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ELECTROMAGNETIC FORM FACTOR OF THE NEUTRAL PION

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The possibility of determining the electromagnetic form factor of the π^0 meson in the process $e^+ + e^- \to \pi^0 + \gamma$ is discussed.

ONE may hope that it will soon be possible to perform experiments using head-on electron and positron beams from the accelerator. In this connection it is interesting to note that the electromagnetic form factor of the π^0 meson can be measured with the help of the reaction

$$e^+ + e^- \to \pi^0 + \gamma. \tag{1}$$

Below we shall consider the dependence of this process on the form factor of the π^0 meson. Recognizing the fact that the effective Hamiltonian for the interaction of the pseudoscalar π^0 meson with electromagnetic fields must be invariant under spatial translations and rotations, we write it in the form

$$H_i = \iiint d^4x\, d^4y\, d^4z \widetilde{F}\left((x-z)^2,\ (y-z)^2,\right.$$
$$\left.\times (x-y)^2\right) \varepsilon_{\alpha\beta\rho\sigma} \frac{\partial A_\alpha(x)}{\partial x_\beta} \frac{\partial A_\rho(y)}{\partial y_\sigma} \Phi_0(z), \tag{2}$$

where Φ_0 and A_α are the field of the pseudoscalar π^0 meson and the electromagnetic field, and $\epsilon_{\alpha\beta\rho\sigma}$ is the completely antisymmetric unit tensor of fourth rank.

Restricting ourselves to lowest order perturbation theory for the electromagnetic interaction between the electron and the positron, we can draw the Feynman graph for process (1) as shown in Fig. 1. The matrix element corresponding to this graph has the form

$$\langle q, k | S | p_e, p_p \rangle$$
$$= -\frac{e}{(2\pi)^2} \frac{1}{\sqrt{q_0 k_0}} \varepsilon_{\mu\beta\rho\sigma} (\bar{v}(-\mathbf{p}_p) \gamma_\mu u(\mathbf{p}_e)) e_\rho$$
$$\times (p_p + p_e)_\beta k_\sigma (p_e + p_p)^{-2}$$
$$\times F((p_e + p_p)^2, 0, m_{\pi^0}^2)\, \delta(p_p + p_e - q - k), \tag{3}$$

where p_p, p_e, k and q are the four-momenta of the positron, electron, photon, and π^0 meson, e_ρ is the polarization vector of the photon,

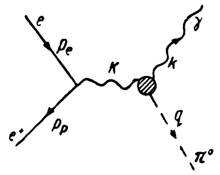

FIG. 1

$F(k_1^2, k_2^2, k_3^2)$ is the Fourier transform of the function $\widetilde{F}$ in (2) in momentum space and is, by definition, the electromagnetic form factor of the π^0 meson.

The total cross section for process (1) is

$$\sigma(E) = \frac{e^2}{4\pi} \frac{1}{6} \frac{(1-x)^3 (1+2y)}{(1-4y)^{1/2}} F^2(-E^2, 0, m_{\pi^0}^2), \tag{4}$$

where E is the total energy in the center of mass system, $x = m_{\pi^0}^2/E^2$, $y = m_e^2/E^2$, and m_{π^0} and m_e are the masses of the π^0 meson and the electron. We see from (4) that the measurement of $\sigma(E)$ yields information on the electromagnetic form factor of the π^0 meson.

In order to estimate the magnitude of $\sigma(E)$, we replace $F^2(-E^2, 0, m_{\pi^0}^2)$ in (4) by $F^2(0, 0, m_{\pi^0}^2)$, which is connected with the lifetime of the π^0 meson, τ, through the relation

$$F^2(0, 0, m_{\pi^0}^2) = 8\pi / m_{\pi^0}^3 \tau. \tag{5}$$

Taking $\tau = (2.3 \pm 0.8) \times 10^{-16}$ sec,[1] we find from (4)

$$\sigma(E) = f(E)\,\sigma, \qquad \sigma = (1.4^{+0.7}_{-0.4}) \cdot 10^{-35}\,\text{cm}^2, \tag{6}$$

the function f(E) is shown in Fig. 2.

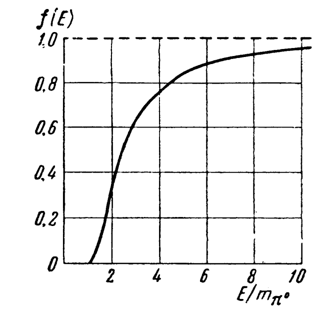

FIG. 2. Dependence of the function f(E) on the energy of the electron-positron pair.

