}{2}[|f_3|^2 + |g_3|^2]m_\mu^2(m_\Lambda^2 \\
& + m_p^2 - Q^2) + m_p m_\Lambda\{2[|g_1|^2 - |f_1|^2] \\
& + (2Q^2 - m_\mu^2)[|g_2|^2 - |f_2|^2] + m_\mu^2[|g_3|^2 - |f_3|^2]\} \\
& + 2\,\mathrm{Re}[f_1 g_2^*(m_\Lambda - m_p) - g_1 f_2^*(m_\Lambda + m_p)](\mathcal{E}_1^2 - m_\Lambda^2) \\
& - \mathrm{Re}(f_2 f_3^* + g_2 g_3^*)[Q^2(\Delta_1 - m_\Lambda^2) + m_\mu^2(m_\Lambda^2 - m_p^2)] \\
& + \mathrm{Re}(f_1 f_2^* + g_1 g_2^*)m_p(\mathcal{E}_1^2 - Q^2 - 2m_\Lambda^2 + m_p^2) \\
& + \mathrm{Re}(f_1 f_2^* - g_1 g_2^*)m_\Lambda(\Delta_1^2 - 2Q^2 - 2m_p^2 + m_\Lambda^2 + m_\mu^2) \\
& + \mathrm{Re}(f_1 f_3^* + g_1 g_3^*)m_p(-\Delta_1^2 + m_\Lambda^2 + m_\mu^2) \\
& + \mathrm{Re}(f_1 f_3^* - g_1 g_3^*)m_\Lambda(-\Delta_1^2 + m_\Lambda^2 - m_\mu^2),
\end{aligned} \qquad \text{(A.7)}$$

C is obtained from (A.7) by reversing the sign of g_1, f_3, and g_3 and by the substitution $m_\Lambda \longleftrightarrow m_p$ and $\mathcal{E}_1^2 \longleftrightarrow \Delta_1^2$;

$$\begin{aligned}
D = {} & \mathrm{Im}\,[(f_1 f_2^* - g_1 g_3^*)(m_\Lambda - m_p) + (f_1 f_3^* - g_1 g_2^*)(m_\Lambda + m_p) \\
& - (f_2 f_3^* + g_2 g_3^*)\,Q^2].
\end{aligned} \qquad \text{(A.8)}$$

D is different from zero only when time parity is not conserved, since otherwise all the form factors are real.

In conclusion let us observe that if it is possible to neglect m_μ^2 in (A.3), (A.7), and (A.6),

$$(\mathcal{E}_1^2 - m_p^2)B_1 + (\Delta_1^2 - m_\Lambda^2)C = A_1, \qquad \text{(A.9)}$$

so that in (A.4)

$$A_1 - 2m_p B\,\mathbf{p}_\mu\boldsymbol{\zeta} = A_1(1 - \mathbf{v}_\mu\boldsymbol{\zeta}). \qquad \text{(A.10)}$$

Note added in proof (July 13, 1961). Analogously to the situation occurring in the annihilation of an electron-positron pair (recently noted by Baĭer and Sokolov[17]), there is a resonance in the reaction $\bar{\nu} + e^- \to \bar{\nu} + \mu^-$, connected with the possibility of a process going via an intermediate π^- meson. Near resonance the cross section is described by the formula

$$\sigma = 16\pi\lambda^2 \left(\frac{m_e}{m_\mu}\right)^2 \frac{m_\pi^4}{(m_\pi^2 - m_\mu^2)^2} \frac{(\Gamma m_\pi)^2}{(\mathscr{E}^2 - m_\pi^2)^2 + (\Gamma m_\pi)^2},$$

where λ is the Compton length of the π meson and Γ is the probability of its decay. The height of the peak is 1.3×10^{-4} mb, but unfortunately its width is insignificantly small: $\Gamma \approx 3 \times 10^{-8}$ ev.

The same formula is correct for the cross section of the process $\bar{\nu} + e^- \to \bar{\nu} + e^-$ with the additional small factor

$$(m_e/m_\mu)^2 [1 - (m_\mu/m_\pi)^2]^{-2} = 1.3 \cdot 10^{-4}.$$

[1] B. M. Pontecorvo, JETP **37**, 1751 (1959), Soviet Phys. JETP **10**, 1236 (1960).

[2] M. Schwartz, Phys. Rev. Lett. **4**, 306 (1960).

[3] T. D. Lee and C. N. Yang, Phys. Rev. Lett. **4**, 307 (1960).

[4] N. Cabibbo and R. Gatto, Nuovo cimento **15**, 304 (1960).

[5] Y. Yamaguchi, Progr. Theoret. Phys. **23**, 1117 (1960).

[6] Collection, K fizike neĭtrino vysokikh énergii (On the Physics of High Energy Neutrinos), Dubna, 1960.

[7] R. P. Feynman and M. Gell-Mann, Phys. Rev. **109**, 193 (1958).

[8] V. M. Shekhter, JETP **34**, 257 (1958), Soviet Phys. JETP **7**, 179 (1958).

[9] V. B. Berestetskii and I. Ya. Pomeranchuk, JETP **36**, 1321 (1959), Soviet Phys. JETP **9**, 936L (1959).

[10] V. M. Shekhter, JETP **38**, 1343 (1960), Soviet Phys. JETP **11**, 967L (1960).

[11] D. I. Blokhintsev, Usp. Fiz. Nauk **62**, 381 (1957).

[12] W. R. Frazer and J. R. Fulco, Phys. Rev. Lett. **2**, 365 (1959).

[13] L. B. Okun', Usp. Fiz. Nauk **68**, 449 (1959), Ann. Rev. Nuc. Sci. **9**, 61 (1959).

[14] V. M. Shekhter, JETP **36**, 1299 (1959), Soviet Phys. JETP **9**, 920L (1959).

[15] Hofstadter, Bumiller, and Yearian, Revs. Modern Phys. **30**, 482 (1958).

[16] M. L. Goldberger and S. B. Treiman, Phys. Rev. **111**, 354 (1958).

[17] V. N. Baĭer and V. V. Sokolov, JETP **40**, 1233 (1961), Soviet Phys. JETP **13**, 866 (1961).

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