It is also interesting to note that the abovementioned experiment yields some information on the contribution of the intermediate 3π state to the form factor of the π^0 meson.

Using the standard technique of dispersion relations, we can easily show (see, for example,[2]) that

$$\frac{1}{2\pi\sqrt{\pi q_0}}\varepsilon_{\mu\beta\rho\sigma}\bar{v}\gamma_\mu u\cdot e_\rho K_\beta k_\sigma F(K^2,0,m_{\pi^0}^2)$$
$$= e_\rho\bar{v}\gamma_\mu u\int d^4z e^{-i(k+K)z/2}\left\langle \mathbf{q}\left|T\left(I_\mu\left(\frac{z}{2}\right)I_\rho\left(-\frac{z}{2}\right)\right)\right|0\right\rangle, \quad (7)$$

where $I_\rho(z/2)$ is the current of strongly interacting particles. To simplify the formulas, we write further

$$F(K^2,0,m_{\pi^0}^2)\equiv F(\nu), \quad (8)$$

where $\nu = -K^2$. Using (7), it is easily shown that $F(\nu)$ is an analytic function in the ν plane with a cut from $4m_{\pi^0}^2$ to ∞. The dispersion relation for $F(\nu)$ is then written in the form

$$F(\nu) = F(0) + \frac{\nu}{\pi}\int_{4m_{\pi^0}^2}^{\infty}\frac{\operatorname{Im} F(\nu')}{\nu'(\nu'-\nu)}d\nu', \quad (9)$$

where $\operatorname{Im} F(\nu')$ is given by

$$\frac{1}{2\pi\sqrt{\pi q_0}}\varepsilon_{\mu\beta\rho\sigma}\bar{v}\gamma_\mu u\cdot e_\rho K_\beta k_\sigma \operatorname{Im} F(K^2,0,m_{\pi^0}^2)$$
$$= (2\pi)^4\bar{v}\gamma_\mu u\sum_n\langle \mathbf{q}\,|\,I_\rho(0)\,|\,n\rangle\langle n\,|\,I_\mu(0)\,|\,0\rangle\,\delta^4(K-P_n), \quad (10)$$

where $n = 2\pi, 3\pi, \ldots$ We shall restrict our discussion to the two intermediate states $n = 2\pi$ and 3π.

The contribution from the intermediate 2π state can be determined from the knowledge of: 1) the structure of the nucleon (the vertex function $\langle 2\pi\,|\,I_\mu(0)\,|\,0\rangle$) and 2) the photoproduction of the pion on a nucleon (the vertex function $\langle \mathbf{q}\,|\,I_\rho(0)\,|\,2\pi\rangle$). It can then be expected that we obtain an estimate of the contribution of the intermediate 3π state by subtracting the contribution of the intermediate 2π state from $F(\nu)$. Since the problems 1) and 2) have not yet been solved with sufficient accuracy, we are unable to make this estimate. We shall only discuss what conclusions can be drawn from a measurement of $F(\nu)$ in this situation.

It is seen at once from (9) that for small ν ($\nu \ll 4m_{\pi^0}^2$)

$$F(\nu) \sim F(0) + a\nu, \quad (11)$$

where a is some contsant. For larger ν ($\nu < 4m_{\pi^0}^2$), when the contributions from the intermediate 2π and 3π states are both in resonance and have about the same resonance energy, we have

$$F(\nu) \sim F(0) + b\nu/(\nu_0-\nu), \quad (12)$$

where b is a constant and ν_0 is the square of the resonance energy of the intermediate 2π state. If the experiment indicates that the form factor of the π^0 meson has the form (12), we conclude that either the intermediate 3π state gives a negligible contribution to the form factor or its contribution is appreciable but its resonance energy is about equal to the resonance energy of the 2π state. If, on the other hand, the experiment shows that $F(\nu)$ has a form which is very different from (12), this will mean that the intermediate 3π state gives a large contribution and either has no resonance character or, if it does have one, then the resonance energy is far away from ν_0.

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[1] Glasser, Seeman, and Stiller, Proc. of the 1960 Ann. Int. Conf. on High Energy Phys. at the University of Rochester, p. 30 (1960).

[2] S. M. Berman and D. A. Geffen, Nuovo cimento 18, 1192 (1960).

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